An integrated just-in-time inventory system with stock-dependent demand

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Abstract

This paper considers a manufacturing system in which a single vendor procures raw materials from a single supplier in single/multiple instalments, processes them and ships the finished products to a single buyer in single/multiple shipments who stores them at a warehouse before presenting them to the end customers in a display area in single/multiple transfers. The demand is assumed to be deterministic and positively dependent on the level of items displayed. In a just-in-time (JIT) system, the manufacturer must deliver the products in small quantities to minimize the buyer's holding cost at the warehouse and accept the supply of small quantities of raw materials to minimize its own holding cost. Similarly, to minimize the display area holding cost, the buyer transfers the finished products in small lot sizes. We develop a mathematical model for this problem and illustrate the effectiveness of the model with numerical examples.

Keywords: Inventory; Stock-dependent Demand; Integrated; Just-in-time;Unequal policy.

1 Introduction

Much attention has been paid in recent years to the management of supply chains. Since 1980, there have been numerous studies discussing the implementation of a JIT system and its effectiveness in US manufacturing (Kim and Ha, 2003). In a JIT integrated manufacturing system, the raw material supplier, the manufacturer and the buyer work in a cooperative manner to synchronize JIT purchasing and selling in small lot sizes as a means of minimizing the total supply chain cost. In a competitive environment, companies are forced to take advantage of any policy to optimize their business processes.

Goyal (1977) was probably one of the first to introduce the idea of joint optimization for a two-level supply chain with single vendor and single buyer at an infinite replenishment rate. Banerjee (1986) developed a model where the vendor manufactures at a finite rate and follows a lot-for-lot policy. Other related two-level vendor-buyer problems are investigated, for example, in Goyal (1988, 1995), Hill (1997, 1999), Goyal and Nebebe (2000), Valentini and Zavanella (2003), Zanoni and Grubbstrom (2004) and Hill and Omar (2006). Sarker et al. (1995) studied a raw material supplier-vendor problem and considered continuous supply at a constant rate. The model was extended by Sarker and Khan (1999, 2001) and they considered a periodic delivery policy. Omar and Smith (2002) and Omar (2009) extended supplier-vendor and vendor-buyer problems by considering time-varying demand rates for a finite planning time horizon.

For a three-level supply chain, Banerjee and Kim (1995) developed an integrated JIT inventory model where the demand rate, production rate and delivery time are constant and deterministic. Munson and Rosenblatt (2001), Lee (2005), Lee and Moon (2006) and Jaber and Goyal (2008) also considered a similar problem consisting of a single raw material supplier, a single vendor and a single buyer. Jaber et al. (2006) extended the work of Munson and Rosenblatt (2001) by considering ordered quantity and price as decision variables.

A common assumption for all the papers above is that the demand is exogenous. However, it has been recognized in the marketing literature that demand for certain items, for example in a supermarket, is influenced by the amount of stock displayed in the shelves. Gupta and Vrat (1986) were among the first to incorporate this observation into a single-level inventory model where the demand rate is a function of the initial stock level. They were followed by Baker and Urban (1988). Datta and Pal (1990) extended Baker and Urban's model by considering shortages, Goh (1994) considered non-linear holding cost and Dye and Ouyang (2005) considered lost sales. Recently, Sarker (2012) considered the EOQ model for stock-dependent demand with delay in payments and imperfect production.

For a two-level system, Wang and Gerchak (2001) developed an integrated model with demand rate dependent on the initial stock level. Zhou et al. (2008) also considered a two-level system by following the Stackelberg game structure. Goyal and Chang (2009) considered a similar inventory model and determined the shipment and transfer schedules based on the buyer's cost. Recently, Sajadieh et al. (2010) proposed an integrated inventory model with limited display area and demand rate dependent on the displayed stock level. They assumed that the vendor delivers the finished products in multiple shipments of equal lot sizes to the buyer. Glock (2012a) gave a comprehensive review of joint economic lot size problems.

In this paper we extend a two-level system as in Sajadieh et al. (2010) and propose a three-level integrated joint economic lot-sizing model for raw material supplier, vendor and buyer. The basic model considered here consists of a single raw material supplier and a single vendor who manufactures the products in batches at a finite rate and sends them to the buyer's warehouse in multiple shipments before they are transferred to the display area. We assume that the demand rate depends on the displayed stock level. We also assume that there is a maximum number of on-display items due to limited shelf capacity. In order to find the most effective coordination, we also extended Sajadieh et al. (2010) policy by considering an unequal shipment size policy. When production starts, the inventory at the manufacturer is equal to zero. However, the inventory level at the buyer is just enough to satisfy their demand at the display area until the next delivery arrives. In this system, we assumed the stock value increases as a product moves down the distribution chain, and therefore the associated holding cost also increases. Consequently, we want as little stock as possible at the buyer's warehouse and display area and so the optimal policy is to order when the buyer is just about to run out of stock and to transfer when the stock at the display area is also about to run out. The total cost for this system includes all costs from both buyer and manufacturer. The buyer's cost consists of shipment cost, holding cost at the warehouse, transfer cost and holding cost at the display area. The manufacturer's cost includes set up and holding costs of finished products and instalment and holding costs of raw materials.

In Section 2 we summarize the assumptions and notations required to state the problem. In Section 3 we present a general mathematical formulation for the problem, while in Section 4 we formulate geometric shipment policies. Section 5 presents numerical examples and the conclusions are drawn in Section 6.

2 Assumptions and notations

To develop a JIT three-level integrated inventory model, the following assumptions are used:

1. Demand is dependent on the amount of items displayed. The functional relationship is given by
$$D(t) = \alpha[I(t)]^{\beta},$$
where $D(t)$ is the demand rate at time t, $\alpha > 0$ is the scale parameter, $I(t)$ is the inventory level at time t and $\beta \in (0, 1)$ is the shape parameter and is a measure of responsiveness of the demand rate to changes in the inventory level.

2. Shortages at the buyer's warehouse and display area are not allowed.

3. The time horizon is infinite.

4. There is a limited capacity C_d in the display area, i.e. $I \leq C_d$. This limitation could be interpreted as a given shelf space allocated for the product.

5. The vendor has a finite production rate P which is greater than the maximum possible demand rate, i.e. $P > \alpha C_d^{\beta}$.

6. Only one type of raw material is required to produce one unit of a finished product .

The following notational scheme is adopted :

- A_v Set up cost per production for the vendor.
- A_b Fixed shipment cost for the buyer.
- A_r Fixed instalment cost for raw material.
- S Fixed transferring cost from the warehouse to the display area.
- c The net unit purchasing price (charged by the vendor to the buyer).
- σ The net unit selling price (charged by the buyer to the consumer).
- h_r The raw material holding cost per unit time.

- h_v The inventory holding cost per unit time for the vendor.
- h_w The inventory holding cost per unit time at the buyer's warehouse where $h_w > h_v$.
- h_d The inventory holding cost per unit time at the buyer's display area where $h_d > h_w$.
- n_r The raw material's lot size factor, equivalent to the number of raw material instalments.
- n_v The number of shipments.
- n_b The number of transfers.
- Q_i The shipment lot size from the vendor to the buyer warehouse, where $i = 1, 2, ..., n_v$.
- q_i The transfer lot size from the warehouse to the display area where, $i = 1, 2, ..., n_v$.

3 Mathematical formulation

The inventory level time plot of the model for a geometric shipment size with $n_r = 3, n_v = 2$ and $n_b = 4$ is depicted in Figure 1. The top part of the figure shows the inventory level of raw materials delivered in n_r equal lot instalments during production uptime. The bottom part of the figure shows the inventory level of the buyer at the display area with the products transferred in n_b equal lots of size q_i where $q_i = Q_i/n_b$. The inventory levels at the vendor and warehouse with the shipment size Q_i are shown in the middle part of the figure.

3.1 The total cost at the buyer

The buyer's cost consists of shipment cost, holding cost at the warehouse, transfer cost and holding cost at the display area. Let $I_i(t)$ be the display area inventory level at time t. Then

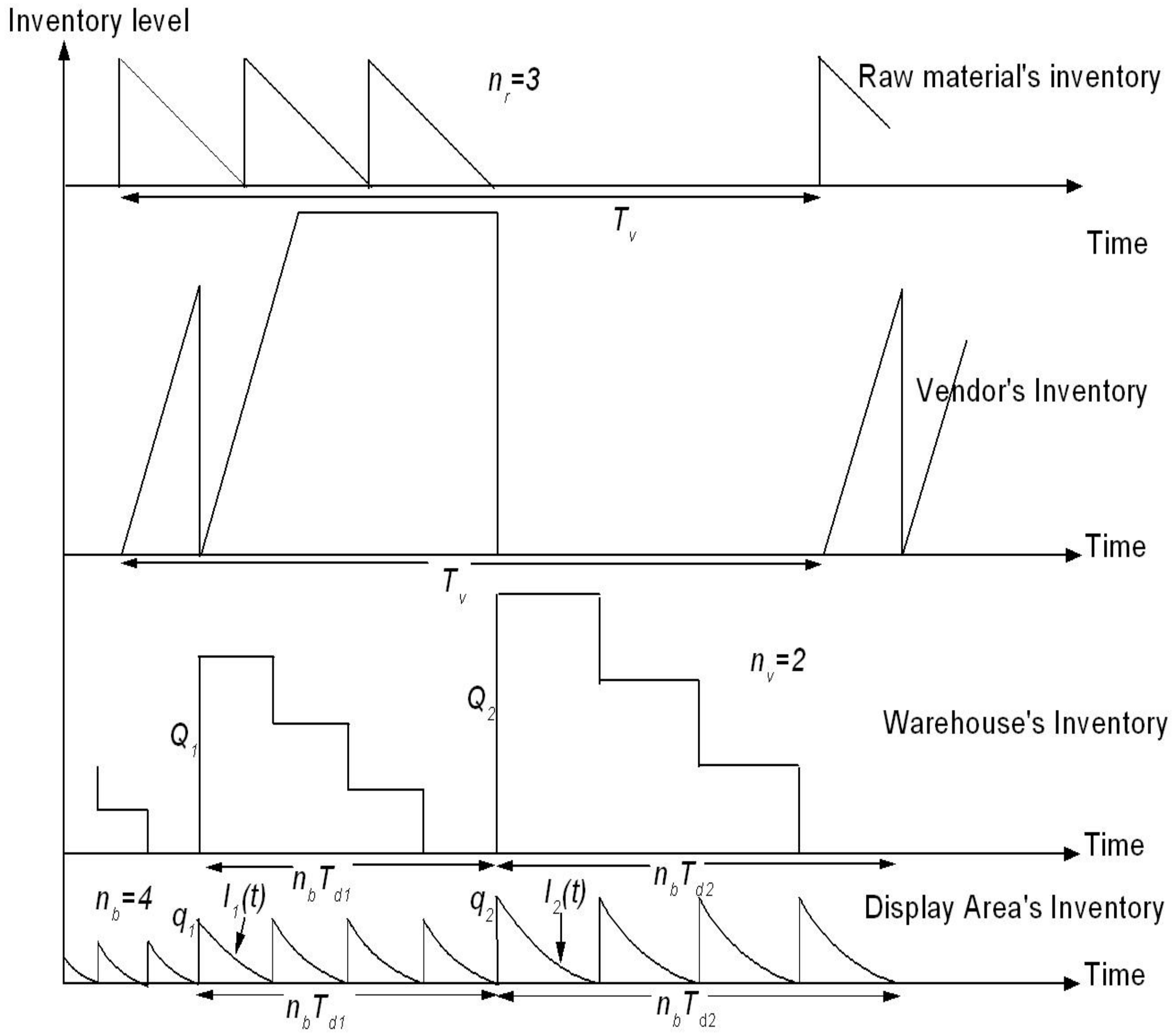

Figure 1: Inventory level at four stocking points of the supply chain.

we have

$$\frac{dI_i(t)}{dt} = -\alpha[I_i(t)]^{\beta}, \qquad 0 \le t \le T_{di}, i = 1, 2, ..., n_v, \tag{1}$$

where T_{di} is the period time defined in Figure 1 with $I_i(0) = q_i$ and $I_i(T_{di}) = 0$. Solving differential equation (1), we get

$$I_i(t) = \Big[-\alpha(1-\beta)t + q_i^{1-\beta} \Big]^{\frac{1}{1-\beta}}.$$

It follows that $T_{di} = \frac{1}{\alpha(1-\beta)}\big(\frac{Q_i}{n_b}\big)^{1-\beta}$.

The total holding cost at the display area is given by

$$HC_{bd} = \frac{h_d n_b}{T_v}\Big[\int_0^{T_{d1}} I_1(t)\,dt + \int_0^{T_{d2}} I_2(t)\,dt + ... + \int_0^{T_{dn_v}} I_{n_v}(t)\,dt\Big], \tag{2}$$

where T_v is the total cycle time. From Figure 1, we have

$$T_v = n_b \sum_{i=1}^{n_v} T_{di} = \frac{n_b}{\alpha(1-\beta)} \sum_{i=1}^{n_v} \big(\frac{Q_i}{n_b}\big)^{1-\beta}.$$

Solving equation (2), we get, after simplification,

$$HC_{bd} = \frac{h_d(1-\beta)\sum_{i=1}^{n_v}\left(\frac{Q_i}{n_b}\right)^{2-\beta}}{(2-\beta)\sum_{i=1}^{n_v}\left(\frac{Q_i}{n_b}\right)^{1-\beta}}. \tag{3}$$

The total holding cost at the buyer's warehouse is

$$\begin{aligned} HC_{bw} &= \frac{h_w}{2T_v}\big[n_b(n_b-1)q_1T_{d1} + n_b(n_b-1)q_2T_{d2} + ... + n_b(n_b-1)q_{n_v}T_{dn_v}\big] \\ &= \frac{h_w n_b(n_b-1)}{2T_v}(q_1T_{d1} + q_2T_{d2} + ... + q_{n_v}T_{dn_v}). \end{aligned} \tag{4}$$

which simplifies to

$$HC_{bw} = \frac{h_w(n_b-1)\sum_{i=1}^{n_v}\left(\frac{Q_i}{n_b}\right)^{2-\beta}}{2\sum_{i=1}^{n_v}\left(\frac{Q_i}{n_b}\right)^{1-\beta}}. \tag{5}$$

Finally, the total cost at the buyer is

$$TRC_b = \frac{1}{T_v}(n_vA_b + n_vn_bS) + HC_{bw} + HC_{bd}. \tag{6}$$

3.2 The total cost at the manufacturer

The manufacturer's cost includes the set up and holding costs of the finished products and the instalment and holding costs of raw materials.

Let ψ be the total production quantity. Then from Figure 1, the maximum inventory of raw materials for each instalment is ψ/n_r, lasting for $\psi/(n_rP)$ units of time. It follows that the holding cost of raw materials is

$$HC_r = \frac{h_r}{T_v}(\psi/n_r)(\psi/2n_rP)n_r = \frac{h_r}{2n_rPT_v}(\psi)^2. \tag{7}$$

where $\psi = \sum_{i=1}^{n_v} Q_i$, which simplifies to

$$HC_r = \frac{h_r\alpha(1-\beta)\sum_{i=1}^{n_v}\left(Q_i\right)^2}{2n_rPn_b\sum_{i=1}^{n_v}\left(\frac{Q_i}{n_b}\right)^{1-\beta}}. \tag{8}$$

Following the same method as in Hill and Omar (2006), the average finished product at the vendor is $\frac{\psi}{2} - \frac{\psi^2}{2T_vP} + \frac{\psi Q_1}{T_vP} - \frac{n_b}{2T_v}\sum_{i=1}^{n_v} Q_iT_{di}$, where the first three terms represent the average system stock and the final term represents the average stock with shipment sizes Q_i for $i = 1, 2, ..., n_v$. For equal shipment sizes, say Q, then this equation reduces to equation (6) in Sajadieh et al. (2010). It follows that the total holding cost at the vendor is

$$HC_v = h_v\Big(\frac{\psi}{2} - \frac{\psi^2}{2T_vP} + \frac{\psi Q_1}{T_vP} - \frac{n_b}{2T_v}\sum_{i=1}^{n_v} Q_iT_{di}\Big). \tag{9}$$

By performing the relevant substitutions, we obtain

$$HC_v = h_v\Big[\frac{\sum_{i=1}^{n_v} Q_i}{2} - \frac{\alpha(1-\beta)(\sum_{i=1}^{n_v} Q_i)^2}{2Pn_b\sum_{i=1}^{n_v}(\frac{Q_i}{n_b})^{1-\beta}} + \frac{\alpha(1-\beta)Q_1\sum_{i=1}^{n_v} Q_i}{Pn_b\sum_{i=1}^{n_v}(\frac{Q_i}{n_b})^{1-\beta}} - \frac{\sum_{i=1}^{n_v} Q_i(\frac{Q_i}{n_b})^{1-\beta}}{2\sum_{i=1}^{n_v}(\frac{Q_i}{n_b})^{1-\beta}}\Big]. \tag{10}$$

Finally, the total cost at the vendor is

$$TRC_v = \frac{1}{T_v}(n_rA_r + A_v) + HC_r + HC_v. \tag{11}$$

3.3 Vendor-buyer sales revenue

The vendor produces $\sum_{i=1}^{n_v} Q_i$ products and sells to the buyer at price c per unit product. Thus, the vendor's sales revenue per unit time is $\frac{c\sum_{i=1}^{n_v} Q_i}{T_v}$. For each order with a quantity of Q_i, the buyer is charged cQ_i by the vendor, and receives the amount γQ_i from the customers. Therefore the buyer's total sales revenue per unit time is $\frac{(\gamma-c)\sum_{i=1}^{n_v} Q_i}{T_v}$. Once the vendor and buyer have established a long-term strategic partnership and contracted to commit to the relationship, they will jointly determine the best policy for the whole supply chain system. Therefore, the total joint sales revenue for vendor and buyer is

$$TR = \frac{c\sum_{i=1}^{n_v} Q_i}{T_v} + \frac{(\gamma-c)\sum_{i=1}^{n_v} Q_i}{T_v} = \frac{\gamma\alpha(1-\beta)\sum_{i=1}^{n_v} Q_i}{n_b\sum_{i=1}^{n_v}(\frac{Q_i}{n_b})^{1-\beta}}. \tag{12}$$

Finally the total joint profit per unit time for the integrated model is

$$TP = TR - TRC_b - TRC_v. \tag{13}$$

4 Geometric shipment policies

The general shipment lot size for such a policy is $Q_i = \lambda^{i-1} Q_1, \quad i = 2, 3, ..., n_v$. It follows that $q_i = \lambda^{i-1} q_1$, where $q_i = Q_i/n_b$. For equal shipment sizes, we have $\lambda = 1$.

Substituting into equations (3), (5) and (6), we have

$$HC_{bd} = h_d \frac{q_1(1-\beta)(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{(2-\beta)(\lambda^\beta - \lambda^2)(\lambda^{n_v} - \lambda^{n_v\beta})}, \tag{14}$$

$$HC_{bw} = h_w \frac{q_1(n_b - 1)(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{2(\lambda^\beta - \lambda^2)(\lambda^{n_v} - \lambda^{n_v\beta})} \tag{15}$$

and

$$\begin{aligned} TRC_b &= \frac{1}{T_v}(n_v A_b + n_v n_b S) + h_d \frac{q_1(1-\beta)(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{(2-\beta)(\lambda^\beta - \lambda^2)(\lambda^{n_v} - \lambda^{n_v\beta})} \\ &+ h_w \frac{q_1(n_b - 1)(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{2(\lambda^\beta - \lambda^2)(\lambda^{n_v} - \lambda^{n_v\beta})}. \end{aligned} \tag{16}$$

Similarly, from equations (8), (10) and (11), we have

$$HC_r = h_r \frac{n_b(-1+\lambda^{n_v})^2 q_1^{1+\beta} \alpha(1-\beta)\lambda^{\beta(n_v-1)}(\lambda^\beta - \lambda)}{2n_r P(-1+\lambda)^2(\lambda^{n_v\beta} - \lambda^{n_v})}, \tag{17}$$

$$\begin{aligned} HC_v &= h_v \frac{n_b q_1}{2}\Big[- \frac{(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{(-\lambda^2 + \lambda^\beta)(\lambda^{n_v} - \lambda^{n_v\beta})} \\ &+ \frac{-1+\lambda^{n_v}}{-1+\lambda}\Big(1 - \frac{q_1^\beta \alpha(1-\beta)\lambda^{\beta(n_v-1)}(-\lambda+\lambda^\beta)}{P(\lambda^{n_v} - \lambda^{n_v\beta})}\big(\frac{-1+\lambda^{n_v}}{-1+\lambda} - 2\big)\Big)\Big] \end{aligned} \tag{18}$$

and

$$\begin{aligned} TRC_v &= \frac{1}{T_v}(n_r A_r + A_v) + h_r \frac{n_b(-1+\lambda^{n_v})^2 q_1^{1+\beta}\alpha(1-\beta)\lambda^{\beta(n_v-1)}(\lambda^\beta - \lambda)}{2n_r P(-1+\lambda)^2(\lambda^{n_v\beta} - \lambda^{n_v})} \\ &+ h_v \frac{n_b q_1}{2}\Big[- \frac{(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{(-\lambda^2 + \lambda^\beta)(\lambda^{n_v} - \lambda^{n_v\beta})} \\ &+ \frac{-1+\lambda^{n_v}}{-1+\lambda}\Big(1 - \frac{q_1^\beta \alpha(1-\beta)\lambda^{\beta(n_v-1)}(-\lambda+\lambda^\beta)}{P(\lambda^{n_v} - \lambda^{n_v\beta})}\big(\frac{-1+\lambda^{n_v}}{-1+\lambda} - 2\big)\Big)\Big]. \end{aligned} \tag{19}$$

For this policy, we have $T_v = \frac{n_b q_1 \lambda^\beta q_1^{-\beta}(1-\lambda^{n_v(1-\beta)})}{\alpha(\lambda^\beta - \lambda)(1-\beta)}$.

Hence from equations (12) and (13), we have

$$TR = \gamma \frac{q_1^\beta \alpha(1-\beta)\lambda^{\beta(n_v-1)}(1-\lambda^{n_v})(\lambda^\beta - \lambda)}{n_b(\lambda - 1)(\lambda^{n_v} - \lambda^{n_v\beta})} \tag{20}$$

and

$$
\begin{aligned}
TP &= \gamma \frac{q_1^{\beta}\alpha(1-\beta)\lambda^{\beta(n_v-1)}(1-\lambda^{n_v})(\lambda^{\beta}-\lambda)}{n_b(\lambda-1)(\lambda^{n_v}-\lambda^{n_v\beta})} \\
&- \frac{n_v q_1^{-1+\beta}\alpha(\beta-1)\lambda^{\beta(n_v-1)}(\lambda^{\beta}-\lambda)}{\lambda^{n_v}-\lambda^{n_v\beta}}\Big[\frac{A_v+n_rA_r}{n_bn_v}+\frac{A_b}{n_b}+S\Big] \\
&- h_r\frac{n_b(-1+\lambda^{n_v})^2 q_1^{1+\beta}\alpha(1-\beta)\lambda^{\beta(n_v-1)}(\lambda^{\beta}-\lambda)}{2n_rP(-1+\lambda)^2(\lambda^{n_v\beta}-\lambda^{n_v})} \\
&- h_d\frac{q_1(1-\beta)(\lambda^{\beta}-\lambda)(\lambda^{2n_v}-\lambda^{n_v\beta})}{(2-\beta)(\lambda^{\beta}-\lambda^2)(\lambda^{n_v}-\lambda^{n_v\beta})} - h_w\frac{q_1(n_b-1)(\lambda^{\beta}-\lambda)(\lambda^{2n_v}-\lambda^{n_v\beta})}{2(\lambda^{\beta}-\lambda^2)(\lambda^{n_v}-\lambda^{n_v\beta})} \\
&- h_v\frac{n_bq_1}{2}\Big[-\frac{(\lambda^{\beta}-\lambda)(\lambda^{2n_v}-\lambda^{n_v\beta})}{(-\lambda^2+\lambda^{\beta})(\lambda^{n_v}-\lambda^{n_v\beta})} \\
&+ \frac{-1+\lambda^{n_v}}{-1+\lambda}\Big(1-\frac{q_1^{\beta}\alpha(1-\beta)\lambda^{\beta(n_v-1)}(-\lambda+\lambda^{\beta})}{P(\lambda^{n_v}-\lambda^{n_v\beta})}(\frac{-1+\lambda^{n_v}}{-1+\lambda}-2)\Big)\Big], \qquad (21)
\end{aligned}
$$

where $(1/T_v)(n_vA_b+n_vn_bS+n_rA_r+A_v) = \frac{n_vq_1^{-1+\beta}\alpha(\beta-1)\lambda^{\beta(n_v-1)}(\lambda^{\beta}-\lambda)}{\lambda^{n_v}-\lambda^{n_v\beta}}\Big[\frac{A_v+n_rA_r}{n_bn_v}+\frac{A_b}{n_b}+S\Big]$.

We note that TP is a function of n_r, n_v, n_b, q_1 and λ. In this paper, we consider two cases, namely, where the value of λ is fixed and equal to P/α, and where λ is variable with $1 < \lambda \leq P/\alpha$. The special case where $\lambda = 1$ gives us an equal shipment size policy.

5 Numerical examples and sensitivity analysis

To demonstrate the effectiveness of this model, this section presents a numerical example and sensitivity analysis. By putting the value of λ close to one (to avoid division by zero) and $h_r = A_r = 0$ in equation (21) , we will obtain numerical results similar to those in Sajadieh et al. (2010). We explore our numerical results by using Matematica 7.0. We use some optimization module such as $NMaximize[\{f, cons\}, \{x, y, ...\}]$ where f is the objective function (TP) subject to constraints *cons* used in this model such as q_1, n_v, n_b, n_r and λ.

Assumed to be continuous, by taking the second partial derivative of equation (21), we can shown easily that TP is concave in n_b and n_r. However, we are unable to prove concavity analytically for other decision variables. Figures 2,3 and 4 give the concavity behaviour of the total joint profit against λ, q_1 and n_v. For Figure 2, we fixed the values of n_v, n_b, n_r and q_1, and evaluated numerically TP while varying λ. Assuming concavity, we compute numerically until the first maximum for each variable is found.

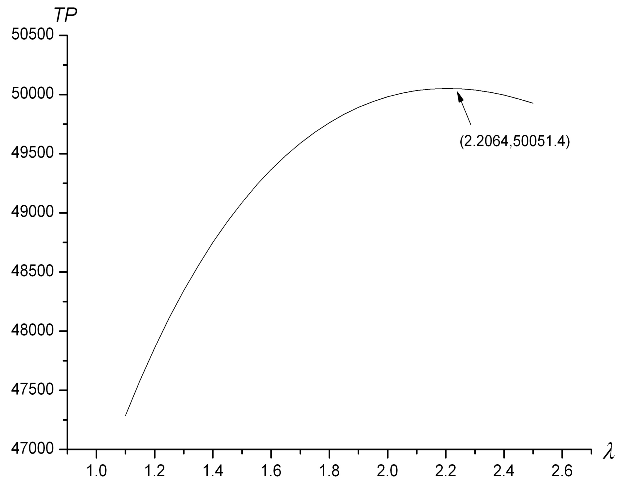

Figure 2: Total joint profit when $n_v = 3, n_b = 1, n_r = 2$, and $q = 77.1$ against λ

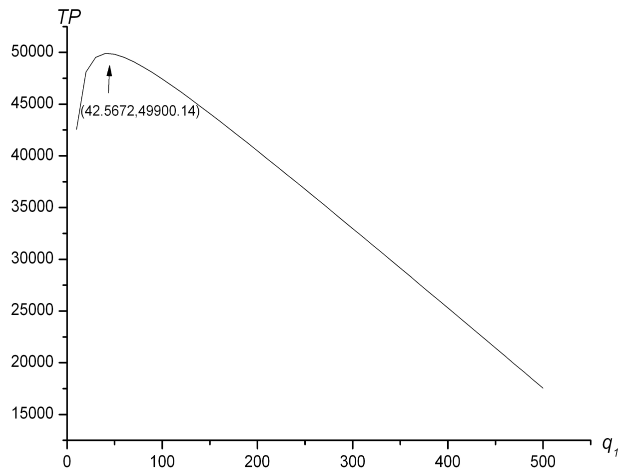

Figure 3: Total joint profit when $n_v = 3, n_b = 2, n_r = 2$, and $\lambda = 2.2064$ against q_1

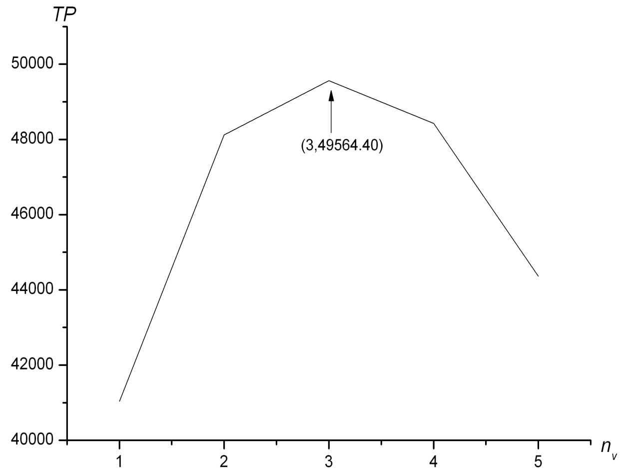

Figure 4: Total joint profit when $n_b = 3, n_r = 2, q_1 = 35.6$ and $\lambda = 1.8657$ against n_v

Table 1.

Optimal solution.

	$\lambda = \frac{P}{\alpha}$			Variable λ				$\lambda = 1$		
β	$n_v; n_b; n_r$	q_1	TP	$n_v; n_b; n_r$	q_1	λ	TP	$n_v; n_b; n_r$	q_1	TP
0	2;2;1	66.8	47830.5	3;2;2	37.9	2.21613	47864.4	3;2;2	98.3	47590.9
0.01	3;1;2	63.5	50046.1	3;1;2	77.1	2.20642	50051.4	3;1;2	201.0	49761.5
0.02	3;1;2	71.6	52617.3	3;1;2	71.6	2.5	52617.3	2;1;2	282.7	52190.4
0.03	2;1;2	190.2	55402.0	2;1;2	190.2	2.5	55402.0	2;1;2	315.2	54884.5
0.04	2;1;2	215.3	58459.8	2;1;2	215.3	2.54	58459.8	2;1;2	352.8	57792.1
0.05	3;1;3	114.8	61834.4	3;1;3	114.8	2.5	61834.4	2;1;2	396.2	60936.5

Table 1 gives an optimal solution for the extended example from Sajadieh et al. (2010) where $P = 4500, S = 25, A_b = 100, A_v = 400, h_d = 17, h_v = 9, h_w = 11, \sigma = 30, c = 20, C_d = 500, \alpha = 1800, h_r = 7$ and $A_r = 100$. In order to analyse the effect of stock-dependent demand, we used $\beta \in [0.00, 0.01, ..., 0.05]$. The geometric model with varying λ is always superior. For example, when $\beta = 0.01$, the optimal lot size transfer items, q_1 is equal to 63.5 for the geometric policy with $\lambda = P/\alpha$. The maximum TP for this policy is 50046.1. On the other hand, the geometric model with λ as a variable gives $q_1 = 77.1$ with the maximum

TP equal to 50051.6. Following Sajadieh et al. (2010) policy, $q_1 = 201.0$ with the maximum TP is 49761.5.

We perform a sensitivity analysis by solving many sample problems in order to determine how the maximum total joint profit responds to parameter changes.

Table 2.

Optimal solution for varying A_b.

A_b	n_v	n_b	n_r	q_1	λ	TP
100	3	1	2	74.8	2.5	51018.1
150	2	1	2	187.4	2.5	50684.8
200	2	1	2	196.1	2.5	50403.0
250	2	1	2	204.5	2.5	50133.2

Table 3.

Optimal solution for varying A_v.

A_v	n_v	n_b	n_r	q_1	λ	TP
350	3	1	2	73.1	2.5	51149.0
400	3	1	2	74.8	2.5	51018.1
450	3	1	2	76.5	2.5	50890.2
500	3	1	2	78.2	2.5	50765.1

Table 4.

Optimal solution for varying A_r.

A_r	n_v	n_b	n_r	q_1	λ	TP
100	3	1	2	74.8	2.5	51018.1
200	2	1	1	165.1	2.5	50539.8
300	2	1	1	173.7	2.5	50221.2
400	3	1	1	74.5	2.5	49965.5

Table 5.

Optimal solution for varying h_d.

h_d	n_v	n_b	n_r	q_1	λ	TP
14	3	1	2	68.5	2.5	50512.3
17	3	1	2	63.5	2.5	50046.1
	(3)	(1)	(2)	(77.1)	(2.2064)	(50051.4)
20	3	2	2	33.9	2.5	49680.5
23	2	2	2	77.1	2.5	49421.0
	(3)	(2)	(2)	(40.8)	(2.1678)	(49452.3)

Table 6.

Optimal solution for varying S when $h_d = 23$.

S	n_v	n_b	n_r	q_1	λ	TP
25	2	2	2	77.1	2.5	49421.0
	(3)	(2)	(2)	(40.8)	(2.1678)	(49452.3)
30	2	2	2	77.9	2.5	49352.0
	(3)	(2)	(2)	(41.5)	(2.1601)	(49365.4)
35	2	2	2	78.7	2.5	49283.6
40	2	2	2	79.4	2.5	49215.8

Tables 2 to 6 show that the maximum TP decreases with an increase of A_b, A_v, A_r, h_d and S. In most cases, the maximum TP is the same irrespective of whether λ is fixed or varying except when $h_d = 17, 23$ and $S = 25, 30$. For these cases, the result when λ varies is given in parentheses. In all our examples, the policy with varying λ is always superior to that with fixed λ.

6 Conclusion

Adopting the best policy is obviously an important decision affecting the economics of the integrated production-inventory systems. This paper considered an integrated supplier-production-inventory economic lot-sizing model for maximizing the total joint profit of the supplier, vendor and buyer. The present model assumed unequal shipments or transfer lot sizes. We developed a mathematical formulation for this problem and carried out some numerical comparative studies to highlight the managerial insight and the sensitivity of the solutions to changes in various parameter values. For example, when the fixed shipment cost for the buyer (A_b) increases, the system favours a smaller number of shipments (n_v). Similarly, when A_r increases, the number of raw material instalments also decreases. It will help the manager to make correct decisions regarding the system policy in different situations.

There are several possible directions our model could take for future research. One immediate extension would be to investigate the effect when the inventory at the displayed area deteriorates with time. It might be interesting to consider an inventory system with stock-dependent selling rate demand. We also might consider unit cash discount and delay payment (see, for example Teng et al., 2011) or supply chain involving reverse logistic (see, for example Omar, M., Yeo, I., 2012). Finally, we may extend the proposed model to account for multiple vendors and buyers (see Zahir and Sarker, 1991; Glock, 2011, 2012b, 2012c).

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