Powers of A-m-Isometric Operators and Their Supercyclicity

M. Faghih-Ahmadi

November 16, 2013

Department of Mathematics, College of Sciences, Shiraz University, Shiraz 71454, Iran

$faghiha@shirazu.ac.ir$

Abstract

We obtain some results on A-m-isometric operators. The paper consists of two parts. At first, we consider the products of A-m-isometries. It will be proved that the iterates of an A-m-isometry are all of this type. We discuss when an A-m-isometry becomes an A-isometry. Also, we consider the products of A-m-isometries for different values of m. In the second part, we are going to investigate the supercyclicity of these operators and prove that they are never supercyclic.

2010 Mathematics Subject Classification: 46C05, 47A05, 47A62.
Key words and phrases: semi-Hilbertian space, A-m-isometric operators, supercyclic operator.

1. Introduction and Preliminaries

Throughout this paper, $\mathcal{H}$ denotes a complex Hilbert space with inner product $\langle , \rangle$ and $\mathcal{B}(\mathcal{H})$ stands for a Banach algebra of all bounded linear operators on $\mathcal{H}$. Also, $\mathcal{B}(\mathcal{H})^+$ is the cone of positive semi-definite operators; *i.e.*,

$$\mathcal{B}(\mathcal{H})^+ = \{A \in \mathcal{B}(\mathcal{H}) : \langle Ah, h\rangle \geq 0, \forall h \in \mathcal{H}\}.$$

In all that follows, by an operator we mean a bounded linear operator. The range of every operator is denoted by $R(T)$, and its null space by $ker(T)$; furthermore T^* is the adjoint of T.

Any operator $A \in \mathcal{B}(\mathcal{H})^+$ defines a positive semi-definite sesquilinear form, denoted by

$$\langle , \rangle_A : \mathcal{H} \times \mathcal{H} \longrightarrow \mathbb{C}$$
$$\langle h, k\rangle_A = \langle Ah, k\rangle.$$

We remark that $\langle h, k\rangle_A = \langle A^{1/2}h, A^{1/2}k\rangle$. The semi-norm induced by $\langle , \rangle_A$, which is denoted by $\|.\|_A$, is given by $\|h\|_A = \langle h, h\rangle_A^{1/2}$.

Observe that $\|h\|_A = 0$ if and only if $h \in ker(A)$, and so $\|.\|_A$ is a norm if and only if A is injective. Besides the semi-normed space $(\mathcal{H}, \|.\|_A)$is complete if and only if $R(A^{1/2})$ is closed.

Recall that for a positive integer m an operator $T \in \mathcal{B}(\mathcal{H})$ is called an m-isometry, if

$$\sum_{k=0}^{m}(-1)^k \binom{m}{k} T^{*k}T^k = 0;$$

or equivalently,

$$\sum_{k=0}^{m}(-1)^k \binom{m}{k} \|T^{m-k}h\|^2 = 0, \quad \forall h \in \mathcal{H}.$$

The class of m-isometric operators on a complex Hilbert space has been studied intensively; see for example, [1-3] and [5-14].

An extension of these operators, called A-m-isometries on semi-Hilbertian spaces, are introduced by the authors in [16].

Definition 1. *Let m be a positive integer and $A \in \mathcal{B}(\mathcal{H})^+$. An operator T is called an A-m-isometry, if*

$$\sum_{k=0}^{m}(-1)^k \binom{m}{k} T^{*m-k}AT^{m-k} = 0.$$

It is obvious that T is an A-m-isometry if and only if for every $h \in \mathcal{H}$,

$$\sum_{k=0}^{m}(-1)^k \binom{m}{k} \|T^{m-k}h\|_A^2 = 0.$$

Thus, T is an m-isometry relative to the semi-norm on $\mathcal{H}$ induced by A.

In [16], the authors have extended some properties for A-m-isometric operators which are generalizations of well-known assertions of m-isometries; also they have obtained some spectral properties of them.

Recently, the products of m-isometries are discussed in [7] and [8]. Our main purpose in the first part of this paper, is to study the products of A-m-isometries. It will be shown that the product of such two operators is again an operator of this type. We remark that our techniques of proofs for A-m-isometries are different from the ones used for m-isometries. The supercyclicity of m-isometries has been studied by Faghih-Ahmadi and Hedayatian ([11], [12]). **Also, in [15] the authors give sufficient conditions under which an A-m-isometry is not supercyclic.** In the second part of this paper, we shall prove that A-m-isometries are all non-supercyclic.

2. Products of A-m-Isometric Operators

At first, integer powers of an A-m-isometry are considered. We begin with a lemma. By convention, take $0^0 = 1$.

Lemma 1. *If n is any positive integer then*

$$\sum_{k=0}^{n}(-1)^{n-k}k^i \begin{pmatrix} n \\ k \end{pmatrix} = 0, \tag{1}$$

for $i = 0, 1, \cdots, n-1$.

Proof. Using the binomial Theorem, (1) holds for $i = 0$. Suppose that $i \geq 1$. We prove (1) by induction on n. Clearly, (1) holds for $n = 1$. Suppose that it is true for $i = 0, 1, \cdots, n-1$. Then for $i = 0, 1, \cdots, n$

$$\begin{aligned}
\sum_{k=0}^{n+1}(-1)^{n+1-k}k^i \begin{pmatrix} n+1 \\ k \end{pmatrix} &= -\sum_{k=1}^{n+1}(-1)^{n-k}k^{i-1}\frac{(n+1)!}{(k-1)!(n+1-k)!} \\
&= (n+1)\sum_{k=0}^{n}(-1)^{n-k}\begin{pmatrix} n \\ k \end{pmatrix}(k+1)^{i-1} \\
&= (n+1)\sum_{k=0}^{n}(-1)^{n-k}\begin{pmatrix} n \\ k \end{pmatrix}\sum_{s=0}^{i-1}\begin{pmatrix} i-1 \\ s \end{pmatrix}k^s \\
&= (n+1)\sum_{s=0}^{i-1}\begin{pmatrix} i-1 \\ s \end{pmatrix}\sum_{k=0}^{n}(-1)^{n-k}k^s\begin{pmatrix} n \\ k \end{pmatrix} = 0.
\end{aligned}$$

□

Theorem 1. *If T is an A-m-isometry on a Hilbert space $\mathcal{H}$, then so is T^n for each positive integer n.*

Two proofs for this theorem will be given.
The First Proof. Fix a positive integer n. Let $x_0 = 1$ and x_i $(1 \leq i \leq m(n-1))$ be real numbers so that

$$(\sum_{i=0}^{n-1} t^i)^m = \sum_{i=0}^{m(n-1)} x_i t^i,$$

for every $t \in \mathbb{R}$. Furthermore, define x_i to be zero for $i > m(n-1)$. Take $\left(\begin{array}{c} m \\ i \end{array}\right) = 0$, for $i > m$. Note that if

$$s_k = \sum_{i=0}^{k} (-1)^i \left(\begin{array}{c} m \\ i \end{array}\right) x_{k-i} \qquad (0 \leq k \leq nm)$$

then $s_k = (-1)^{k_1} \left(\begin{array}{c} m \\ k_1 \end{array}\right)$, whenever $k = nk_1$ for some positive integer k_1, and otherwise $s_k = 0$. Indeed,

$$\begin{aligned} \sum_{k=0}^{mn} s_k t^k &= (\sum_{k=0}^{m} (-1)^k \left(\begin{array}{c} m \\ k \end{array}\right) t^k)(\sum_{k=0}^{m(n-1)} x_k t^k) \\ &= (1-t)^m (\sum_{i=0}^{n-1} t^i)^m \\ &= (1-t^n)^m \\ &= \sum_{k=0}^{m} (-1)^k \left(\begin{array}{c} m \\ k \end{array}\right) t^{kn}. \end{aligned}$$

Since T is an A-m-isometry, for any $h \in \mathcal{H}$

$$\sum_{j=0}^{m} (-1)^j \left(\begin{array}{c} m \\ j \end{array}\right) \|T^j h\|_A^2 = 0.$$

Consequently,

$$\begin{aligned}
0 &= \sum_{i=0}^{m(n-1)} x_i \sum_{j=0}^{m} (-1)^j \begin{pmatrix} m \\ j \end{pmatrix} \|T^{j+i}h\|_A^2 \\
&= \sum_{k=0}^{mn} s_k \|T^k h\|_A^2 \\
&= \sum_{k=0}^{m} s_{kn} \|T^{kn} h\|_A^2 \\
&= \sum_{k=0}^{m} (-1)^k \begin{pmatrix} m \\ k \end{pmatrix} \|T^{kn} h\|_A^2.
\end{aligned}$$

Hence, T^n is an A-m-isometry. □

For non-negative integers n and k, we denote

$$n^{(k)} = \begin{cases} 1 & (n = 0 \text{ or } k = 0), \\ n(n-1)\cdots(n-k+1) & (n \neq 0 \text{ and } k \neq 0). \end{cases}$$

If $T \in \mathcal{B}(\mathcal{H})$ and k is a non-negative integer, the operator $\beta_k(T)$ is defined by

$$\beta_k(T) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \begin{pmatrix} k \\ j \end{pmatrix} T^{*j} A T^j.$$

Also, similar to [1] it can be proved that $T^{*n}AT^n = \sum_{k=0}^{\infty} n^{(k)} \beta_k(T)$. Note that if T is an A-m-isometry, then $\beta_k(T) = 0$ for every $k \geq m$; moreover,

$$T^{*n}AT^n = \sum_{k=0}^{m-1} n^{(k)} \beta_k(T),$$

and consequently, for any $h \in \mathcal{H}$,

$$\|T^n h\|_A^2 = \sum_{k=0}^{m-1} n^{(k)} \langle \beta_k(T)h, h \rangle.$$

Considering these notations, we can bring another proof of Theorem 1, based upon Lemma 1.

The Second Proof of Theorem 1 . For each $n \geq 1$,

$$T^{*n}AT^n = \sum_{k=0}^{m-1} n^{(k)} \beta_k(T).$$

So

$$\sum_{k=0}^{m}(-1)^{m-k}\begin{pmatrix} m \\ k \end{pmatrix}T^{*nk}AT^{nk} = \sum_{k=0}^{m}(-1)^{m-k}\begin{pmatrix} m \\ k \end{pmatrix}\sum_{j=0}^{m-1}(nk)^{(j)}\beta_j(T)$$
$$= \sum_{j=0}^{m-1}[\sum_{k=0}^{m}(-1)^{m-k}\begin{pmatrix} m \\ k \end{pmatrix}(nk)^{(j)}]\beta_j(T).$$

Using Lemma 1, we observe that each inner summation in the above equality is zero. Hence T^n is an A-m-isometry. □

It is known that the inverse of any invertible A-m-isometry is again an A-m-isometry [16]. Since the identity operator is also an A-m-isometry, the following result holds.

Corollary 1. *Any integer power of an invertible A-m-isometry is an A-m-isometry.*

In general, the class of A-isometric operators is a strict subclass of A-m-isometries [16]. However, we can obtain the following result **(see also Page 83 of [15])**.

Theorem 2. *Suppose that $T \in B(\mathcal{H})$ is an A-m-isometry and there is a sequence $\{n_i\}_i$ of positive integers so that $\sup_i \|T^{n_i}\|_A < \infty$. Then T is an A-isometry.*

Proof. The result is obvious for $m = 1$. Suppose that $m > 1$. For every h in the closure of $R(A)$, denoted by $\overline{R(A)}$, and every non-negative integer n

$$||T^n h||_A^2 = \sum_{k=0}^{m-1} n^{(k)}\langle\beta_k(T)h, h\rangle. \tag{2}$$

Assume that k_0 is the largest integer so that $1 \leq k_0 \leq m-1$ and $\langle\beta_{k_0}(T)h, h\rangle \neq 0$. Then taking for granted that $\lim_{n\longrightarrow\infty}\frac{n^{(k)}}{n^{(k_0)}} = 0$ for $i = 0, \cdots, k_0 - 1$, we see that

$$\lim_{n\longrightarrow\infty}\sum_{k=0}^{m-1} n^{(k)}\langle\beta_k(T)h, h\rangle = \lim_{n\longrightarrow\infty} n^{(k_0)}\sum_{k=0}^{k_0}\frac{n^{(k)}}{n^{(k_0)}}\langle\beta_k(T)h, h\rangle = +\infty.$$

On the other hand, there is a real number $M > 0$ so that for every i and every $h \in \overline{R(A)}$,

$$||T^{n_i}h||_A \leq M||h||_A;$$

consequently, $\lim_{i\longrightarrow\infty}||T^{n_i}h||_A$ cannot be $+\infty$, which is a contradiction. Thus,

$$\langle\beta_k(T)h, h\rangle = 0$$

for $k = 1, \cdots, m-1$. This coupled with (2) for $n = 1$ implies that $||Th||_A = ||h||_A$ for every $h \in \overline{R(A)}$.

Now, an arbitrary h in H can be written as $h = h_1 + h_2$ for some $h_1 \in kreA^{\frac{1}{2}}$ and $h_2 \in \overline{R(A)}$. Taking into account that $||h||_A = ||A^{\frac{1}{2}}h|| = ||h_2||_A$ and $||Th||_A = ||Th_2||_A$, we conclude that $||Th||_A = ||h||_A$ for every $h \in H$. Thus, T is an A-isometry. □

Recall that an operator T is said to be A-power bounded, if

$$\sup_n \|T^n\|_A < \infty,$$

or equivalently, there exists $M > 0$ so that for every n and every $u \in \overline{R(A)}$, one has $\|T^n u\|_A \leq M\|u\|_A$ (see [16] for details). As a consequence of the above theorem, it is remarkable that every A-power bounded A-m-isometry is an A-isometry. Moreover, if T is an A-m-isometry so that T^k is an A-isometry for some k, then T is an A-isometry.

Lemma 2. *An operator $T \in B(\mathcal{H})$ is an A-m-isometry if and only if for every non-negative integer n and every $h \in H$,*

$$||T^n h||_A^2 = \sum_{j=0}^{m-1}(\sum_{k=j}^{m-1}(-1)^{k-j}.\frac{1}{k!}n^{(k)}\left(\begin{array}{c} k \\ j \end{array}\right))||T^j h||_A^2. \qquad (3)$$

Proof. Suppose that T is an A-m-isometry and $h \in \mathcal{H}$. Then

$$\begin{aligned}
||T^n h||_A^2 &= \sum_{j=0}^{m-1} n^{(k)}\langle\beta_k(T)h, h\rangle \\
&= \sum_{k=0}^{m-1} n^{(k)}.\frac{1}{k!}(\sum_{j=0}^{k}(-1)^{k-j}\left(\begin{array}{c} k \\ j \end{array}\right))||T^j h||_A^2 \\
&= [\sum_{k=0}^{m-1} n^{(k)}(-1)^k\left(\begin{array}{c} k \\ 0 \end{array}\right).\frac{1}{k!}]||h||_A^2 + [\sum_{k=1}^{m-1} n^{(k)}(-1)^{k-1}\left(\begin{array}{c} k \\ 1 \end{array}\right).\frac{1}{k!}]||Th||_A^2 \\
&\quad + \cdots + [\sum_{k=m-1}^{m-1} n^{(k)}(-1)^{k-m+1}\left(\begin{array}{c} k \\ m-1 \end{array}\right).\frac{1}{k!}]||T^{m-1}h||_A^2 \\
&= \sum_{j=0}^{m-1}(\sum_{k=j}^{m-1}(-1)^{k-j}.\frac{1}{k!}n^{(k)}\left(\begin{array}{c} k \\ j \end{array}\right))||T^j h||_A^2.
\end{aligned}$$

Thus, (3) holds.

To prove the converse note that equality (3) shows that $||T^n h||_A^2$ is, indeed, a polynomial in n, and so we can write

$$||T^n h||_A^2 = a_0 + a_1 n + \cdots + a_{m-1}n^{m-1}$$

for some scalars $a_0, \cdots a_{m-1}$. Consequently, applying Lemma 1 we observe that

$$\sum_{n=0}^{m}(-1)^{m-n}\left(\begin{array}{c} m \\ n \end{array}\right)\|T^n h\|_A^2 = 0.$$

Hence T is an A-m-isometry. $\square$

Lemma 3. *Suppose that T is an A-m-isometry, $n \geq m$ is an integer number and $h \in \mathcal{H}$. Then*

$$\sum_{k=0}^{n}(-1)^k \left(\begin{array}{c} n \\ k \end{array} \right) k^i ||T^{n-k}h||_A^2 = 0, \tag{4}$$

for $i = 0, 1, \cdots, n-m$.

Proof. It is known that if T is an A-m-isometry, then it is an A-n-isometry for each $n \geq m$ (see Proposition 2.4 of [16]). Thus, the hypotheses imply that (4) is valid for $i = 0$. Suppose that $i \geq 1$. We are going to prove (4) by applying induction on n. The result clearly holds if $n = m$. Suppose that (4) holds for $i = 1, \cdots, n-m$. Then for $i = 1, \cdots, n-m+1$,

$$\begin{aligned}
\sum_{k=0}^{n+1} \quad (-1)^k \quad & \left(\begin{array}{c} n+1 \\ k \end{array} \right) k^i ||T^{n-k+1}h||_A^2 = \sum_{k=0}^{n}(-1)^{k+1} \left(\begin{array}{c} n+1 \\ k+1 \end{array} \right) (k+1)^i ||T^{n-k}h||_A^2 \\
= \quad & -(n+1)\sum_{k=0}^{n}(-1)^k \frac{n!}{k!(n-k)!}(k+1)^{i-1}||T^{n-k}h||_A^2 \\
= \quad & -(n+1)\sum_{k=0}^{n}(-1)^k \left(\begin{array}{c} n \\ k \end{array} \right) (\sum_{j=0}^{i-1} \left(\begin{array}{c} i-1 \\ j \end{array} \right) k^j)||T^{n-k}h||_A^2 \\
= \quad & -(n+1)\sum_{j=0}^{i-1} \left(\begin{array}{c} i-1 \\ j \end{array} \right) \sum_{k=0}^{n}(-1)^k \left(\begin{array}{c} n \\ k \end{array} \right) k^j ||T^{n-k}h||_A^2 = 0.
\end{aligned}$$

□

Theorem 3. *Suppose that T is an A-m-isometry and S is an A-n-isometry. If $TS = ST$ then ST is an A-$m+n-1$-isometry.*

Proof. For any $h \in \mathcal{H}$, using Lemma 2 we observe that

$$\begin{aligned}
& \sum_{j=0}^{m+n-1} (-1)^j \left(\begin{array}{c} m+n-1 \\ j \end{array} \right) ||(TS)^{m+n-1-j}h||_A^2 \\
= & \sum_{j=0}^{m+n-1} (-1)^j \left(\begin{array}{c} m+n-1 \\ j \end{array} \right) ||T^{m+n-1-j}S^{m+n-1-j}h||_A^2 \\
= & \sum_{j=0}^{m+n-1} (-1)^j \left(\begin{array}{c} m+n-1 \\ j \end{array} \right) R_{m,j}
\end{aligned}$$

where

$$R_{m,j} = \sum_{i=0}^{m-1}\sum_{k=i}^{m-1}(-1)^{k-i}(m+n-1-j)^{(k)} \left(\begin{array}{c} k \\ i \end{array} \right) \cdot \frac{1}{k!}||S^{m+n-1-j}(T^i h)||_A^2.$$

So to prove the theorem, it is enough to show that for $i = 0, 1, ..., m-1$,

$$\sum_{j=0}^{m+n-1} Q_{m,n,j}^{(i)} = 0 \tag{5}$$

where

$$Q_{m,n,j}^{(i)} = \sum_{k=i}^{m-1} (-1)^{j+k-i} \left(\begin{array}{c} m+n-1 \\ j \end{array} \right) (m+n-1-j)^{(k)} \left(\begin{array}{c} k \\ i \end{array} \right) \cdot \frac{1}{k!} ||S^{m+n-1-j}(T^i h)||_A^2.$$

Notice that $(m+n-1-j)^{(k)} = \sum_{l=0}^{k} a_l j^l$, where each a_l is a scalar in terms of m and n. So the left hand side of (5) is

$$\sum_{k=i}^{m-1} (-1)^{k-i} \frac{1}{k!} \left(\begin{array}{c} k \\ i \end{array} \right) \sum_{l=0}^{k} a_l \sum_{j=0}^{m+n-1} (-1)^j j^l \left(\begin{array}{c} m+n-1 \\ j \end{array} \right) ||S^{m+n-1-j}(T^i h)||_A^2.$$

Now, taking into account that S is an A-n-isometry and applying Lemma 3, we see that

$$\sum_{j=0}^{m+n-1} (-1)^j j^l \left(\begin{array}{c} m+n-1 \\ j \end{array} \right) ||S^{m+n-1-j}(T^i h)||_A^2 = 0, \tag{6}$$

when $k = i, i+1, ..., m-1$, for $l = 0, 1, ..., k$. □

Theorem 1 along with the preceding theorem leads to the following result.

Corollary 2. *Let S, T be operators satisfying $ST = TS$. If T is an A-m-isometry and S is an A-n-isometry, then the operators $S^p T^q$, $(p, q = 0, 1, 2, \cdots)$ are A-$m + n - 1$-isometries.*

3. Supercyclicity of A-m-Isometric Operators

In this section, we show that an A-m-isometric operator cannot be supercyclic. This generalizes a similar result obtained for m-isometries in [12]. The following lemma will be useful.

Lemma 4. *The null space $kerA$ is a closed subspace of $(\mathcal{H}, ||.||_A)$.*

Proof. Suppose that $\{u_n\}_n$ is a sequence in $kerA$ and $u \in \mathcal{H}$. Note that

$$||u_n - u||_A^2 = \langle A(u_n - u), u_n - u\rangle = -\langle u, A(u_n - u)\rangle = \langle u, Au\rangle.$$

Consequently, if $||u_n - u||_A \longrightarrow 0$ then $\langle u, Au\rangle = 0$. Taking into account that A is a positive operator, we get $Au = 0$, and so $u \in kerA$. Hence $kerA$ is closed in $(\mathcal{H}, ||.||_A)$. □

Define $||.||_A$ on the quotient space $\mathcal{H}/kerA$ via

$$||u + kerA||_A = \inf\{||u + x||_A : x \in kerA\}. \tag{7}$$

The above lemma implies that $||u + kerA||_A$ defines a norm on the space $\mathcal{H}/kerA$.

Theorem 4. *Every A-m-isometric operator is not supercyclic.*

Proof. Let T be an A-m-isometric operator on a Hilbert space $\mathcal{H}$. For each $u \in \mathcal{H}$,

$$||u||_A^2 = |\langle Au, u\rangle| \leq ||A||.||u||^2.$$

Thus, if $T : (\mathcal{H}, ||.||) \longrightarrow (\mathcal{H}, ||.||)$ is a supercyclic operator, then so is $T : (\mathcal{H}, ||.||_A) \longrightarrow (\mathcal{H}, ||.||_A)$.

Now, define the operator $\tilde{T} : \mathcal{H}/kerA \longrightarrow \mathcal{H}/kerA$ by

$$\tilde{T}(u + kerA) = Tu + kerA.$$

For any $u \in \mathcal{H}$, $||u + kerA||_A = ||u||_A$, which implies that $\tilde{T}$ is an m-isometric operator. Indeed,

$$\begin{aligned} & \sum_{k=0}^{m}(-1)^k \begin{pmatrix} m \\ k \end{pmatrix} ||\tilde{T}^{m-k}(u + kerA)||_A^2 \\ = & \sum_{k=0}^{m}(-1)^k \begin{pmatrix} m \\ k \end{pmatrix} ||T^{m-k}u + kerA||_A^2 \\ = & \sum_{k=0}^{m}(-1)^k \begin{pmatrix} m \\ k \end{pmatrix} ||T^{m-k}u||_A^2 = 0. \end{aligned}$$

Denote the completion of $\mathcal{H}/kerA$ by $\mathcal{K}$, and let S be the extension of $\tilde{T}$ on the Hilbert space $\mathcal{K}$. Then define the operator $Q : \mathcal{H} \longrightarrow \mathcal{H}/kerA$ by $Q(x) = x + kerA$, and consider

$$\begin{array}{ccc} \mathcal{H} & \xrightarrow{T} & \mathcal{H} \\ Q\downarrow & & \downarrow Q \\ \mathcal{H}/kerA & \xrightarrow{\tilde{T}} & \mathcal{H}/kerA \\ I\downarrow & & \downarrow I \\ \mathcal{K} & \xrightarrow{S} & \mathcal{K} \end{array}$$

The comparison principle [4] states that if T is supercyclic then so is $\tilde{T}$, which, in turn, implies that S is supercyclic. But the operator S, being an m-isometry on a Hilbert space $\mathcal{K}$ cannot be supercyclic [12]. This leads to a contradiction. □

Acknowledgments. This research was in part supported by a grant from Shiraz University Research Council.

References

[1] J. Agler and M. Stankus, m-isometric transformation of Hilbert space I, *Integr. Equ. Oper. Theory.* **21**, 383-429 (1995).

[2] —-: m-isometric transformation of Hilbert space II, *Integr. Equ. Oper. Theory.* **23**, 1-48 (1995).

[3] —-: m-isometric transformation of Hilbert space III, *Integr. Equ. Oper. Theory.* **24**, 379-421 (1996).

[4] F. Bayart and E. Matheron, Dynamics of linear operators, Cambridge University Press 2009.

[5] T. Bermúdes, I. Marrero and A. Martionón, On the orbit of an m-isometry, *Integr. Equ. Oper. Theory* **64**, 487-494 (2009).

[6] T. Bermúdes, A. Martionón and E. Negrín, Weighted shift operators which are m-isometries, *Integr. Equ. Oper. Theory* **68**, 301-312 (2010).

[7] T. Bermúdes, C.D. Mendoza and A. Martionón, Powers of m-isometries, *Studia Mathematica* **208**, 249-255 (2012).

[8] T. Bermúdes, A. Martionón and J.A. Noda, Product of m-isometries, *Linear Algebra Appl.* **438**, 80-86 (2013).

[9] F. Botelho, and J. Jamison, Isometric properties of elementary operators, *Linear Algebra Appl.* **432**, 357-365 (2010).

[10] M. Chō, S. Ôta, K. Tanahashi and A. Uchiyama, Spectral properties of m-isometric operators, *Functional Analysis, Approximation and Computation* **4:2**, , 33-39 (2012).

[11] M. Faghih-Ahmadi and K. Hedayatian, Supercyclicity of two-isometries, *Honam Math. J.* **30**, 115-118 (2008).

[12] —–.: Hypercyclicity and **supercyclicity** of m-isometric operators, *Rocky Mountain J. Math.* **42**, 15-24 (2012).

[13] —-.: m-isometric weighted shifts and reflexivity of some operators, *Rocky Mountain J. Math.* **43**, 123-133 (2013).

[14] J. Gleason and S. Richter, m-isometric commuting tuples of operators on a Hilbert space, *Integr. Equ. Oper. Theory*, **56**, 181-196 (2006).

[15] R. Rabaoui and A.Saddi, On the orbit of an A-m-isometry, *Annales Mathematicae Silesianae*, **26, 75-91 (2012).**

[16] O.A.M. Sid Ahmed and A. Saddi, A-m-isometric operators in semi-Hilbertian spaces, *Linear Algebra Appl.* **436**, 3930-3942 (2012).