BULLETIN *of the*
MALAYSIAN
MATHEMATICAL
SCIENCES
SOCIETY

Bull. Malaysian Math. Sc. Soc. (Second Series) **23** (2000) 59-68

Coincidence Point Theorems in Pseudocompact Tichonov Spaces

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Abstract. In this paper some coincidence point theorems in pseudocompact Tichonov spaces have been established which generalize the corresponding results of Fisher [2], Harinath [3], Jain and Dixit [4], Liu[6,7], Rao [9] and Taniguchi [11].

1. Introduction

A topological space X is said to be pseudocompact if and only if each real valued continuous function on X is bounded. By Tichonov space we mean a completely regular Hausdorff space. It may be noted that every compact space is pseudocompact. Example 112 in [10] reveals that pseudocompact Tichonov spaces contain all compact metric spaces as a proper subclass. If X is an arbitrary Tichonov space, then X is pseudocompact if and only if every real valued continuous function over X is bounded and attains its bounds.

In [3], Harinath first established fixed point theorems for contractive type mappings in pseudocompact Tichonov spaces. Afterwards, Jain and Dixit [4,5], Liu [6,7], Naidu and Rao [8], Rao [9] and others extended Harinath's results in various directions. The purpose of this paper is to extend the results of Harinath, Jain, Dixit, Liu and Rao to much wider classes of mappings. In section 2, we prove coincidence point theorems of contractive type mappings. In section 3, we obtain coincidence point theorems of expansive type mappings.

Throughout this paper, X stands for a pseudocompact Tichonov space and F denotes a nonnegative real valued function on $X \times X$.

2. Coincidence point theorems for contractive type mappings

Theorem 2.1. *Let* F *satisfy that* $F(x, y) = 0$ *if and only if* $x = y$. *Suppose that* f, g, s *and* t *are four self mappings on* X *such that* *(a1)* $f(X) \subseteq t(X)$, $g(X) \subseteq s(X)$;

(a2) *either* $a(x) = F(fx, sx)$ *or* $b(x) = F(tx, gx)$ *is continuous on* X *;*

(a3) *for* $fx \neq gy$, $ty \neq sx$, $fx \neq sx$, $ty \neq gy$,

$$F(fx, gy) < \max\left\{F(ty, sx), F(fx, sx), F(ty, gy), \frac{F(fx, sx)F(ty, gy)}{F(ty, sx)}, \frac{F(fx, ty)F(sx, gy)}{F(ty, sx)}, \frac{F(fx, ty)F(sx, gy)}{F(fx, gy)}, \frac{[F(ty, sx)]^2}{F(fx, gy)}, \frac{[F(fx, sx)]^2}{F(fx, gy)}, \frac{[F(ty, gy)]^2}{F(fx, gy)}\right\}. \tag{2.1}$$

Then either f *and* s *or* g *and* t *have a coincidence point in* X*.*

Proof. First of all we assume that $b(x)$ is continuous. Then there exists x_0 in X such that $b(x_0) = \inf\{b(x) \mid x \in X\}$. It follows from (a1) that there exist x_1 and x_2 in X such that $gx_0 = sx_1$, $fx_1 = tx_2$. Suppose that neither f and s nor g and t have a coincidence point in X. In view of (2.1), we have

$$\begin{aligned} b(x_2) &= F(tx_2, gx_2) = F(fx_1, gx_2) \\ &< \max\left\{F(fx_1, sx_1), F(fx_1, sx_1), F(fx_1, gx_2), \frac{F(fx_1, sx_1)F(fx_1, gx_2)}{F(fx_1, sx_1)}, \frac{F(fx_1, fx_1)F(sx_1, gx_2)}{F(fx_1, sx_1)}, \frac{F(fx_1, fx_1)F(sx_1, gx_2)}{F(fx_1, gx_2)}, \frac{[F(fx_1, sx_1)]^2}{F(fx_1, gx_2)}, \frac{[F(fx_1, sx_1)]^2}{F(fx_1, gx_2)}, \frac{[F(fx_1, gx_2)]^2}{F(fx_1, gx_2)}\right\} \\ &= \max\left\{F(fx_1, sx_1), F(fx_1, gx_2), \frac{[F(fx_1, sx_1)]^2}{F(fx_1, gx_2)}\right\} \\ &= \max\left\{F(fx_1, sx_1), \frac{[F(fx_1, sx_1)]^2}{F(fx_1, gx_2)}\right\}, \end{aligned}$$

which implies that

$$F(fx_1, gx_2) = b(x_2) < F(fx_1, sx_1). \tag{2.2}$$

(*) This project is supported in part by the National Natural Science Foundation of China (69973019).

By virtue of (2.1), we conclude that

$$\begin{aligned} F(fx_1, sx_1) &= F(fx_1, gx_0) \\ &< \max\left\{F(tx_0, gx_0), F(fx_1, gx_0), F(tx_0, gx_0), \frac{F(fx_1, gx_0)F(tx_0, gx_0)}{F(tx_0, gx_0)},\right. \\ &\quad \frac{F(fx_1, tx_0)F(gx_0, gx_0)}{F(tx_0, gx_0)}, \frac{F(fx_1, tx_0)F(gx_0, gx_0)}{F(fx_1, gx_0)}, \frac{[F(tx_0, gx_0)]^2}{F(fx_1, gx_0)}, \\ &\quad \left.\frac{[F(fx_1, gx_0)]^2}{F(fx_1, gx_0)}, \frac{[F(tx_0, gx_0)]^2}{F(fx_1, gx_0)}\right\} \\ &= \max\left\{F(tx_0, gx_0), F(fx_1, gx_0), \frac{[F(tx_0, gx_0)]^2}{F(fx_1, gx_0)}\right\} \\ &= \max\left\{F(tx_0, gx_0), \frac{[F(tx_0, gx_0)]^2}{F(fx_1, gx_0)}\right\}, \end{aligned}$$

which yields that

$$F(fx_1, gx_0) < F(tx_0, gx_0). \tag{2.3}$$

It follows from (2.2) and (2.3) that $b(x_2) < F(fx_1, gx_0) < F(tx_0, gx_0) = b(x_0)$, which is a contradiction. So either f and s or g and t have a coincidence point. Similarly we can prove the theorem when $a(x)$ is continuous. This completes the proof.

As a consequent of Theorem 2.1, we have

Corollary 2.1. *Let* F *satisfy that* $F(x, y) = 0$ *if and only if* $x = y$. *Assume that f and s are self mappings on* X *satisfying*

(a4) $f(X) \subseteq s(X)$;

(a5) $a(x) = F(fx, sx)$ *is continuous;*

(a6) *for* $fx \neq fy,\ sy \neq sx,\ fx \neq sx,\ fy \neq sy$,

$$\begin{aligned} F(fx, fy) < \max\Bigg\{&F(sy, sx), F(fx, sx), F(sy, fy), \frac{F(fx, sx)F(sy, fy)}{F(sy, sx)}, \\ &\frac{F(fx, sy)F(sx, fy)}{F(sy, sx)}, \frac{F(fx, sy)F(sx, fy)}{F(fx, fy)}, \\ &\frac{[F(sy, sx)]^2}{F(fx, fy)}, \frac{[F(fx, sx)]^2}{F(fx, fy)}, \frac{[F(sy, fy)]^2}{F(fx, fy)}\Bigg\}. \end{aligned}$$

Then f and s have a coincidence point in X.

Reviewing the proof of Theorem 2.1 and omitting the condition that $F(x, y) = 0$ if and only if $x = y$, we can see that the following result is true.

Theorem 2.2. *Suppose that f, g, s and t are four self mappings on X satisfying (a1) and (a2) in Theorem 2.1 and*

(a7) *for* $fx \neq gy,\ ty \neq sx,\ fx \neq sx,\ ty \neq gy,$
$$F(fx, gy) < \max\{F(ty, sx), F(fx, sx), F(ty, gy)\}.$$

Then either f and s or g and t have a coincidence point in X.

Theorem 2.3. *Let F, f and s be as in Corollary 2.1 and satisfy conditions (a4), (a5) and the inequality*

$$\begin{aligned} F(fx, fy) < \max\Bigg\{ & F(sx, sy), F(fx, sx), F(fy, sy), \frac{F(fx, sx)F(fy, sy)}{F(sx, sy)}, \\ & \frac{F(fx, sy)F(sx, fy)}{F(sx, sy)}, \frac{F(fx, sy)F(sx, fy)}{F(fx, fy)}, \\ & \frac{[F(sx, sy)]^2}{F(fx, fy)}, \frac{[F(fx, sx)]^2}{F(fx, fy)}, \frac{[F(fy, sy)]^2}{F(fx, fy)} \Bigg\}, \end{aligned} \tag{2.4}$$

for $fx \neq fy, sx \neq sy$. *Then f and s have a coincidence point in X. Moreover, if f and s commute at the coincidence point, then f and s have a unique common fixed point in X.*

Proof. As in the proof of Theorem 2.1, we can find a point $x_0 \in X$ such that $a(x_0) = \inf\{a(x) \mid x \in X\}$. Suppose $fx_0 \neq sx_0$. It follows that there exists $x_1 \in X$ satisfying $fx_0 = sx_1$. So $sx_1 \neq sx_0$, $fx_1 \neq fx_0$. Using (2.4), we conclude that

$$
\begin{aligned}
a(x_1) &= F(fx_1, sx_1) = F(fx_1, fx_0) \\
&< \max\Big\{ F(sx_1, sx_0),\ F(fx_1, sx_1),\ F(fx_0, sx_0),\ \frac{F(fx_1, sx_1)F(fx_0, sx_0)}{F(sx_1, sx_0)}, \\
&\quad \frac{F(fx_1, sx_0)F(sx_1, fx_0)}{F(sx_1, sx_0)},\ \frac{F(fx_1, sx_0)F(sx_1, fx_0)}{F(fx_1, fx_0)}, \\
&\quad \frac{[F(sx_1, sx_0)]^2}{F(fx_1, fx_0)},\ \frac{[F(fx_1, sx_1)]^2}{F(fx_1, fx_0)},\ \frac{[F(fx_0, sx_0)]^2}{F(fx_1, fx_0)} \Big\} \\
&= \max\Big\{ F(fx_0, sx_0),\ F(fx_1, sx_1),\ \frac{[F(fx_0, sx_0)]^2}{F(fx_1, fx_0)} \Big\} \\
&= \max\Big\{ F(fx_0, sx_0),\ \frac{[F(fx_0, sx_0)]^2}{F(fx_1, fx_0)} \Big\},
\end{aligned}
$$

which means that $a(x_0) \le a(x_1) < a(x_0)$. This is a contradiction. Hence $fx_0 = sx_0$. Moreover, if f and s commute at x_0, that is, $fsx_0 = sfx_0$, then

$$
ffx_0 = fsx_0 = sfx_0 = ssx_0 . \tag{2.5}
$$

Put $\upsilon = fx_0$. Now we show that υ is a common fixed point of f and s. Suppose that $f\upsilon \ne \upsilon$. According to (2.4) and (2.5), we have

$$
\begin{aligned}
F(f\upsilon, \upsilon) &< \max\Big\{ F(sfx_0, sx_0), F(ffx_0, sfx_0), F(fx_0, sx_0), \frac{F(ffx_0, sfx_0)F(fx_0, sx_0)}{F(sfx_0, sx_0)}, \\
&\quad \frac{F(ffx_0, sx_0)F(sfx_0, fx_0)}{F(sfx_0, sx_0)},\ \frac{F(ffx_0, sx_0)F(sfx_0, fx_0)}{F(ffx_0, fx_0)}, \\
&\quad \frac{[F(sfx_0, sx_0)]^2}{F(ffx_0, fx_0)},\ \frac{[F(ffx_0, sfx_0)]^2}{F(ffx_0, fx_0)},\ \frac{[F(fx_0, sx_0)]^2}{F(ffx_0, fx_0)} \Big\} \\
&= F(ffx_0, fx_0) = F(f\upsilon, \upsilon),
\end{aligned}
$$

which is a contradiction. Hence υ is a fixed point of f. Clearly, $s\upsilon = \upsilon$. That is, υ is a common fixed point of f and s.

To prove uniqueness, if possible, suppose that ω is another common fixed point of f and s, and is different from υ. It follows from (2.4) that

$$
\begin{aligned}
F(\omega,\upsilon) &= F(f\omega, f\upsilon) \\
&< \max\left\{ F(s\omega,s\upsilon),\, F(f\omega,s\omega),\, F(f\upsilon,s\upsilon),\, \frac{F(f\omega,s\omega)F(f\upsilon,s\upsilon)}{F(s\omega,s\upsilon)}, \right. \\
&\quad \frac{F(f\omega,s\upsilon)F(s\omega,f\upsilon)}{F(s\omega,s\upsilon)}, \frac{F(f\omega,s\upsilon)F(s\omega,f\upsilon)}{F(f\omega,f\upsilon)}, \\
&\quad \left. \frac{[F(s\omega,s\upsilon)]^2}{F(f\omega,f\upsilon)}, \frac{[F(f\omega,s\omega)]^2}{F(f\omega,f\upsilon)}, \frac{[F(f\upsilon,s\upsilon)]^2}{F(f\omega,f\upsilon)} \right\} \\
&= F(\omega,\upsilon),
\end{aligned}
$$

which is a contradiction. Hence υ is the only common fixed point of f and s. This completes the proof.

Similarly we can prove the following result.

Corollary 2.2. *Let F be as in Corollary 2.1 and $f : X \to X$ be a mapping satisfying*

(a8) $a(x) = F(fx, x)$ *is continuous;*
(a9) *for* $x \neq y$ *,*

$$F(fx, fy) < \max\left\{ F(x,y), F(fx,x), F(fy,y), \frac{F(fx,x)F(fy,y)}{F(x,y)}, \frac{F(fx,y)F(x,fy)}{F(x,y)} \right\}.$$

Then f has a unique fixed point in X .

Remark 2.1. Theorem 2.1 extends Theorem 1 of Liu [6] with $s = t$ and Theorem 1 of Liu [7].

The following example shows that Theorem 2.1 generalizes properly the corresponding results of Liu [6, 7] .

Example 2.1. Let $X = \{1,2,3,4\}$ with F defined by $F(x,x) = 0$, $F(x,y) = F(y,x)$ for all $x, y \in X$, and

$$F(1,2) = F(1,3) = F(1,4) = F(3,4) = 1, F(2,3) = 2, F(2,4) = 1.5.$$

Clearly X is a compact metric space with metric F. So X is a pseudocompact Tichonov space. Define f, g, s and $t : X \to X$ by

$$f1 = f2 = f4 = 1,\ f3 = 4;\ \ g1 = g2 = 4, g3 = g4 = 1;$$
$$s1 = 3, s3 = 4, s4 = 2, s2 = 1;\ \ t1 = 4, t2 = t4 = 2, t3 = 1.$$

It is easy to verify that the conditions of Theorem 2.1 are satisfied, but Theorem 1 of Liu [7] is not applicable since

$$F(fx, fy) < \max\left\{F(x,y), F(x,fx), F(y,fy), \frac{F(x,fx)F(y,fy)}{F(x,y)}, \frac{F(x,fy)F(fx,y)}{F(x,y)}, \frac{F(x,fy)F(fx,y)}{F(fx,fy)}, \frac{[F(x,y)]^2}{F(fx,fy)}, \frac{[F(x,fx)]^2}{F(fx,fy)}, \frac{[F(y,fy)]^2}{F(fx,fy)}\right\}$$

does not hold for $fx \neq fy$ with $x = 1, y = 3$.

On the other hand, let $h = t$. Then

$$F(fx, gy) < \max\left\{F(hy,hx), F(fx,hx), F(hy,gy), \frac{F(fx,hx)F(hy,gy)}{F(hy,hx)}, \frac{F(fx,hy)F(hx,gy)}{F(hy,hx)}\right\}$$

is not satisfied for $x = 3$, $y = 4$. That is, Theorem 1 of Liu [6] is unavailable.

Remark 2.2. Corollary 2.2 extends Theorem 2 of Jain and Dixit [4], Theorem 1 of Harinath [3] and Theorem 2 of Fisher [2].

3. Coincidence point theorems for expansive type mappings

The following result is a cousin to Theorem 2.1, but "reverses" the inequality in (2.1).

Theorem 3.1. *Let F satisfy that $F(x,y) = 0$ if and only if $x = y$. Suppose f, g, s and t are four self mappings on X satisfying conditions (a1) and (a2) of Theorem 2.1 and*

(b1) for $fx \neq gy, ty \neq sx, fx \neq sx, ty \neq gy$

$$F(fx, gy) > \min\left\{F(ty,sx), F(fx,sx), F(ty,gy), \frac{F(fx,sx)F(ty,gy)}{F(ty,sx)}, \frac{[F(ty,sx)]^2}{F(fx,gy)}, \frac{[F(fx,sx)]^2}{F(fx,gy)}, \frac{[F(ty,gy)]^2}{F(fx,gy)}\right\}. \tag{3.1}$$

Then either f and s or g and t have a coincidence point in X.

Proof. Assume that $b(x) = F(tx, gx)$ is continuous on X. Since X is a pseudoompact Tichonov space, it follows from (a2) that there exists x_0 in X such that

$b(x_0) = \sup\{b(x) \mid x \in X\}$. By (a1), there exist x_1, x_2 in X such that $gx_0 = sx_1$, $fx_1 = tx_2$. Suppose neither f and s nor g and t have a coincidence point. Then from (b1), we have

$$\begin{aligned} b(x_2) &= F(tx_2, gx_2) = F(fx_1, gx_2) \\ &> \min\Big\{F(tx_2, sx_1), F(fx_1, sx_1), F(tx_2, gx_2), \frac{F(fx_1, sx_1)F(tx_2, gx_2)}{F(tx_2, sx_1)}, \\ &\quad \frac{[F(tx_2, sx_1)]^2}{F(fx_1, gx_2)}, \frac{[F(fx_1, sx_1)]^2}{F(fx_1, gx_2)}, \frac{[F(tx_2, gx_2)]^2}{F(fx_1, gx_2)}\Big\} \\ &= \min\Big\{F(fx_1, sx_1), F(fx_1, gx_2), \frac{[F(fx_1, sx_1)]^2}{F(fx_1, gx_2)}\Big\} \\ &= \min\Big\{F(fx_1, sx_1), \frac{[F(fx_1, sx_1)]^2}{F(fx_1, gx_2)}\Big\}, \end{aligned}$$

so that $b(x_2) > F(fx_1, sx_1)$. Using (b1), we infer that

$$\begin{aligned} F(fx_1, sx_1) &= F(fx_1, gx_0) \\ &> \min\Big\{F(tx_0, sx_1), F(fx_1, sx_1), F(tx_0, gx_0), \frac{F(fx_1, sx_1)F(tx_0, gx_0)}{F(tx_0, sx_1)}, \\ &\quad \frac{[F(tx_0, sx_1)]^2}{F(fx_1, gx_0)}, \frac{[F(fx_1, sx_1)]^2}{F(fx_1, gx_0)}, \frac{[F(tx_0, gx_0)]^2}{F(fx_1, gx_0)}\Big\} \\ &= \min\Big\{F(tx_0, gx_0), F(fx_1, gx_0), \frac{[F(tx_0, gx_0)]^2}{F(fx_1, gx_0)}\Big\} \\ &= \min\Big\{F(tx_0, gx_0), \frac{[F(tx_0, gx_0)]^2}{F(fx_1, gx_0)}\Big\}, \end{aligned}$$

which means that $b(x_2) > F(fx_1, sx_1) > F(tx_0, gx_0) = b(x_0)$. It is a contradiction. Hence either f and s or g and t have a coincidence point. Similarly we can prove the theorem when $a(x)$ is continuous.

The example below shows that the coincidence points of f and s or g and t may not be unique.

Example 3.1. Let X, f, g, s and t be as in Example 2.1. Define $F : X \times X \to [0, +\infty)$ by

$$F(x, x) = 0, F(x, y) = F(y, x),$$

for all $x, y \in X$, and

$$F(1,2) = F(1,4) = F(3,4) = 1, F(2,3) = 2, F(1,3) = 1.2, F(2,4) = 1.5.$$

It is easy to see that the conditions of Theorem3.1 are satisfied. However, 2 and 3 are coincidence points of f and s, and 1 and 3 are coincidence points of g and t.

From Theorem 3.1, we have

Corollary 3.1. *Let F, f and s be as Theorem 3.1 and satisfy conditions (a4) and (a5) in Corollary 2.1 and*

(b2) for $fx \neq fy, sx \neq sy, fx \neq sx, fy \neq sy$,

$$F(fx, fy) > \min\left\{F(sx, sy), F(fx, sx), F(fy, sy), \frac{F(fx, sx)F(fy, sy)}{F(sx, sy)}, \frac{[F(sx, sy)]^2}{F(fx, fy)}, \frac{[F(fx, sx)]^2}{F(fx, fy)}, \frac{[F(fy, sy)]^2}{F(fx, fy)}\right\}. \quad (3.2)$$

Then f and s have a coincidence point in X.

Corollary 3.2. [8,Theorem 2]. *Suppose that f, g, s and t are four self mappings on X satisfying (a1) and (a2) in Theorem 2.1 and*

(b3) for $fx \neq gy, ty \neq sx, fx \neq sx, ty \neq gy$,

$$F(fx, gy) > \min\{F(ty, sx), F(fx, sx), F(ty, gy)\}. \quad (3.3)$$

Then either f and s or g and t have a coincidence point in X.

Remark 3.1. Theorem 3 of Taniguchi [11] is a special case of Corollary 3.2.

Theorem 3.2. *Let F be as in Corollary 2.1 and* $f : X \to X$ *be a mapping satisfying condition (a8) in Corollary 2.2 and*

(b4) for $fx \neq fy$

$$F(fx, fy) > \min\left\{F(x, y), F(fx, x), F(fy, y), \frac{F(fx, x)F(fy, y)}{F(x, y)}, \frac{[F(x, y)]^2}{F(fx, fy)}, \frac{[F(fx, x)]^2}{F(fx, fy)}, \frac{[F(fy, y)]^2}{F(fx, fy)}\right\}. \quad (3.4)$$

Then f has a fixed point in X.

Proof. Let $a(x_0) = \sup\{a(x) \mid x \in X\}$. If $f^2x_0 \neq fx_0$, then $fx_0 \neq x_0$. By (3.4), we have

$$F(f^2x_0, fx_0) > \min\left\{F(fx_0, x_0), F(f^2x_0, fx_0), F(fx_0, x_0), \frac{F(f^2x_0, fx_0)\,F(fx_0, x_0)}{F(fx_0, x_0)},\right.$$
$$\left.\frac{[F(fx_0, x_0)]^2}{F(f^2x_0, fx_0)}, \frac{[F(f^2x_0, fx_0)]^2}{F(f^2x_0, fx_0)}, \frac{[F(fx_0, x_0)]^2}{F(f^2x_0, fx_0)}\right\}$$
$$= \min\left\{F(fx_0, x_0), \frac{[F(fx_0, x_0)]^2}{F(f^2x_0, fx_0)}\right\},$$

which implies that $a(fx_0) > a(x_0)$. It is a contradiction. Hence $f^2x_0 = fx_0$. This completes the proof.

Remark 3.2. Observe that the identity mapping on *X* satisfies the assumptions of Theorem 3.2. It follows that the fixed points of f in Theorem 3.2 may not be unique.

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Keywords and phrases: Coincidence point, psedocompact Tichonov space.

AMS subject classification: 54H25.