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Almost Contra-Precontinuous Functions

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Abstract. In this paper, we present and study almost contra-precontinuity as a new generalization of regular set-connectedness, contra-precontinuity, contra-continuity, almost *s*-continuity and perfectly continuity. Furthermore, we obtain basic properties and preservation theorems of almost contra-precontinuity and relationships between almost contra-precontinuity and *P*-regular graphs.

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1. Introduction

In 1996, Dontchev introduced contra-continuous functions. Recently, Dontchev, Ganster and Reilly introduced a new class of functions called regular set-connected functions (in 1999) and Jafari and Noiri introduced and studied a new form of functions called contra-precontinuous functions (in 2002). We introduced and studied a new class of functions called almost contra-precontinuous functions which generalize classes of regular set-connected [5], contra-precontinuous [9], contra-continuous [4], almost *s*-continuous [19] and perfectly continuous [17] functions. Moreover, we obtain basic properties and preservation theorems of almost contra-precontinuous functions and relationships between almost contra-precontinuity and *P*-regular graphs.

2. Preliminaries

Throughout this paper, all spaces X and Y (or (X,τ) and (Y,υ)) are always topological spaces.

A subset A of a space X is said to be regular open (respectively regular closed) if $A = \operatorname{int}(cl(A))$ (respectively $A = cl(\operatorname{int}(A))$) where $cl(A)$ and $\operatorname{int}(A)$ denote the closure and interior of A [26]. A subset A of a space is called preopen if $A \subset \operatorname{int}(cl(A))$ [14]. The complement of a preopen set is said to be preclosed.

The family of all regular open (respectively regular closed, preopen, preclosed) sets of X is denoted by $RO(X)$ (respectively $RC(X)$, $PO(X)$, $PC(X)$).

A subset A of a space X is said to be semi open if $A \subset cl(\text{int}(A))$. The complement of a semi open set is called semi closed [2]. The intersection of all semi closed sets containing A is called the semi closure [2] of A and is denoted by $scl(A)$. The semi interior of A is defined by the union of all semi open sets contained in A and is denoted by s-int(A).

Definition 1. *A function $f : X \to Y$ is called contra-precontinuous if $f^{-1}(V)$ is preclosed in X for each open set V of Y* [9].

Definition 2. *A function $f : X \to Y$ is called contra-continuous if $f^{-1}(V)$ is closed in X for each open set V of Y* [4].

Definition 3. *A function $f : X \to Y$ is said to be regular set-connected if $f^{-1}(V)$ is clopen for every $V \in RO(Y)$* [5].

Definition 4. *A function $f : X \to Y$ is said to be perfectly continuous if $f^{-1}(V)$ is clopen in X for every open set V of Y* [17].

Definition 5. *A function $f : X \to Y$ is called almost s-continuous if for each $x \in X$ and each $V \in SO(Y)$ with $f(x) \in V$, there exists an open set U in X containing x such that $f(U) \subset scl(V)$* [19].

Definition 6. *A function $f : X \to Y$ is said to be almost precontinuous if $f^{-1}(V)$ is preopen in X for every regular open set V of Y* [16].

Definition 7. *A function $f : X \to Y$ is said to be precontinuous if $f^{-1}(V)$ is preopen in X for every open set V of Y* [13].

Definition 8. *A function $f : X \to Y$ is called M-preopen (M-preclosed) if image of each preopen (resp. preclosed) set is preopen (resp. preclosed)* [14].

3. Almost contra-precontinuous functions

Definition 9. *A function $f : X \to Y$ is said to be almost contra-precontinuous if $f^{-1}(V) \in PC(X)$ for each $V \in RO(Y)$.*

Theorem 1. *Let (X, τ) and (Y, υ) be topological spaces. The following statements are equivalent for a function $f : X \to Y$:*

(1) *f is almost contra-precontinuous;*

(2) $f^{-1}(F) \in PO(X)$ *for every* $F \in RC(Y)$;

(3) *for each* $x \in X$ *and each regular closed set F in Y containing* $f(x)$, *there exists a preopen set U in X containing x such that* $f(U) \subset F$;

(4) *for each* $x \in X$ *and each regular open set V in Y non-containing* $f(x)$, *there exists a preclosed set K in X non-containing* x *such that* $f^{-1}(V) \subset K$;

(5) $f^{-1}(\text{int}(cl(G))) \in PC(X)$ *for every open subset G of Y;*

(6) $f^{-1}(cl(\text{int}(F))) \in PO(X)$ *for every closed subset F of Y.*

Proof. (1) $\Leftrightarrow$ (2) : Let $F \in RC(Y)$. Then $Y \setminus F \in RO(Y)$. By (1), $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F) \in PC(X)$. We have $f^{-1}(F) \in PO(X)$.

Reverse can be obtained similarly.

(2) $\Rightarrow$ (3) : Let F be any regular closed set in Y containing $f(x)$. By (2), $f^{-1}(F) \in PO(X)$ and $x \in f^{-1}(F)$. Take $U = f^{-1}(F)$. Then $f(U) \subset F$.

(3) $\Rightarrow$ (2) : Let $F \in RC(Y)$ and $x \in f^{-1}(F)$. From (3), there exists a preopen set U_x in X containing x such that $U \subset f^{-1}(F)$. We have $f^{-1}(F) = \bigcup_{x \in f^{-1}(F)} U_x$. Thus, $f^{-1}(F)$ is preopen.

(3) $\Leftrightarrow$ (4) : Let V be any regular open set in Y non-containing $f(x)$. Then, $Y \setminus V$ is a regular closed set containing $f(x)$. By (3), there exists a preopen set U in X containing x such that $f(U) \subset Y \setminus V$. Hence, $U \subset f^{-1}(Y \setminus V) \subset X \setminus f^{-1}(V)$ and then $f^{-1}(V) \subset X \setminus U$. Take $H = X \setminus U$. We obtain that H is a preclosed set in X non-containing x.

The converse can be shown easily.

(1) $\Leftrightarrow$ (5) : Let G be open subset of Y. Since $\text{int}(cl(G))$ is regular open, then by (1), it follows that $f^{-1}(\text{int}(cl(G))) \in PC(X)$.

The converse can be shown easily.

(2) $\Leftrightarrow$ (6) : It can be obtained smilar as (1) $\Leftrightarrow$ (5).

Remark 1. The following diagram holds:

perfectly continuous $\Rightarrow$ contra-continuous $\Rightarrow$ contra-precontinuous

$\Downarrow$ $\Downarrow$

regular set-connected $\Rightarrow$ almost contra-precontinuous

$\Uparrow$

almost *s*-continuous

None of the implications is reversible for almost contra-precontinuity as shown by the following examples.

Example 1. Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, c\}, \{a, b\}\}$ and $\upsilon = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$. Then the identity function $f : (X, \tau) \to (X, \upsilon)$ is almost contra-precontinuous. But it is not regular set-connected.

Example 2. Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$ and $\upsilon = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $f : (X, \tau) \to (X, \upsilon)$ be the identity function. Then *f* is almost contra-precontinuous function which is not contra-precontinuous.

The other implications are not reversible as shown in [5, 6, 9].

Theorem 2. *If $f : X \to Y$ is almost contra-precontinuous function and A is a semi open subset of X, then the restriction $f\mid_A : A \to Y$ is almost contra-precontinuous.*

Proof. Let $F \in RC(Y)$. Since f is almost contra-precontinuous, then $f^{-1}(F) \in PO(X)$. Since *A* is semi open in *X*, it follows from ([13], Lemma 2.1) that $(f\mid_A)^{-1}(F) = A \cap f^{-1}(F) \in PO(A)$. Therefore, $f\mid_A$ is a almost contra-precontinuous function.

Remark 2. It should be noted that every restriction of an almost contra-precontinuous function is not necessarily almost contra-precontinuous.

Example 3. Let $X = \{a, b, c, d\}$, $\sigma = \{X, \emptyset, \{a, b\}\}$, and $\tau = \{X, \emptyset, \{a\}, \{b, c, d\}\}$. The identity function $f : (X, \sigma) \to (X, \tau)$ is almost contra-precontinuous, but, if $A = \{a, c, d\}$ where *A* is not semi open in (X, σ) and σ_A is the relative topology on A induced by σ, then $f\mid_A : (A, \sigma_A) \to (X, \tau)$ is not almost contra-precontinuous.

Note that $\{b, c, d\}$ is regular closed in (X, τ), but that $(f \mid_A)^{-1}(\{b, c, d\}) = \{c, d\}$ is not preopen in (A, σ_A).

Definition 10. *A cover* $\sum = \{U_\alpha : \alpha \in I\}$ *of subsets of X is called a p-cover if* U_α *is preopen for each* $\alpha \in I$.

Lemma 1. *If* $U \in PO(X)$ *and* $V \in PO(U)$, *then* $V \in PO(X)$ [13].

Theorem 3. *Let* $f : X \to Y$ *be a function and* $\sum = \{U_\alpha : \alpha \in I\}$ *be a p-cover of X. If for each* $\alpha \in I$, $f \mid_{U\alpha}$ *is almost contra-precontinuous, then* $f : X \to Y$ *is an almost contra-precontinuous function.*

Proof. Let $V \in RC(Y)$. Since $f \mid_{U\alpha}$ is almost contra-precontinuous for each $\alpha \in I, (f \mid_{U\alpha})^{-1}(V) \in PO(U_\alpha)$. Since $U_\alpha \in PO(X)$, by the previous lemma, $(f \mid_{U\alpha})^{-1}(V) \in PO(X)$ for each $\alpha \in I$. Then $f^{-1}(V) = \bigcup_{\alpha \in I} (f \mid_{U_\alpha})^{-1}(V) \in PO(X)$. This gives f is an almost contra-precontinuous.

Theorem 4. *Let* $f : X \to Y$ *be a function and let* $g : X \to X \times Y$ *be the graph function of f, defined by* $g(x) = (x, f(x))$ *for every* $x \in X$. *If* g *is almost contra-precontinuous, then* f *is almost contra-precontinuous*.

Proof. Let $V \in RC(Y)$, then $X \times V = X \times cl(\text{int}(V)) = cl(\text{int}(X)) \times cl(\text{int}(V)) = cl(\text{int}(X \times V))$. Therefore, $X \times V \in RC(X \times Y)$. Since g is almost contra-precontinuous, then $f^{-1}(V) = g^{-1}(X \times V) \in PO(X)$. Thus, f is almost contra-precontinuous.

Theorem 5. *Let* $f : X \to Y$ *and* $g : Y \to Z$ *be functions. Then, the following properties hold:*

(1) *If* f *is almost contra-precontinuous and* g *is regular set-connected, then* $g \, o \, f : X \to Z$ *is almost contra-precontinuous and almost precontinuous.*

(2) *If* f *is almost contra-precontinuous and* g *is perfectly continuous, then* $g \, o \, f : X \to Z$ *is precontinuous and contra-precontinuous.*

(3) *If* f *is contra-precontinuous and g is regular set-connected, then* $g \, o \, f : X \to Z$ *is almost contra-precontinuous and almost precontinuous.*

Proof. (1) Let V be any regular open set in Z. Since g is regular set-connected, $g^{-1}(V)$ is clopen. Since f is almost contra-precontinuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is preopen and preclosed. Therefore, $g \circ f$ is almost contra-precontinuous and almost precontinuous.

(2) and (3) can be obtained similarly.

Theorem 6. *If $f : X \to Y$ is a surjective M-preopen and $g : Y \to Z$ is a function such that $g \circ f : X \to Z$ is almost contra-precontinuous, then g is almost contra-precontinuous.*

Proof. Let V be any regular closed set in Z. Since $g \circ f$ is almost contra-precontinuous, $(g \circ f)^{-1}(V) = f^{1}(g^{-1}(V))$ is preopen. Since f is surjective M-preopen, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is preopen. Therefore, g is almost contra-precontinuous.

Theorem 7. *If $f : X \to Y$ is a surjective M-preclosed and $g : Y \to Z$ is a function such that $g \circ f : X \to Z$ is almost contra-precontinuous, then g is almost contra-precontinuous.*

Proof. Similarly as the previous theorem.

Definition 11. *A function $f : X \to Y$ is called almost continuous if $f^{-1}(V)$ is open in X for each regular open set V of Y* [20].

Theorem 8. *If a function $f : X \to Y$ is almost contra-precontinuous and almost continuous, then f is regular set-connected.*

Proof. Let $V \in RO(Y)$. Since f is almost contra-precontinuous and almost continuous, $f^{-1}(V)$ is preclosed and open. Hence, $f^{-1}(V)$ is clopen. We obtain that f is regular set-connected.

Definition 12. *A filter base Λ is said to be p-convergent (resp. rc-convergent) to a point x in X if for any $U \in PO(X)$ containing x (resp. $U \in RC(X)$ containing x), there exists a $B \in \Lambda$ such that $B \subset U$.*

Theorem 9. *If a function $f : X \to Y$ is almost contra-precontinuous, then for each point $x \in X$ and each filter base Λ in X p-converging to x, the filter base $f(\Lambda)$ is rc-convergent to $f(x)$.*

Proof. Let $x \in X$ and Λ be any filter base in X p-converging to x. Since f is almost contra-precontinuous, then for any $V \in RC(Y)$ containing $f(x)$, there exists $U \in PO(X)$ containing x such that $f(U) \subset V$. Since Λ is p-converging to x, there exists a $B \in \Lambda$ such that $B \subset U$. This means that $f(B) \subset V$ and therefore the filter base $f(\Lambda)$ is rc-convergent to $f(x)$.

Note that a function $f : X \to Y$ is said to be almost contra-precontinuous at x if each regular closed set F in Y containing $f(x)$, there exists a preopen set U in X containing x such that $f(U) \subset F$.

Theorem 10. *Let $f : X \to Y$ be a function and $x \in X$. If there exists $U \in PO(X)$ such that $x \in U$ and the restriction of f to U is a almost contra-precontinuous at x, then f is almost contra-precontinuous at x.*

Proof. Suppose that $F \in RC(Y)$ containing $f(x)$. Since $f|_U$ is almost contra-precontinuous at x, there exists $V \in PO(U)$ containing x such that $f(V) = (f|_U)(V) \subset F$. Since $U \in PO(X)$ containing x, it follows from ([13] 1982, Lemma 2.2) that $V \in PO(X)$ containing x. This shows clearly that f is almost contra-precontinuous at x.

4. The preservation theorems

In this section, we investigate the relationships among almost contra-precontinuous functions, separation axioms, connectedness and compactness.

Definition 13. *A space X is said to be weakly Hausdorff if each element of X is an intersection of regular closed sets* [23].

Definition 14. *A space X is said to be pre-T_0 if for each pair of distinct points in X, there exists a preopen set of X containing one point but not the other* [1, 11].

Definition 15. *A space X is said to be pre-T_1 if for each pair of distinct points x and y of X, there exist preopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$* [1, 11].

Theorem 11. *If $f : X \to Y$ is an almost contra-precontinuous injection and Y is weakly Hausdorff, then X is pre-T_1.*

Proof. Suppose that Y is weakly Hausdorff. For any distinct points x and y in X, there exist $V, W \in RC(Y)$ such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is almost contra-precontinuous, $f^{-1}(V)$ and $f^{-1}(W)$ are preopen subsets of X such that $x \in f^{-1}(V), y \notin f^{-1}(V), x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is pre-T_1.

Definition 16. *A topological space X is called p-ultra-connected if every two non-void preclosed subsets of X intersect.*

Definition 17. *A topological space X is called hyperconnected if every open set is dense* [25].

Theorem 12. *If X is p-ultra-connected and $f: X \to Y$ is almost contra-precontinuous and surjective, then Y is hyperconnected.*

Proof. Assume that Y is not hyperconnected. Then there exists an open set V such that V is not dense in Y. Then there exist disjoint non-empty regular open subsets B_1 and B_2 in Y, namely $\text{int}(cl(V))$ and $Y \setminus cl(V)$. Since f is almost contra-precontinuous and onto, $A_1 = f^{-1}(B_1)$ and $A_2 = f^{-1}(B_2)$ are disjoint non-empty preclosed subsets of X. By assumption, the p-ultra-connectedness of X implies that A_1 and A_2 must intersect. By contradiction, Y is hyperconnected.

Definition 18. *A space X is called preconnected provided that X is not the union of two disjoint nonempty preopen sets* [18].

Theorem 13. *If $f: X \to Y$ is almost contra-precontinuous surjection and X is preconnected, then Y is connected.*

Proof. Suppose that Y is not connected space. There exist nonempty disjoint open sets V_1 and V_2 such that $Y = V_1 \cup V_2$. Therefore, V_1 and V_2 are clopen in Y. Since f is almost contra-precontinuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are preopen in X. Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are nonempty disjoint and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that X is not preconnected. This contradicts that Y is not connected assumed. Hence, Y is connected.

Definition 19. *A space X is said to be*

(1) *strongly compact if every preopen cover of X has a finite subcover* [8, 15].

(2) *strongly countably compact if every countable cover of X by preopen sets has a finite subcover.*

(3) *strongly Lindelof if every preopen cover of X has a countable subcover* [15].

(4) *S-Lindelof if every cover of X by regular closed sets has a countable subcover* [12].

(5) *countably S-closed if every countable cover of X by regular closed sets has a finite subcover* [3].

(6) *S-closed if every regular closed cover of X has a finite subcover* [27].

Theorem 14. *Let* $f : X \to Y$ *be an almost contra-precontinuous surjection. Then the following statements hold:*

(1) *if X is strongly compact, then Y is S-closed.*

(2) *if X is strongly Lindelof, then Y is S-Lindelof.*

(3) *if X is strongly countably compact, then Y is countably S-closed.*

Proof. We prove only (1), the proofs of (2) and (3) being entirely analogous.

Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of *Y*. Since *f* is almost contra-precontinuous, then $\{f^{-1}(V_\alpha : \alpha \in I\}$ is a preopen cover of *X* and hence there exists a finite subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, we have $Y = \bigcup\{V_\alpha : \alpha \in I_0\}$ and *Y* is *S*-closed.

Definition 20. *A space X is said to be*

(1) *P-closed if every preclosed cover of X has a finite subcover.*

(2) *countably P-closed if every countable cover of X by preclosed sets has a finite subcover.*

(3) *P-Lindelof if every cover of X by preclosed sets has a countable subcover.*

(4) *nearly compact if every regular open cover of X has a finite subcover* [21].

(5) *nearly countably compact if every countable cover of X by regular open sets has a finite subcover* [7, 22].

(6) *nearly Lindelof if every cover of X by regular open sets has a countable subcover.*

Theorem 15. *Let* $f : X \to Y$ *be an almost contra-precontinuous surjection. Then the following statements hold:*

(1) *if X is P-closed, then Y is nearly compact.*

(2) *if X is P-Lindelof, then Y is nearly Lindelof.*

(3) *if X is countably P-closed, then Y is nearly countably compact.*

Proof. We prove only (1), the proofs of (2) and (3) being entirely analogous.

Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of *Y*. Since *f* is almost contra-precontinuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a preclosed cover of *X*. Since *X* is *P*-closed, there exists a finite subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Thus, we have $Y = \bigcup\{V_\alpha : \alpha \in I_0\}$ and Y is nearly compact.

Definition 21. *A space X is said to be mildly compact (mildly countably compact, mildly Lindelof) if every clopen cover (respectively, clopen countable cover, clopen cover) of X has a finite (respectively, a finite, a countable) subcover* [24].

Theorem 16. *If $f : X \to Y$ is an almost contra-precontinuous and almost continuous surjection and X is mildly compact (resp. mildly countably compact, mildly Lindelof), then Y is nearly compact (resp. nearly countably compact, nearly Lindelof) and S-closed (resp. countably S-closed, S-Lindelof).*

Proof. Let $V \in RC(Y)$. Then since f is almost contra-precontinuous and almost continuous, $f^{-1}(V)$ is preopen and closed in X and hence $f^{-1}(V)$ is clopen. Let $\{V_\alpha : \alpha \in I\}$ be any regular closed (respectively regular open) cover of Y. Then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a clopen cover of X and since X is mildly compact, there exists a finite subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, we obtain $Y = \bigcup\{V_\alpha : \alpha \in I_0\}$. This shows that Y is S-closed (respectively nearly compact).

The other proofs can be obtained similarly.

5. *P*-regular graphs

In this section, we define *P*-regular graphs and investigate the relationships between *P*-regular graphs and almost contra-precontinuous functions.

Recall that for a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 22. *A graph $G(f)$ of a function $f : X \to Y$ is said to be P-regular if for each $\{x, y\} \in (X \times Y) \setminus G(f)$, there exist a preclosed set U in X containing x and $V \in RO(Y)$ containing y such that $(U \times V) \cap G(f) = \emptyset$.*

Lemma 2. *The following properties are equivalent for a graph $G(f)$ of a function:*

(1) *$G(f)$ is P-regular;*

(2) *for each point $(x, y) \in (X \times Y) \setminus G(f)$, there exist a preclosed set U in X containing x and $V \in RO(Y)$ containing y such that $f(U) \cap V = \emptyset$.*

Proof. It is an immediate consequence of definition of P-regular graph and the fact that for any subsets $A \subset X$ and $B \subset Y$, $(A \times B) \cap G(f) = \varnothing$ if and only if $f(A) \cap B = \varnothing$.

Theorem 17. *If $f : X \to Y$ is almost contra-precontinuous and Y is T_2, then $G(f)$ is P-regular graph in $X \times Y$.*

Proof. First, suppose that Y is T_2. Let $(x, y) \in (X \times Y) \setminus G(f)$. It follows that $f(x) \neq y$. Since Y is T_2, there exist open sets V and W containing $f(x)$ and y, respectively, such that $V \cap W = \varnothing$. We have $\text{int}(cl(V)) \cap \text{int}(cl(W)) = \varnothing$. Since f is almost contra-precontinuous, $f^{-1}(\text{int}(cl(V)))$ is preclosed in X containing x. Take $U = f^{-1}(\text{int}(cl(V)))$. Then $f(U) \subset \text{int}(cl(V))$. Therefore, $f(U) \cap \text{int}(cl(W)) = \varnothing$ and $G(f)$ is P-regular in $X \times Y$.

Theorem 18. *Let $f : X \to Y$ have a P-regular graph $G(f)$. If f is injective, then X is pre-T_1.*

Proof. Let x and y be any two distinct points of X. Then, we have $(x, f(y)) \in (X \times Y) \setminus G(f)$. By definition of P-regular graph, there exist a preclosed set U of X and $V \in RO(Y)$ such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \varnothing$; hence $U \cap f^{-1}(V) = \varnothing$. Therefore, we have $y \notin U$. Thus, $y \in X \setminus U$ and $x \notin X \setminus U$. We obtain that $X \setminus U \in PO(X)$. This implies that X is pre-T_1.

Theorem 19. *Let $f : X \to Y$ have a P-regular graph $G(f)$. If f is surjective, then Y is weakly T_2.*

Proof. Let y_1 and y_2 be any distinct points of Y. Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in (X \times Y) \setminus G(f)$. By definition of P-regular graph, there exist a preclosed set U of X and $F \in RO(Y)$ such that $(x, y_2) \in U \times F$ and $f(U) \cap F = \varnothing$; hence $y_1 \notin F$. Then $y_2 \notin Y \setminus F \in RC(Y)$ and $y_1 \in Y \setminus F$. This implies that Y is weakly T_2.

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