

On Fuzzy Semi-Pre-Generalized Closed Sets

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Abstract. In this paper, a new class of sets called fuzzy semi-pre-generalized closed sets is introduced and its properties are studied. As an application of this set we also introduce the notions of Fsp $T_{1/2}$ -space, Fspg-continuity and Fspg-irresolute mappings.

2000 Mathematics Subject Classification: 54A40

Key words and phrases: Fuzzy topology, Fspg-closed set, fuzzy semiopen set, Fspg-continuity, Fsp $T_{1/2}$ -space.

1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh in his classical paper [19]. Subsequently several authors have applied various basic concepts from general topology to fuzzy sets and developed the theory of Fuzzy topological spaces. The notion of fuzzy sets naturally plays a very significant role in the study of fuzzy topology introduced by Chang [6]. Pu and Liu [10] introduced the concept of quasi-coincidence and q -neighbourhoods by which the extensions of functions in fuzzy setting can very interestingly and effectively be carried out.

Thakur *et al.* [16] defined fuzzy semi-preopen sets. Saraf *et al.* [12] generalized the concept of fuzzy semi-preopen sets and introduced fuzzy semi-pre- $T_{1/2}$ spaces, Fgsp-continuity and Fgsp-irresoluteness. The aim of this paper is to introduce the notion of fuzzy semi-pre-generalized closed sets, an alternative generalization of fuzzy semi-preopen set in fuzzy topological spaces. Moreover, as applications, we introduce a class of fuzzy topological spaces, called fuzzy semi-pre- $T_{1/2}$ (i.e. Fsp $T_{1/2}$)-spaces and obtain some of its characterizations. Further, we also introduce Fspg-continuity and Fspg-irresoluteness.

2. Preliminaries

A family τ of fuzzy sets of X is called a fuzzy topology [6] on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open sets and their complements are fuzzy closed sets.

Throughout this paper, (X, τ) , (Y, σ) and (Z, γ) (or simply X , Y and Z) always mean fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy set A of (X, τ) , $\text{Cl}(A)$ and $\text{Int}(A)$ denote the closure and the interior of A respectively. By 0_X and 1_X we will mean the fuzzy sets with constant function 0 (Zero function) and 1 (Unit function) respectively.

The following definitions are useful in the sequel.

Definition 2.1. A fuzzy set A of (X, τ) is called:

- (1) Fuzzy semiopen (briefly, Fs-open) if $A \leq \text{Cl}(\text{Int}(A))$ and a fuzzy semiclosed (briefly, Fs-closed) if $\text{Int}(\text{Cl}(A)) \leq A$ [1];
- (2) Fuzzy preopen (briefly, Fp-open) if $A \leq \text{Int}(\text{Cl}(A))$ and a fuzzy preclosed (briefly, Fp-closed) if $\text{Cl}(\text{Int}(A)) \leq A$ [5];
- (3) Fuzzy α -open (briefly, $F\alpha$ -open) if $A \leq \text{Int Cl}(\text{Int}(A))$ and a fuzzy α -closed (briefly, $F\alpha$ -closed) if $\text{Cl Int}(\text{Cl}(A)) \leq A$ [5];
- (4) Fuzzy semi-preopen (briefly, Fsp-open) [16] if $A \leq \text{Cl Int}(\text{Cl}(A))$ and a fuzzy semi-preclosed (briefly, Fsp-closed) if $\text{Int Cl}(\text{Int}(A)) \leq A$ [16].

By FSPO (X, τ) , we denote the family of all fuzzy semi-preopen sets of fts X [16].

The semiclosure [18] (resp. α -closure [9], semi-preclosure [16]) of a fuzzy set A of (X, τ) is the intersection of all Fs-closed (resp. $F\alpha$ -closed, Fsp-closed) sets that contain A and is denoted by $\text{sCl}(A)$ (resp. $\alpha\text{Cl}(A)$ and $\text{spCl}(A)$).

Definition 2.2. A fuzzy set A of (X, τ) is called:

- (1) Fuzzy generalized closed (briefly, Fg-closed) [2] if $\text{Cl} A \leq H$, whenever $A \leq H$ and H is fuzzy open set in X ;
- (2) Generalized fuzzy semiclosed (briefly, gFs-closed) [4] if $\text{sCl}(A) \leq H$, whenever $A \leq H$ and H is Fs-open set in X . In [8], Hakeim called this set as generalized fuzzy weakly semiclosed set;
- (3) Fuzzy generalized semiclosed (briefly, Fgs-closed) [13] if $\text{sCl}(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X ;
- (4) Fuzzy α -generalized closed (briefly, $F\alpha$ g-closed) [14] if $\alpha\text{Cl}(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X ;
- (5) Fuzzy generalized α -closed (briefly, $Fg\alpha$ -closed) [11] if $\alpha\text{Cl}(A) \leq H$, whenever $A \leq H$ and H is $F\alpha$ -open set in X ;
- (6) Fuzzy generalized semi-preclosed (briefly, Fgsp-closed) [12] if $\text{spCl}(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X .

Definition 2.3. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be:

- (1) Fs-continuous [1] if $f^{-1}(V)$ is Fs-open in X , for each fuzzy open set V in Y ;
- (2) Fuzzy- irresolute [18] if $f^{-1}(V)$ is Fs-open in X , for each Fs-open set V in Y ;
- (3) Fp-continuous [5] if $f^{-1}(V)$ is Fp-open in X , for each fuzzy open set V in Y ;
- (4) $F\alpha$ -continuous [5] if $f^{-1}(V)$ is $F\alpha$ -open in X , for each fuzzy open set V in Y ;
- (5) gFs-continuous [4] if $f^{-1}(V)$ is gFs-closed in X , for each fuzzy closed set V in Y ;

- (6) *Fgs-continuous* [13] if $f^{-1}(V)$ is *Fgs-closed* in X , for each fuzzy closed set V in Y ;
- (7) *Fsp-continuous* [16] if $f^{-1}(V)$ is *Fsp-open* in X , for each fuzzy open set V in Y ;
- (8) *Fuzzy M-semiprecontinuous* [17] if $f^{-1}(V)$ is *Fsp-open* in X , for each *Fsp-open* set V in Y ;
- (9) *Fgsp-continuous* [12] if $f^{-1}(V)$ is *Fgsp-closed* in X , for every fuzzy closed set V in Y ;
- (10) *Fgsp-irresolute* [12] if $f^{-1}(V)$ is *Fgsp-closed* set in X , for every *Fgsp-closed* set V in Y ;
- (11) *Fuzzy M-semi-preclosed* [15] if $f(V)$ is *Fsp-closed* set in Y , for every *Fsp-closed* set V in Y .

Definition 2.4. A fuzzy point $x_p \in A$ is said to be quasi-coincident with the fuzzy set A denoted by $x_p q A$ iff $p + A(x) > 1$. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by $A q B$ iff there exists $x \in X$ such that $A(x) + B(x) > 1$. If A and B are not quasi-coincident then we write $A \bar{q} B$. Note that $A \leq B \Leftrightarrow A \bar{q} (1 - B)$ [10].

Definition 2.5. A fuzzy topological space (X, τ) is said to be fuzzy semiconnected (briefly, *Fs-connected*) iff the only fuzzy sets which are both *Fs-open* and *Fs-closed* sets are 0_X and 1_X [7].

Definition 2.6. [6] Let f be a mapping from X into Y . If A is a fuzzy set of X and B is a fuzzy set of Y , then

- (i) $f(A)$ is a fuzzy set of Y , where

$$f(A) = \begin{cases} \sup_{x \in f^{-1}(y)} A(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

for every $y \in Y$.

- (ii) $f^{-1}(B)$ is fuzzy set of X , where $f^{-1}(B)(x) = B(f(x))$ for each $x \in X$.
- (iii) $f^{-1}(1 - B) = 1 - f^{-1}(B)$.

3. Fspg-closed sets

Definition 3.1. A fuzzy set A of (X, τ) is called fuzzy semi-pre-generalized closed (briefly, *Fspg-closed*) if $\text{spCl}(A) \leq H$, whenever $A \leq H$ and H is *Fs-open* in X .

By *FSPGC* (X, τ) , we denote the family of all fuzzy semi-pre-generalized closed sets of fts X .

Observation 3.1. Every *Fp-closed*, *gFs-closed*, *Fsp-closed* sets are *Fspg-closed* and every *Fspg-closed* set is *Fgsp-closed* but the converse may not be true in general. For,

Example 3.1. Let $X = \{a, b\}$ and $Y = \{x, y, z\}$ and fuzzy sets A, B, E, H, K and M be defined by:

$$\begin{aligned} A(a) = 0.3, \quad A(b) = 0.4; \quad B(a) = 0.4, \quad B(b) = 0.5; \\ E(a) = 0.3, \quad E(b) = 0.7; \quad H(a) = 0.7, \quad H(b) = 0.6; \end{aligned}$$

$$\begin{aligned} K(x) &= 0.1, & K(y) &= 0.2, & K(z) &= 0.7; \\ M(x) &= 0.9, & M(y) &= 0.2, & M(z) &= 0.5. \end{aligned}$$

Let $\tau = \{0, A, 1\}$, $\sigma = \{0, E, 1\}$ and $\gamma = \{0, K, 1\}$. Then B is Fspg-closed in (X, τ) but not Fp-closed; M is Fspg-closed in (Y, γ) but not gFs-closed because: If we consider a fuzzy set $T(x) = 0.9, T(y) = 0.2, T(z) = 0.7$, then clearly $\text{sCl}(M) \not\leq T$ where as $M \leq T$ and T is fuzzy semiopen in (Y, γ) and H is Fgsp-closed in (X, σ) but neither Fspg-closed because: If we consider a fuzzy set $L(a) = 0.8, L(b) = 0.7$, then clearly $\text{spCl}(H) \not\leq L$ where as $H \leq L$ and L is fuzzy semiopen in (X, σ) nor Fsp-closed because $\text{Int}(\text{Cl}(\text{Int}(H))) \not\leq H$.

Theorem 3.1. *If A is fuzzy semiopen and Fspg-closed in (X, τ) , then A is a Fsp-closed in (X, τ) .*

Proof. Since $A \leq A$ and A is fuzzy semiopen and Fspg-closed, then $\text{spCl}(A) \leq A$. Since $A \leq \text{spCl}(A)$, we have $A = \text{spCl}(A)$ and thus A is a Fsp-closed set in X . \square

Theorem 3.2. *A fuzzy set A of (X, τ) is Fspg-closed iff $A\bar{q}E \Rightarrow \text{spCl}(A)\bar{q}E$, for every Fs-closed set E of X .*

Proof. (Necessity.) Let E be a Fs-closed set of X an $A\bar{q}E$. Then $A \leq 1 - E$ and $1 - E$ is Fs-open in X which implies that $\text{spCl}(A) \leq 1 - E$ as A is Fspg-closed. Hence, $\text{spCl}(A)\bar{q}E$.

(Sufficiency.) Let H be a Fs-open set of X such that $A \leq H$. Then $A\bar{q}(1 - H)$ and $1 - H$ is Fs-closed in X . By hypothesis, $\text{spCl}(A)\bar{q}(1 - H)$ implies $\text{spCl}(A) \leq H$. Hence, A is Fspg-closed in X . \square

Theorem 3.3. *Let A be a Fspg-closed set of (X, τ) and x_p be a fuzzy point of X such that $x_p q \text{spCl}(A)$ then $\text{spCl}(x_p)qA$.*

Proof. If $\text{spCl}(x_p)\bar{q}A$ then $A \leq 1 - \text{spCl}(x_p)$ and so $\text{spCl}(A) \leq 1 - \text{spCl}(x_p) \leq 1 - x_p$ because $1 - \text{spCl}(x_p)$ is Fs-open and A is Fspg-closed in X . Hence, $x_p \bar{q} \text{spCl}(A)$, a contradiction. \square

Theorem 3.4. *If A is a Fspg-closed set of (X, τ) and $A \leq B \leq \text{spCl}(A)$, then B is a Fspg-closed set of (X, τ) .*

Proof. Let H be a Fs-open set of (X, τ) such that $B \leq H$. Then $A \leq H$. Since A is Fspg-closed, it follows that $\text{spCl}(A) \leq H$. Now, $B \leq \text{spCl}(A)$ implies $\text{spCl}(B) \leq \text{spCl}(\text{spCl}(A)) = \text{spCl}(A)$. Thus, $\text{spCl}(B) \leq H$. This proves that B is also a Fspg-closed set of (X, τ) . \square

Definition 3.2. *A fuzzy set A of (X, τ) is called fuzzy semi-pre-generalized open (briefly, Fspg-open) iff $(1 - A)$ is Fspg-closed in X . That is, A is Fspg-open iff $E \leq \text{spInt}(A)$ whenever $E \leq A$ and E is a Fs-closed set in X .*

By FSPGO (X, τ) , we denote the family of all fuzzy semi-pre-generalized open sets of fts X .

Observation 3.2. Every Fp-open, gFs- open, Fsp-open sets are Fspg-open and every Fspg-open set is Fgsp-open but not conversely. Example 3.1 serves the purpose.

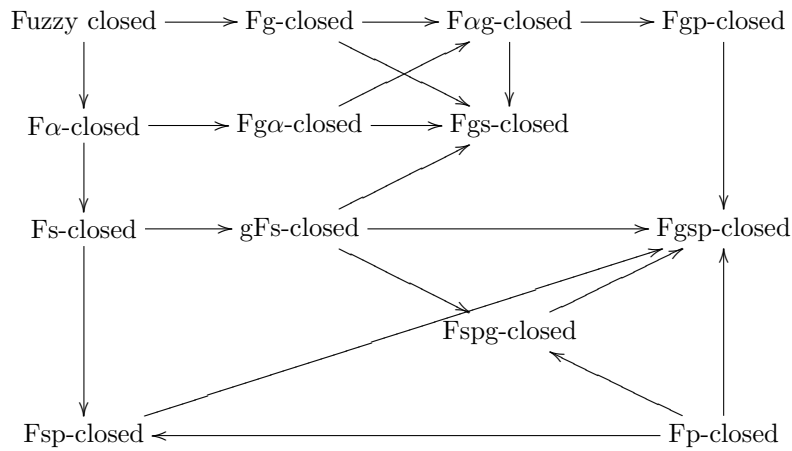
Theorem 3.5. $FSPO(X, \tau) \leq FSPGO(X, \tau)$.

Proof. Let A be any fuzzy semi-preopen set in X . Then, $1 - A$ is Fspg-closed and hence Fspg-closed by Observation 3.1. This implies that A is Fspg-open. Hence, $FSPO(X, \tau) \leq FSPGO(X, \tau)$. \square

Theorem 3.6. Let A be Fspg-open in X and $spInt(A) \leq B \leq A$, then B is Fspg-open.

Proof. Suppose A is Fspg-open in X and $spInt(A) \leq B \leq A$. Then $1 - A$ is Fspg-closed and $1 - A \leq 1 - B \leq spCl(1 - A)$. Then $1 - B$ is Fspg-closed set by Theorem 3.4. Hence, B is Fspg-open set in X . \square

The following “diagram” is the enlargement of diagram from [11].



Here $A \rightarrow B$ means “ A implies B ” but “ B does not imply A ”.

4. Fspg-continuous and Fspg-irresolute mappings

Definition 4.1. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called fuzzy semi-pre-generalized continuous (briefly, Fspg-continuous) if $f^{-1}(V)$ is Fspg-closed in (X, τ) for every fuzzy closed set V of (Y, σ) .

Definition 4.2. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called fuzzy semi-pre-generalized irresolute (briefly, Fspg-irresolute) if $f^{-1}(V)$ is Fspg-closed in (X, τ) for every Fspg-closed set V of (Y, σ) .

Theorem 4.1. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be gFs-continuous. Then f is Fspg-continuous.

Proof. Let V be a fuzzy closed set of Y . Since f is gFs-continuous, then $f^{-1}(V)$ is gFs-closed in X . Since every gFs-closed set is Fspg-closed, then $f^{-1}(V)$ is Fspg-closed. Thus, f is Fspg-continuous. \square

Observation 4.1. The converse of the above theorem is not true in general. For,

Example 4.1. Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$. Fuzzy sets A and B are defined as:

$$(4.1) \quad A(a) = 0.1, \quad A(b) = 0.2, \quad A(c) = 0.7;$$

$$(4.2) \quad B(x) = 0.1, \quad B(y) = 0.8, \quad B(z) = 0.5.$$

Let $\tau = \{0, A, 1\}$ and $\sigma = \{0, B, 1\}$. Then the mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = x$, $f(b) = y$ and $f(c) = z$ is Fspg-continuous but not gFs-continuous.

Theorem 4.2. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be Fspg-irresolute, then f is Fspg-continuous.*

Proof. Proof is immediate as every fuzzy closed set is Fspg-closed and f is Fspg-irresolute map. \square

Observation 4.2. The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.2. Let $X = \{a, b\}$, $Y = \{x, y\}$. The fuzzy set A is defined as: $A(a) = 0.3$, $A(b) = 0.7$. Let $\tau = \{0, A, 1\}$ and $\sigma = \{0, 1\}$. Then the mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is Fspg-continuous but not Fspg-irresolute.

Theorem 4.3. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be Fspg-continuous. Then f is Fgsp-continuous but not conversely.*

Proof. Let V be a fuzzy closed set of Y . Since f is Fspg-continuous, then $f^{-1}(V)$ is a Fspg-closed set of X . Since every Fspg-closed set is Fgsp-closed, $f^{-1}(V)$ is also a Fgsp-closed set of X . Thus, f is Fgsp-continuous. \square

Following example shows that the converse is not true in general:

Example 4.3. Let $X = \{a, b\}$, $Y = \{x, y\}$. Fuzzy sets A and B are defined as:

$$A(a) = 0.3, \quad A(b) = 0.7; \quad B(x) = 0.3, \quad B(y) = 0.4.$$

Let $\tau = \{0, A, 1\}$ and $\sigma = \{0, B, 1\}$. Then the mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is Fgsp-continuous but not Fspg-continuous.

Theorem 4.4. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be Fp-continuous, then f is Fspg-continuous.*

The following example shows that the converse of the above theorem is not true in general:

Example 4.4. Let $X = \{a, b\}$, $Y = \{x, y\}$. Fuzzy sets A and B are defined as:

$$A(a) = 0.3, \quad A(b) = 0.4; \quad B(x) = 0.6, \quad B(y) = 0.5.$$

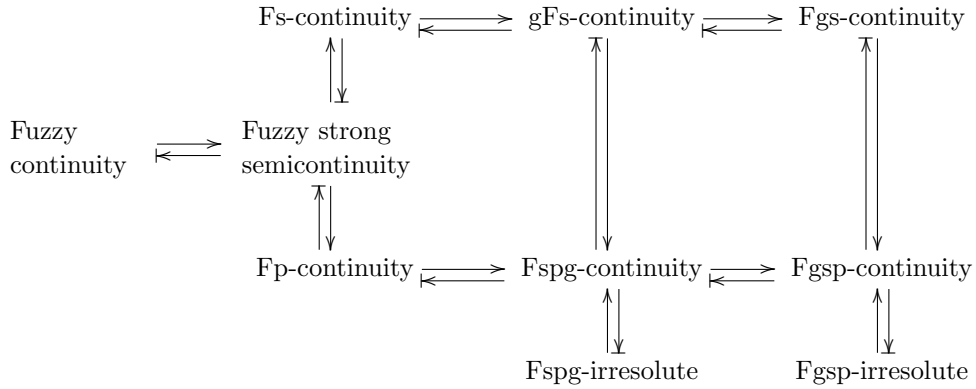
Let $\tau = \{0, A, 1\}$ and $\sigma = \{0, B, 1\}$. Then the mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is Fspg-continuous but not Fp-continuous.

Every Fs-continuous function is gFs-continuous but not conversely [4].

Theorem 4.5. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be Fgs-continuous. Then f is Fgsp-continuous but not conversely.*

Bin Shahna [5] introduced the concept of fuzzy strongly semi continuity and showed that the class of fuzzy strongly semi continuous functions properly contains the class of fuzzy continuous function and is properly contained in the class of Fs-continuous functions as well as the class of Fp-continuous functions.

The following “Diagram” summarizes the above discussions:



Theorem 4.6. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is Fspg-continuous iff inverse image of each fuzzy open set of Y is Fspg-open in X .

Proof. It is obvious because $f^{-1}(1 - H) = 1 - f^{-1}(H)$ for each fuzzy open set H of Y . \square

Theorem 4.7. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is Fspg-continuous then for each fuzzy point x_p of X and each $A \in \sigma$ such that $f(x_p) \in A$, there exists a Fspg-open set B of X such that $x_p \in B$ and $f(B) \leq A$.

Proof. Let x_p be a fuzzy point of X and $A \in \sigma$ such that $f(x_p) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis B is a Fspg-open set of X such that $x_p \in B$ and $f(B) = f(f^{-1}(A)) \leq A$. \square

Theorem 4.8. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is Fspg-continuous, then for each fuzzy point x_p of X and each $A \in \sigma$ such that $f(x_p)qA$, there exists a Fspg-open set B of X such that x_pqB and $f(B) \leq A$.

Proof. Let $x_p \in X$ and $A \in \sigma$ such that $f(x_p)qA$. Put $B = f^{-1}(A)$. Then by hypothesis B is a Fspg-open set of X such that x_pqB and $f(B) = f(f^{-1}(A)) \leq A$. \square

Recall that a fuzzy topological space (X, τ) is fuzzy $T_{1/2}$ -space if every Fg-closed set in X is fuzzy closed [2].

Theorem 4.9. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is Fspg-continuous and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ is Fg-continuous and Y is a fuzzy $T_{1/2}$ -space. Then $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ will be Fspg-continuous.

Proof. Let A is a fuzzy closed set in Z , then $g^{-1}(A)$ is Fg-closed in Y . Since Y is a fuzzy $T_{1/2}$ -space, $g^{-1}(A)$ is Fg-closed in Y implies $f^{-1}(g^{-1}(A))$ is a Fspg-closed set in X . Thus, $g \circ f$ is Fspg-continuous. \square

Now, we define the following.

Definition 4.3. If every Fspg-closed set in X is Fsp-closed in X , then the space can be denoted as Fsp $T_{1/2}$ -space.

Next, we prove the following:

Theorem 4.10. *A fuzzy topological space (X, τ) is Fsp $T_{1/2}$ -space iff $FSPO(X, \tau) = FSPGO(X, \tau)$.*

Proof. (Necessity) Let (X, τ) be Fsp $T_{1/2}$ -space. Let $A \in FSPGO(X, \tau)$. Then, $1 - A$ is a Fspg-closed. By hypothesis, $1 - A$ is a Fsp-closed set and thus $A \in FSPO(X, \tau)$. Hence, $FSPO(X, \tau) = FSPGO(X, \tau)$.

(Sufficiency) Let $FSPO(X, \tau) = FSPGO(X, \tau)$. Let A is a Fspg-closed. Then, $1 - A$ is a Fspg-open. Hence, $1 - A \in FSPO(X, \tau)$. Thus, A is a Fsp-closed set. Therefore, (X, τ) is a Fsp $T_{1/2}$ -space. \square

Theorem 4.11. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be any two functions. Then,*

- (i) $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is Fspg-continuous, if g is fuzzy continuous and f is Fspg-continuous.
- (ii) $g \circ f$ is Fspg-irresolute, if f and g both are Fspg-irresolute.
- (iii) $g \circ f$ is Fspg-continuous, if g is Fspg-continuous and f is Fspg-irresolute.
- (iv) Let Y be a Fsp $T_{1/2}$ -space. Then, $g \circ f$ is Fspg-continuous, if g is Fspg-continuous and f is fuzzy M-semi-pre-continuous.

Proof. Obvious. \square

Theorem 4.12. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be Fspg-continuous. Then f is fuzzy semi-pre continuous if (X, τ) is Fsp $T_{1/2}$ -space.*

Proof. Let V be a fuzzy closed set of Y . Since f is Fspg-continuous, $f^{-1}(V)$ is Fspg-closed set of X . Again, X is Fsp $T_{1/2}$ -space and hence $f^{-1}(V)$ is Fsp-closed set of X . This implies that f is fuzzy semi-precontinuous. \square

Theorem 4.13. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be fuzzy irresolute and fuzzy M-semi-pre-closed. Then for every Fspg-closed set A of X , $f(A)$ is a Fspg-closed in Y .*

Proof. Let A be a Fspg-closed set of X . Let V be a fuzzy semiopen set of Y containing $f(A)$. Since f is fuzzy irresolute, $f^{-1}(V)$ is a fuzzy semiopen set of X . As $A \leq f^{-1}(V)$ and A is a Fspg-closed in X , then $\text{spCl}(A) \leq f^{-1}(V)$ implies that $f(\text{spCl}(A)) \leq V$. Since f is fuzzy M-semi-pre-closed, then $f(\text{spCl}(A)) = \text{spCl}(f(\text{spCl}(A)))$. Then, $\text{spCl}(f(A)) \leq \text{spCl}(f(\text{spCl}(A))) = f(\text{spCl}(A)) \leq V$. Therefore, $f(A)$ is a Fspg-closed set in Y . \square

Theorem 4.14. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be onto Fspg-irresolute and fuzzy M-semi-pre-closed. If X is Fsp $T_{1/2}$ -space, then (Y, σ) is also Fsp $T_{1/2}$ -space.*

Proof. Let A be a Fspg-closed set of Y . Since f is Fspg-irresolute, then $f^{-1}(A)$ is Fspg-closed set in X . As X is a Fsp $T_{1/2}$ -space and hence $f^{-1}(A)$ is Fsp-closed in X . Again, f is a fuzzy M-semi-pre-closed map, $f(f^{-1}(A))$ is a Fsp-closed set in Y . Since f is onto, $f(f^{-1}(A)) = A$. Thus, A is a Fsp-closed set in Y or equivalently, (Y, σ) is Fsp $T_{1/2}$ -space. \square

Theorem 4.15. *If the bijective mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is fuzzy pre-semi-open and fuzzy M-semi-pre-continuous, then f is Fspg-irresolute.*

Proof. Let V be a Fspg-closed set in Y and let $f^{-1}(V) \leq H$ where H is a fuzzy semiopen set in X . Clearly, $V \leq f(H)$. Since f is a fuzzy pre-semi-open map, $f(H)$ is a fuzzy semiopen set in Y and V is a Fspg-closed set in Y then $\text{spCl}(V) \leq f(H)$ and thus $f^{-1}(\text{spCl}(V)) \leq H$. Again, f is a fuzzy M-semi-pre-continuous, and $\text{spCl}(V)$ is Fsp-closed set, then $f^{-1}(\text{spCl}(V))$ is a Fsp-closed set in X . Thus, $\text{spCl}(f^{-1}(V)) \leq \text{spCl}(f^{-1}(\text{spCl}(V))) = f^{-1}(\text{spCl}(V)) \leq H$. So $f^{-1}(V)$ is a Fspg-closed set in X . Hence, f is Fspg-irresolute map. \square

5. Fuzzy semi-pre-generalized connectedness

Definition 5.1. A fuzzy topological space (X, τ) is said to be fuzzy semi-pre-generalized connected (in short, Fspg-connected) if and only if the only fuzzy sets which are both Fspg-open and Fspg-closed are 0_X and 1_X .

Example 5.1. Let $X = \{a, b, c\}$ and a fuzzy topology $\tau = \{0, 1, A\}$, where $A : X \rightarrow [0, 1]$ is such that $A(a) = 1$, $A(b) = A(c) = 0$. Then it is clear that (X, τ) is Fspg-connected.

Theorem 5.1. Let (X, τ) be a fuzzy topological space. If X is a Fspg-connected space, then it is Fs-connected.

Proof. Let X be Fspg-connected and X is not Fs-connected. Then there exists a proper fuzzy set E such that $E \neq 0_X$, $E \neq 1_X$ and E is both Fs-open and Fs-closed which implies that E is Fspg-open and Fspg-closed set. Clearly, X is not Fspg-connected, a contradiction. \square

The converse of the above theorem is not true in general:

Example 5.2. Let τ be the indiscrete fuzzy topology on X . Then it is clear that (X, τ) is Fs-connected space, but it is not Fspg-connected.

Theorem 5.2. A fuzzy topological space (X, τ) is Fspg-connected iff X has no non-zero Fspg-open sets A and B such that $A + B = 1_X$.

Proof. (Necessity) Suppose (X, τ) is Fspg-connected. If X has two non-zero Fspg-open sets A and B such that $A + B = 1_X$, then A is proper Fspg-open and Fspg-closed set of X . Hence, X is not Fspg-connected, a contradiction.

(Sufficiency) If (X, τ) is not Fspg-connected then it has a proper fuzzy set A of X which is both Fspg-open and Fspg-closed. So $B = 1 - A$, is a Fspg-open set of X such that $A + B = 1_X$, which is a contradiction. \square

Theorem 5.3. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is Fspg-continuous surjection and (X, τ) is Fspg-connected, then (Y, σ) is fuzzy connected.

Proof. Let X be a Fspg-connected space and Y is not fuzzy connected. As Y is not fuzzy connected, then there exists a proper fuzzy set V of Y such that $V \neq 0_Y$, $V \neq 1_Y$ and V is both fuzzy open and fuzzy closed set. Since, f is Fspg-continuous, $f^{-1}(V)$ is both Fspg-open and Fspg-closed set in X such that $f^{-1}(V) \neq 0_X$ and $f^{-1}(V) \neq 1_X$. Hence, X is not Fspg-connected, a contradiction. \square

Theorem 5.4. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is Fspg-irresolute surjection and X is Fspg-connected, then Y is so.

Proof. Similar to the proof of the above Theorem 5.3. \square

Definition 5.2. A fuzzy topological space (X, τ) is said to be Fspg-connected between fuzzy sets A and B if there is no Fspg-closed Fspg-open set E in X such that $A \leq E$ and $E\bar{q}B$.

Observation 5.1. If a fuzzy topological space (X, τ) is Fspg-connected between fuzzy sets A and B then it is fuzzy connected between A and B but the converse may not be true. For,

Example 5.3. Let $X = \{a, b\}$. Fuzzy sets A, B and H on X are defined as:

$$A(a) = 0.4, \quad A(b) = 0.5; \quad B(a) = 0.5, \quad B(b) = 0.3; \quad H(a) = 0.4, \quad H(b) = 0.3.$$

Let $\tau = \{0, H, 1\}$ be fuzzy topology on X . Then (X, τ) is fuzzy connected between A and B but not Fspg-connected between A and B .

Theorem 5.5. If a fuzzy topological space (X, τ) is Fspg-connected between A and B iff there is no Fspg-closed, Fspg-open set E in X such that $A \leq E \leq 1 - B$.

Theorem 5.6. If a fuzzy topological space (X, τ) is Fspg-connected between fuzzy sets A and B then A and B are non-zero.

Proof. If $A = 0$, then A is Fspg-closed, Fspg-open in X such that $A \leq A$ and $A\bar{q}B$. Hence X cannot be Fspg-connected, which is contradiction. \square

Theorem 5.7. If a fuzzy topological space (X, τ) is Fspg-connected between fuzzy sets A and B and $A \leq A_1$ and $B \leq B_1$, then (X, τ) is Fspg-connected between A_1 and B_1 .

Proof. Suppose (X, τ) is not Fspg-connected between A_1 and B_1 . Then, there is a Fspg-closed, Fspg-open set E in X such that $A_1 \leq E$ and $E\bar{q}B_1$. Clearly, $A \leq E$. Now, we claim that $E\bar{q}B$: If $E\bar{q}B$, then there exists a point $x \in X$ such that $E(x) + B(x) > 1$. Therefore, $E(x) + B_1(x) > E(x) + B(x) > 1$ and $E\bar{q}B_1$, then a contradiction. \square

Theorem 5.8. Let (X, τ) be a fuzzy topological space, A and B are fuzzy sets in X . If $A\bar{q}B$, then (X, τ) is Fspg-connected between A and B .

Proof. If E is any Fspg-closed, Fspg-open set in X such that $A \leq E$, then $A\bar{q}B \Rightarrow E\bar{q}B$. \square

The converse of the above theorem is not true in general.

Example 5.4. Let $X = \{a, b\}$. Fuzzy sets A, B and H on X are defined as:

$$A(a) = 0.3, \quad A(b) = 0.5; \quad B(a) = 0.5, \quad B(b) = 0.4; \quad H(a) = 0.5, \quad H(b) = 0.7.$$

Let $\tau = \{0, H, 1\}$ be fuzzy topology on X . Then (X, τ) is Fspg-connected between A and B but $A\bar{q}B$.

Theorem 5.9. A fuzzy topological space (X, τ) is Fspg-connected iff it is Fspg-connected between every pair of its non-zero fuzzy sets.

Proof. (Necessity) Let A and B be any pair of non-zero fuzzy sets of X . Suppose, (X, τ) is not Fspg-connected between A and B . Then there is a Fspg-closed, Fspg-open set E in X such that $A \leq E$ and $E\bar{q}B$. Since A and B are non-zero, it follows that E is proper Fspg-closed, Fspg-open set of X . This implies that (X, τ) is not Fspg-connected.

(Sufficiency) Suppose (X, τ) is not Fspg-connected. Then there exists a proper fuzzy set E of X which is both Fspg-closed and Fspg-open. Consequently, X is not Fspg-connected between E and $1 - E$, a contradiction. \square

Observation 5.2. If a fuzzy topological space (X, τ) is Fspg-connected between a pair of its subsets then it is not necessarily that (X, τ) is Fspg-connected between every pair of fuzzy sets and so is not necessarily Fspg-connected. For,

Example 5.5. Let $X = \{a, b\}$. Fuzzy sets A, B, C and H on X are defined as:

$$(5.1) \quad A(a) = 0.2, \quad A(b) = 0.7; \quad B(a) = 0.5, \quad B(b) = 0.4;$$

$$(5.2) \quad C(a) = 0.3, \quad C(b) = 0.5; \quad H(a) = 0.3, \quad H(b) = 0.4.$$

Let $\tau = \{0, H, 1\}$ be the fuzzy topology on X . Then (X, τ) is Fspg-connected between A and B , but it is not Fspg-connected B and C . Also, (X, τ) is not Fspg-connected.

Acknowledgment. The authors would like to express their thanks to the referee for his detailed examinations of this paper and his valuable suggestions concerning the Definition 4.3.

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