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Contra-Pre-Semi-Continuous Functions

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Abstract. In this paper, we introduce and investigate contra-pre-semi-continuous functions. This new class is a superclass of the class of contra-β-continuous functions and contra-pre-continuous functions.

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1. Introduction

Dontchev [5] introduced the notion of contra-continuity and obtained some results concerning compactness, S-closedness and strong S-closedness in 1996. Dontchev and Noiri [6] introduced and investigated contra-semi-continuous functions and RC-continuous functions between topological spaces in 1999. Jafari and Noiri [7] introduced contra-pre-continuous functions and obtained their basic properties. Jafari and Noiri [8] also introduced contra-α-continuous functions between topological spaces. Recently author [17] introduced the class of contra-ψ-continuous functions.

Recently author [16] introduced pre-semi-closed sets for topological spaces. In this paper, we introduce and investigate contra-pre-semi-continuous functions. This new class is a superclass of the class of contra-β-continuous functions and contra-pre-continuous functions.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) will denote topological spaces. For a subset A of a space (X, τ), $\mathrm{cl}(A)$ (resp. $\mathrm{int}(A)$ and $C(A)$) will denote the closure (resp. the interior and the complement) of A in (X, τ).

Definition 2.1. *A subset A of a topological space (X, τ) is called*

(1) semi-open [9] *if* $A \subseteq \mathrm{cl}(\mathrm{int}(A))$,
(2) preopen [12] *if* $A \subseteq \mathrm{int}(\mathrm{cl}(A))$,
(3) α-open [13] *if* $A \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$,
(4) β–open [1] *or* semi-pre-open [2] *if* $A \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A)))$,
(5) regular open *if* $A = \mathrm{int}(\mathrm{cl}(A))$,

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(6) regular closed *if* $A = \mathrm{cl}(\mathrm{int}(A))$.

The complement of a semi-open (resp. preopen, α-open, β-open) set is called a semi-closed (resp. preclosed, α-closed, β-closed) set.

Definition 2.2. *A subset A of a space (X, τ) is called a* generalized closed *(briefly* g-closed*) set* [11] *if* $\mathrm{cl}(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is open in* (X, τ). *The complement of a g-closed set is called a* g-open set.

Definition 2.3. *A subset A of a space (X, τ) is called a* pre-semi-closed set [16] *if* $\mathrm{spcl}(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is a g-open set of* (X, τ), *where* $\mathrm{spcl}(A)$ *is the semi-preclosure of A. The complement of a pre-semi-closed set A is called a* pre-semi-open set.

Definition 2.4. *A subset A of a space (X, τ) is called a* semi-generalized closed *(briefly* sg-closed*) set* [3] *if* $\mathrm{scl}(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is semi-open in* (X, τ), *where* $\mathrm{scl}(A)$ *is the semi-closure of A. The complement of a sg-closed set is called a* sg-open set.

Definition 2.5. *A subset A of a space (X, τ) is called a* ψ-closed set [15] *if* $\mathrm{scl}(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is a sg-open set of* (X, τ). *The complement of a ψ-closed set A is called a* ψ-open set.

Definition 2.6. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is called*

(1) perfectly continuous [14] *or* strongly continuous [10] *if* $f^1(V)$ *is clopen in* (X, τ) *for every open set* V *of* (Y, σ),
(2) RC-continuous [6] *if* $f^{-1}(V)$ *is regular closed in* (X, τ) *for every open set* V *of* (Y, σ),
(3) contra-continuous [5] *if* $f^{-1}(V)$ *is closed in* (X, τ) *for every open set* V *of* (Y, σ),
(4) contra-pre-continuous [7] *if* $f^{-1}(V)$ *is pre-closed in* (X, τ) *for every open set* V *of* (Y, σ),
(5) contra-semi-continuous [6] *if* $f^{-1}(V)$ *is semi-closed in* (X, τ) *for every open set* V *of* (Y, σ),
(6) contra-α-continuous [8] *if* $f^{-1}(V)$ *is α-closed in* (X, τ) *for every open set* V *of* (Y, σ),
(7) contra-β-continuous [4] *if* $f^{-1}(V)$ *is β-closed in* (X, τ) *for every open set* V *of* (Y, σ) *and*
(8) contra-ψ-continuous [17] *if* $f^{-1}(V)$ *is ψ-closed in* (X, τ) *for every open set* V *of* (Y, σ).

Definition 2.7. *A topological space (X, τ) is called*

(1) a pre-semi-$\mathrm{T}_{1/2}$ space [16] *if every pre-semi-closed set in it is semi-pre-closed,*
(2) a pre-semi-T_b space [16] *if every pre-semi-closed set in it is semi-closed,*
(3) a pre-semi-$\mathrm{T}_{3/4}$ space [16] *if every pre-semi-closed set in it is pre-closed.*

3. Contra-pre-semi-continuous functions

Definition 3.1. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is called* contra-pre-semi-continuous *if* $f^{-1}(V)$ *is pre-semi-closed in* (X, τ) *for each open set* V *of* (Y, σ).

Theorem 3.1. *Every contra-*β*-continuous function is contra-pre-semi-continuous.*

Proof. It follows from the fact that every semi-preclosed set is pre-semi-closed ([16, Theorem 3.02]). □

Example 3.1. A contra-pre-semi-continuous function need not be contra-β-continuous. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$ and $\sigma = \{\emptyset, Y, \{a\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = a$ and $f(c) = b$. Then f is contra-pre-semi-continuous but not contra-β-continuous.

Thus the class of contra-pre-semi-continuous functions properly contains the class of contra-β-continuous functions.

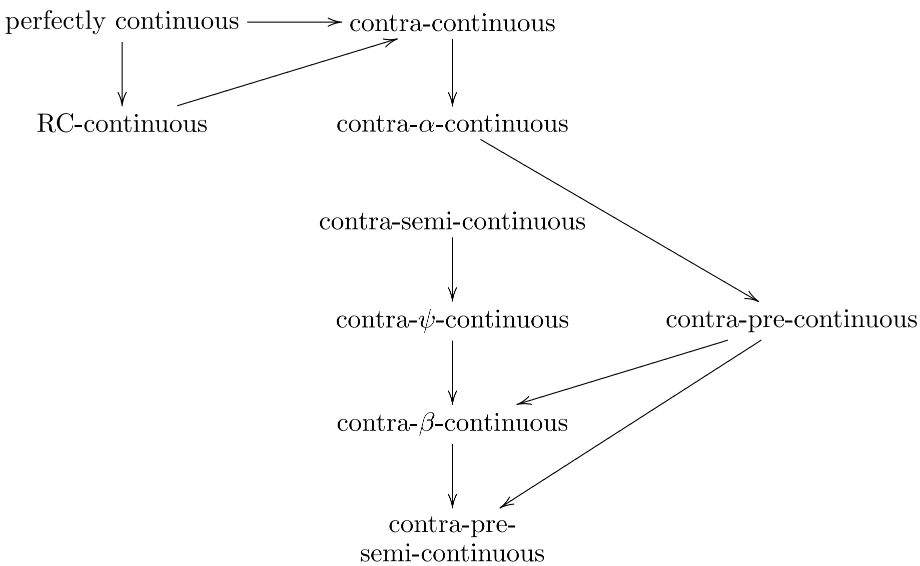
Theorem 3.2. *Every contra-pre-continuous function is contra-pre-semi-continuous.*

Proof. It follows from the fact that every preclosed set is pre-semi-closed ([16, Theorem 3.04]). □

Example 3.2. A contra-pre-semi-continuous function need not be contra-pre-continuous. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a\}\}$. Define $g : (X, \tau) \to (Y, \sigma)$ by $g(a) = a$, $g(b) = b$ and $g(c) = c$. Then g is contra-pre-semi-continuous but not contra-pre-continuous.

Thus the class of contra-pre-semi- continuous functions properly contains the class of contra-pre-continuous functions.

Thus we have the following diagram.



In the above diagram $A \to B$ denotes A implies B but not conversely.

Definition 3.2. *A space* (X, τ) *is called* pre-semi-locally indiscrete *if every pre-semi-open set in it is closed.*

Example 3.3. Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}\}$. Then (X, τ) is a pre-semi-locally indiscrete space.

The space (X, τ) in Example 3.2 is not a pre-semi-locally indiscrete space since $\{a\}$ is a pre-semi-open set but it not closed.

Theorem 3.3. *If a function* $f : (X, \tau) \to (Y, \sigma)$ *is pre-semi-continuous and* (X, τ) *is pre-semi-locally indiscrete, then* f *is contra-continuous.*

Proof. Let V be an open set of (Y, σ). Then $f^{-1}(V)$ is pre-semi-open in (X, τ) since f is pre-semi-continuous. Since (X, τ) is pre-semi-locally indiscrete, $f^{-1}(V)$ is closed. Therefore f is contra-continuous. □

Theorem 3.4. *If a function* $f : (X, \tau) \to (Y, \sigma)$ *is contra-pre-semi-continuous and* (X, τ) *is a pre-semi-*$T_{1/2}$ *space, then* f *is contra-*β*-continuous.*

Proof. Let V be an open set of (Y, σ). Then $f^{-1}(V)$ is pre-semi-closed in (X, τ) since f is contra-pre-semi-continuous. Since (X, τ) is pre-semi-$\mathrm{T}_{1/2}$, $f^{-1}(V)$ is semi-pre-closed. Therefore f is contra-β-continuous. □

Theorem 3.5. *If a function* $f : (X, \tau) \to (Y, \sigma)$ *is contra-pre-semi-continuous and* (X, τ) *is a pre-semi-*T_b *space, then* f *is contra-semi-continuous.*

Proof. Let V be an open set of (Y, σ). Then $f^{-1}(V)$ is pre-semi-closed in (X, τ) since f is contra-pre-semi-continuous. Since (X, τ) is pre-semi-T_b, $f^{-1}(V)$ is semi-closed. Therefore f is contra-semi-continuous. □

Theorem 3.6. *If a function* $f : (X, \tau) \to (Y, \sigma)$ *is contra-pre-semi-continuous and* (X, τ) *is a pre-semi-*$T_{3/4}$ *space, then* f *is contra-pre-continuous.*

Proof. Let V be an open set of (Y, σ). Then $f^{-1}(V)$ is pre-semi-closed in (X, τ) since f is contra-pre-semi-continuous. Since (X, τ) is pre-semi-$\mathrm{T}_{3/4}$, $f^{-1}(V)$ is preclosed. Therefore f is contra-pre-continuous. □

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