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On Strongly Precontinuous Functions

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Abstract. In this paper, we give some characterizations of strongly precontinuous functions. Also we investigate some special properties of these functions.

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1. Introduction

Mashhour *et al.* [20] introduced the notions of preopen sets and precontinuity in topological spaces. Recently, Beceren and Noiri [8] have introduced the notion of strongly precontinuous functions and studied their properties. The purpose of the present paper is to give a set of further characterizations for these functions, to study a few invariant properties under these functions and to characterize extremally disconnected spaces using these functions.

2. Preliminaries

Throughout the present paper, (X, τ) and (Y, σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated and $f : X \to Y$ denotes a single valued function. Let A be a subset of the space X. The closure and interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A is said to be regular open (resp. regular closed) if $A = \mathrm{Int}(\mathrm{Cl}(A))$ (resp. $A = \mathrm{Cl}(\mathrm{Int}(A))$). A subset A is called semi-open [17] (resp. preopen [20], α-open [23], β-open [1]) if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$ (resp. $A \subset \mathrm{Int}(\mathrm{Cl}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$, $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$). The complement of a semi-open (resp. preopen) set is called semi-closed (resp. pre-closed). A point x of a space X is said to be in the preclosure [13] (resp. semi-closure [9]) of a subset A of X, denoted by $p\,\mathrm{Cl}(A)$ (resp. $s\,\mathrm{Cl}(A)$)

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if $A \cap U \neq \emptyset$ for every preopen (resp. semi-open) set containing x. It is known that A is preclosed if and only if $p\,\mathrm{Cl}(A) = A$ and A is semi-closed if and only if $s\,\mathrm{Cl}(A) = A$.

The family of all α-open (resp. preopen, semi-open) subsets of (X, τ) is denoted by τ^α (resp. $PO(X)$, $SO(X)$). It is shown in [23] that τ^α is a topology for X. Moreover, $\tau \subset \tau^\alpha \subset PO(X)$. For a topological space (X, τ), τ_p means the smallest topology on X containing $PO(X)$ due to Andrijevic [5]. The family of semi-open (resp. preopen) sets of X containing x is denoted by $SO(X, x)$ (resp. $PO(X, x)$).

Definition 2.1. *A function $f : X \to Y$ is called*

(1) *irresolute* [10] *if $f^{-1}(V)$ is semi-open in X for every semi-open set V of Y.*
(2) *strongly semi-continuous* [2] *if $f^{-1}(V)$ is open in X for every semi-open set V of Y.*
(3) *precontinuous* [20] *if $f^{-1}(V)$ is preopen in X for every open set V of Y.*
(4) *strongly α-continuous* [7] *if $f^{-1}(V)$ is α-open in X for every semi-open set V of Y.*

Definition 2.2. *A function $f : X \to Y$ is said to be strongly precontinuous* [8] *if $f^{-1}(V)$ is preopen in X for every semi-open set V of Y.*

Recall that a space X is called submaximal [6] if each dense subset of X is open in X.

Remark 2.1. If X is a submaximal space and $f : X \to Y$ is a function, then the following are equivalent:

(i) f is strongly semi-continuous;
(ii) f is strongly α-continuous;
(iii) f is strongly precontinuous.

Theorem 2.1. [8] *A function $f : X \to Y$ is strongly precontinuous if and only if for each $x \in X$ and each semi-open set V of Y containing $f(x)$, there exists a preopen set U of X containing x such that $f(U) \subset V$.*

3. Characterizations

It is shown in [19] that a space X is submaximal if and only if every preopen set of X is open.

A filterbase Γ is said to be s-convergent [12] (resp. p-convergent [19])to a point x in X, if for any semi-open (resp. preopen) set U containing x, there exists $B \in \Gamma$ such that $B \subset U$.

Proposition 3.1. [19] *If Γ is a filterbase in (X, τ), Γ p-converges to x if and only if Γ converges to x in (X, τ_p).*

Theorem 3.1. *For a function $f : X \to Y$, consider the following statements.*

(1) *$f : (X, \tau) \to (Y, \sigma)$ is strongly precontinuous;*
(2) *$f : (X, \tau_p) \to (Y, \sigma)$ is strongly semi-continuous;*
(3) *For each point $x \in X$ and each filterbase Γ in X p-converging to x, the filterbase $f(\Gamma)$ is s-convergent to $f(x)$*

Then (1)⇒(2)⇒(3). Moreover if X is submaximal, then (3) implies (1), and hence the above statements are equivalent.

Proof. (1)⇒(2) Obvious.

(2)⇒(3) Suppose that $x \in X$ and Γ is any filterbase in X which p-converges to x. Let V be any semi-open set of Y with $f(x) \in V$. Since f is strongly semi-continuous, $f^{-1}(V) \in \tau_p$ and $x \in f^{-1}(V)$. Since Γ is p-convergent to x, by Proposition 3.1, then there exists $B \in \Gamma$ such that $B \subset f^{-1}(V)$. Therefore, we have $f(B) \subset V$. This shows that $f(\Gamma)$ is s-convergent to $f(x)$.

(3)⇒(1) Now suppose that X is submaximal. Let x be a point in X and V be any semi-open set containing $f(x)$. Since X is submaximal, every preopen set is open. If we set $\Gamma = \{U \in PO(X) : x \in U\}$, then Γ will be a filterbase which p-converges to x. So there exists U in Γ such that $f(U) \subset V$. This completes the proof. ∎

Let (Y, σ) be a topological space. σ_ψ denotes the topology on Y which has $SO(Y)$ as a subbase [29]. Then, we have the following result.

Proposition 3.2. *If Γ is a filterbase in (X, τ), Γ s-converges to x if and only if Γ converges to x in (X, τ_ψ).*

Proof. Proof is similar to that of Proposition 2 in [19]. ∎

Theorem 3.2. *For a function $f : X \to Y$, consider the following statements.*

(1) $f : (X, \tau) \to (Y, \sigma_\psi)$ *is continuous;*
(2) $f : (X, \tau) \to (Y, \sigma_\psi)$ *is precontinuous;*
(3) $f : (X, \tau) \to (Y, \sigma)$ *is strongly precontinuous.*

Then we have (1)⇒(2)⇒(3). Moreover if X is submaximal, then (3) implies (1), and hence the above statements are equivalent.

Proof. (1)⇒(2)⇒(3) are obvious.

(3)⇒(1) Now suppose that X is submaximal and $f : (X, \tau) \to (Y, \sigma)$ is strongly precontinuous. A basic open set in σ_ψ has the form $V = \cap_{k=1}^n B_k$ where each $B_k \in SO(Y)$. By strongly precontinuity of f, $f^{-1}(B_k)$ is preopen in X. Since X is submaximal, $f^{-1}(V) = \cap_{k=1}^n f^{-1}(B_k)$ is open in X. ∎

4. More Properties

We recall that a space X is said to be extremally disconnected (e.d.) if the closure of each open subset of X is open in X. It is shown in [15] that X is e.d. if and only if $SO(X) \subset PO(X)$. Also $\tau^\alpha = PO(X) \cap SO(X)$ [30].

Theorem 4.1. *The following are equivalent for a topological space (X, τ);*

(1) (X, τ) *is e.d.;*
(2) *For every space (Y, σ), each irresolute $f : (X, \tau) \to (Y, \sigma)$ is strongly precontinuous;*
(3) *The identity function $I : (X, \tau) \to (X, \tau)$ is strongly precontinuous;*
(4) $\tau^\alpha = SO(X)$*;*
(5) *For all $A \subseteq X$, $A - \mathrm{Cl}(\mathrm{Int}(A)) = \emptyset$ implies $A - \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A))) = \emptyset$;*
(6) *For all $B \subseteq X$, $B - \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(B)) \neq \emptyset$ implies $B - \mathrm{Cl}(\mathrm{Int}(B)) \neq \emptyset$;*
(7) $\tau^\alpha = \{R \backslash E : R \in RC(X)$ *and E is nowhere dense*$\} = \{R \cap D : R \in RC(X)$ *and* $\mathrm{Int}(D)$ *is dense*$\}$.

Proof. (1)⇒(2), (2)⇒(3) and (3)⇒(1) are obvious.

(1)⇔(4) In [15].

(4)⇒(5) Let A be a subset of X. If $A - \mathrm{Cl}(\mathrm{Int}(A)) = \emptyset$, then $A \subseteq \mathrm{Cl}(\mathrm{Int}(A))$. By (4), $A \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$ and so $A - \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A))) = \emptyset$.

(5)⇒(6) This is trivial.

(6)⇒(4) It is known that $\tau^\alpha \subset SO(X)$. We will show that the contra inclusion is true. Suppose that $U \notin \tau^\alpha$. Then U is not contained in $\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(U)))$. Hence $U - \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(U))) \neq \emptyset$. By (6), $U - \mathrm{Cl}(\mathrm{Int}(U)) \neq \emptyset$. Then we have that U is not contained in $\mathrm{Cl}(\mathrm{Int}(U))$ and so $U \notin SO(X)$.

(4)⇔(7) It follows from Lemma 3.2 in [4]. ∎

Definition 4.1. *A space X is said to be*

(a) *semi-T_2* [18] *(resp. pre-T_2* [16]*) if for each pair of distinct points x and y in X, there exist disjoint semi-open (resp. preopen) sets U and V in X such that $x \in U$ and $y \in V$.*

(b) *semi compact* [11] *(resp. strongly compact* [21]*) if every semi-open (resp. preopen) cover of X has a finite subcover.*

Theorem 4.2. *If $f : X \to Y$ is a strongly precontinuous injection and Y is semi-T_2, then X is pre-T_2.*

Proof. Let x_1, $x_2 \in X$ and $x_1 \neq x_2$. Then since f is injective and Y is semi-T_2, $f(x_1) \neq f(x_2)$ and there exist V_1, $V_2 \in SO(Y)$ such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$ and $V_1 \cap V_2 = \emptyset$. Since f is strongly precontinuous, $x_i \in f^{-1}(V_i) \in PO(X)$ for $i = 1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$. Thus X is pre-T_2. ∎

Theorem 4.3. *If $f : X \to Y$ is a strongly precontinuous surjection and X is strongly compact, then Y is semi compact.*

Proof. Let $\{V_\alpha : V_\alpha \in SO(Y), \alpha \in I\}$ be a cover of Y. Since f is strongly precontinuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a preopen cover of X and so there is a finite subset I_0 of I such that $X = \cup_{\alpha \in I_0} f^{-1}(V_\alpha)$. Therefore, $Y = \cup_{\alpha \in I_0} V_\alpha$ since f is surjective. Thus Y is semi compact. ∎

Theorem 4.4. [12] *If $f : X \to Y$ is an irresolute injection and Y is semi-T_2, then the graph $G(f)$ of f is semi-closed in the product space $X \times Y$.*

A space X is said to be PS-space [3] if $PO(X) \subset SO(X)$.

Corollary 4.1. *Let X be a PS-space. If $f : X \to Y$ is a strongly precontinuous injection and Y is semi-T_2, then the graph $G(f)$ of f is semi-closed in the product space $X \times Y$.*

Theorem 4.5. *Let f, $g : X \to Y$ be functions. If f and g are strongly semi-continuous and if Y is semi-T_2, then $A = \{x \in X : f(x) = g(x)\}$ is closed in X.*

Proof. Let $x \notin A$, then $f(x) \neq g(x)$. Since Y is semi-T_2, there exist disjoint semi-open sets V_1 and V_2 in Y such that $f(x) \in V_1$ and $g(x) \in V_2$. Since f and g are strongly semi-continuous, $f^{-1}(V_1)$ and $g^{-1}(V_2)$ are open sets in X. Put $U = f^{-1}(V_1) \cap g^{-1}(V_2)$. Then U is an open set with $x \in f^{-1}(V_1) \cap g^{-1}(V_2)$ and $U \cap A = \emptyset$ and so $x \notin \mathrm{Cl}\, A$. This completes the proof. ∎

Corollary 4.2. *Let f, $g : X \to Y$ be functions. If f and g are strongly precontinuous, Y is semi-T_2 and X is a submaximal space, then $A = \{x \in X : f(x) = g(x)\}$ is closed in X.*

Corollary 4.3. *Let f, g be strongly precontinuous from a submaximal space X into a semi-T_2 space Y. If f, g agree on a dense set of X, then $f = g$ everywhere.*

Theorem 4.6. *Let f, $g : X \to Y$ be functions and Y be a semi-T_2 space. If f is strongly α-continuous and g is strongly precontinuous, then the set $A = \{x \in X : f(x) = g(x)\}$ is preclosed in X.*

Proof. Let $x \notin A$, then $f(x) \neq g(x)$. Since Y is semi-T_2, there exist disjoint semi-open sets V_1 and V_2 in Y such that $f(x) \in V_1$ and $g(x) \in V_2$. Since f is strongly α-continuous and g is strongly precontinuous, $f^{-1}(V_1)$ is α-open in X and $g^{-1}(V_2) \in PO(X)$. By Lemma 4.1 of [27], $x \in f^{-1}(V_1) \cap g^{-1}(V_2) \in PO(X)$. Put $U = f^{-1}(V_1) \cap g^{-1}(V_2)$. Then $U \cap A = \emptyset$ and so $x \notin p\,\mathrm{Cl}(A)$. This completes the proof. ∎

A subset of a space X is said to be predense if its preclosure equals X.

Corollary 4.4. *Let f, $g : X \to Y$ be functions and Y be a semi-T_2 space. If f is strongly α-continuous and g is strongly precontinuous, and if f, g agree on a predense set of X, then $f = g$ everywhere.*

Theorem 4.7. *If $f : X \to Y$ is a strongly precontinuous and Y is semi-T_2, then $A = \{(x_1, x_2) : f(x_1) = f(x_2)\}$ is preclosed in the product space $X \times X$.*

Proof. Let $(x_1, x_2) \notin A$, then $f(x_1) \neq f(x_2)$. Since Y is semi-T_2, there exist V_1, $V_2 \in SO(Y)$ such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$ and $V_1 \cap V_2 = \emptyset$. Since f is strongly precontinuous, $x_i \in f^{-1}(V_i) \in PO(X)$ for $i = 1, 2$. Therefore, $(f^{-1}(V_1) \times f^{-1}(V_2)) \cap A = \emptyset$. Since $(x_1, x_2) \in (f^{-1}(V_1) \times f^{-1}(V_2)) \in PO(X \times X)$, we obtain $(x_1, x_2) \notin p\,\mathrm{Cl}(A)$. ∎

Definition 4.2. *Let A be a subset of X. A mapping $r : X \to A$ is called a strongly precontinuous retraction if r is strongly precontinuous and the restriction $r\,|_A$ is the identity mapping on A.*

In [24], it is shown that for a topological space (X, τ), if $U \in SO(X)$ and $A \in PO(X)$, then $U \cap A \in SO(A)$.

Theorem 4.8. *Let A be a preopen subset of X and $r : X \to A$ be a strongly precontinuous retraction. If X is semi-T_2 and e.d., then A is a preclosed set of X.*

Proof. Suppose that A is not preclosed. Then there exists a point x in X such that $x \in p\,\mathrm{Cl}(A)$ but $x \notin A$. It follows that $r(x) \neq x$ because r is a strongly precontinuous retraction. Since X is semi-T_2, there exist disjoint semi-open sets U and V such that $x \in U$ and $r(x) \in V$. By hypothesis, there exists a preopen set $W \subset X$ containing x such that $r(W) \subset V$. Since X is e.d., U is an α-open set in X and by Lemma 4.1 of [27], $W \cap U$ is a preopen set containing x and since $x \in p\,\mathrm{Cl}(A)$, we have $(W \cap U) \cap A \neq \emptyset$. Let $y \in (W \cap U) \cap A$. Then we have $r(y) = y \in U$, and hence $r(y) \in X - V$. This shows that $r(W)$ is not contained in V. This is a contradiction. Consequently, A is preclosed. ∎

A topological space X is said to be semipreconnected [3] or β-connected [28] (resp. semiconnected [25], preconnected [26]) if X can not be expressed as the union of two non-empty disjoint β-open (resp. semi-open, preopen) sets of X. It is shown in [14] that X is semipreconnected if and only if $pclV = X$ for each non-empty $V \in PO(X)$.

Theorem 4.9. *If X is preconnected, $f : X \to Y$ is strongly precontinuous and surjective, then Y is semiconnected.*

Proof. This is clear. ∎

Definition 4.3. *The graph $G(f)$ of a function $f : X \to Y$ is said to be p-s-closed if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \in PO(X, x)$ and $V \in SO(Y, y)$ such that $(U \times V) \cap G(f) = \emptyset$.*

Lemma 4.1. *The graph $G(f)$ of a function $f : X \to Y$ is p-s-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \in PO(X, x)$ and $V \in SO(Y, y)$ such that $f(U) \cap V = \emptyset$.*

Proof. It follows immediately from the definition. ∎

Theorem 4.10. *If $f : X \to Y$ is strongly precontinuous and Y is semi-T_2, then the graph $G(f)$ of f is p-s-closed in $X \times Y$.*

Proof. Let $(x, y) \notin G(f)$, then $y \neq f(x)$. Since Y is semi-T_2, there exist semi-open sets V_1 and V_2 containing $f(x)$ and y, respectively, such that $V_1 \cap V_2 = \emptyset$. Since f is strongly precontinuous, there exists a preopen set U containing x such that $f(U) \subseteq V_1$. Therefore, $f(U) \cap V_2 = \emptyset$ and $G(f)$ is p-s-closed in $X \times Y$. ∎

Definition 4.4. *The graph $G(f)$ of a function $f : X \to Y$ is said to be s-closed if for each $(x, y) \in (X \times Y) - G(f)$, there exist an open set U in X, $x \in U$ and $V \in SO(Y, y)$ such that $(U \times V) \cap G(f) = \emptyset$, or equivalently, if for each $(x, y) \in (X \times Y) - G(f)$, there exist an open set U in X, $x \in U$ and $V \in SO(Y, y)$ such that $f(U) \cap V = \emptyset$.*

Let $A \subset X$. A is called semi-compact set of X [29] if every cover of A by semi-open sets of X has a finite subcover.

Theorem 4.11. *If a function $f : X \to Y$ has an s-closed graph, then $f^{-1}(K)$ is closed in X for each semi-compact set K of Y*

Proof. Let K be a semi-compact set of Y and $x \notin f^{-1}(K)$. Then for each $y \in K$, we have $(x, y) \notin G(f)$ and by s-closedness of $G(f)$, there exist $U_y \in O(X, x)$ and $V_y \in SO(Y, y)$ such that $f(U_y) \cap V_y = \emptyset$. The family $\{V_y : y \in K\}$ is a semi-open cover of K and there exists a finite subset K_* of K such that $K \subseteq \cup_{y \in K_*} V_y$. Set $U = \cap_{y \in K_*} U_y$. Then U is an open set containing x and $f(U) \cap K \subseteq \cup_{y \in K_*} [f(U) \cap V_y] = \emptyset$. Therefore we have $U \cap f^{-1}(K) = \emptyset$ and hence $x \notin \mathrm{Cl}(f^{-1}(K))$. ∎

Corollary 4.5. *Let X be a submaximal space. If a function $f : X \to Y$ has a p-s-closed graph, then $f^{-1}(K)$ is closed in X for each semi-compact set K of Y.*

Theorem 4.12. *Let X be a submaximal space and Y be a semi-T_2 semi compact space. Then the following properties are equivalent.*

(1) *f is strongly precontinuous;*

(2) *$G(f)$ is p-s-closed in $X \times Y$;*
(3) *f is strongly semi-continuous;*
(4) *f is strongly α-continuous.*

Proof. (1)⇒(2) This is obvious from Theorem 4.18.

(2)⇒(3) Let K be a semi-closed subset of Y. Since every semi-closed subset of a semi compact space is semi compact [Proposition 4, [29]], $f^{-1}(K)$ is a closed set in X by Corollary 4.21. This shows that f is strongly semi-continuous.

(3)⇒(4) Clear.

(4)⇒(1) Since every α-open set is preopen, this is obvious. ∎

Theorem 4.13. *Let X be semipreconnected. If $f : X \to Y$ is a strongly precontinuous function with a p-s-closed graph, then f is constant.*

Proof. Suppose that f is not constant. Then there exist two points x and y of X such that $f(x) \neq f(y)$. Then we have $(x, f(y)) \notin G(f)$. Since $G(f)$ is p-s-closed, there exist $U \in PO(X, x)$ and $V \in SO(Y, f(y))$ such that $f(U) \cap V = \emptyset$; hence $U \cap f^{-1}(V) = \emptyset$. This is a contradiction with the semipreconnectedness of X. ∎

The following corollary follows immediately from Theorem 4.18.

Corollary 4.6. *If X is semipreconnected, Y is semi-T_2 and $f : X \to Y$ is strongly precontinous, then f is constant.*

Definition 4.5. [22] *Let X be a topological space and let A be a subset of X. The prefrontier of A is defined by $pFr(A) = p\,\mathrm{Cl}(A) \cap p\,\mathrm{Cl}(X - A) = (p\,\mathrm{Cl}(A)) - (p\,\mathrm{Int}(A))$.*

Theorem 4.14. *A function $f : X \to Y$ is not strongly precontinuous at a point $x \in X$ if and only if there exist a semi-open subset V of Y such that $f(x) \in V$ and x belongs to the prefrontier of $f^{-1}(V)$.*

Proof. Suppose that f is not strongly precontinuous at $x \in X$. Then there exists a semi-open set V in Y containing $f(x)$ such that $f(U)$ is not contained in V for every $U \in PO(X, x)$. Then $U \cap (X - f^{-1}(V)) \neq \emptyset$ for every $U \in PO(X, x)$. Hence $x \in p\,\mathrm{Cl}(X - f^{-1}(V))$. On the other hand, we have $x \in f^{-1}(V) \subset p\,\mathrm{Cl}(f^{-1}(V))$ and hence $x \in pFr(f^{-1}(V))$.

Conversely, suppose that f is strongly precontinuous at $x \in X$ and let V be any semi-open set in Y containing $f(x)$. Then we have $x \in f^{-1}(V) \in PO(X)$. Therefore, $x \notin pFr(f^{-1}(V))$ for each semi-open sets V containing $f(x)$. This completes the proof. ∎

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