

Analytical Treatment of Generalized Burgers-Huxley Equation by Homotopy Analysis Method

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Abstract. In this paper, the homotopy analysis method (HAM) is applied to obtain approximate analytical solutions of the generalized Burgers-Huxley and Huxley equations. The series solution is developed and given explicitly. The initial approximation can be freely chosen with possible unknown constants which can be determined by imposing the boundary and initial conditions. The comparison of the HAM results with the variational iteration method (VIM) results is made. It is shown, in particular, that the VIM solutions are only special cases of the HAM solutions.

2000 Mathematics Subject Classification: 35Q53, 35L75, 35L75

Key words and phrases: Homotopy analysis method, nonlinear PDEs, Burgers-Huxley equation, Huxley equation.

1. Introduction

There has recently been much attention devoted to the search for better and more efficient solution methods for determining a solution, approximate or exact, analytical or numerical, to nonlinear models, [37, 1, 25, 38, 35]. Finding exact/approximate solutions of these nonlinear equations is interesting and important. Many methods have been developed to solve nonlinear partial differential equations (NPDEs) such as pseudospectral method [22], spectral collocation method [23], Adomian decomposition method (ADM) [14, 15, 16], homotopy perturbation method (HPM) [18] and variational iteration method (VIM) [12, 13, 19, 20, 24].

Another analytic method which has received a great deal of attention is the homotopy analysis method (HAM). Initially proposed by Liao in his PhD thesis [26], HAM is a powerful analytic method for nonlinear problems. A systematic and clear exposition on HAM is given in [27]. In recent years, this method has been successfully employed to solve many types of nonlinear problems in science and engineering [28, 29, 30, 31, 32, 34, 33, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Recently Mustafa [17] presented HAM to solve Burgers' equation. Very recently, Molabahrami *et al.* [2] presented

HAM to solve Burgers-Huxley equation. HAM contains a certain auxiliary parameter \hbar , which provides us with a simple way to adjust and control the convergence region and rate of convergence of the series solution. Thus, through HAM, explicit analytic solutions of nonlinear problems are possible.

In this paper, the HAM is applied to find the approximate solutions of the generalized nonlinear Burgers-Huxley equation. Numerical comparisons are made against the exact and VIM solutions [12, 13]. In so doing, the VIM solutions are shown to be special cases of the HAM solutions.

2. The model problem

The analysis presented in this paper is based upon the generalized nonlinear Burgers-Huxley equation:

$$(2.1) \quad u_t + \alpha u^\delta u_x - u_{xx} = \beta u(1 - u^\delta)(u^\delta - \gamma), \quad 0 \leq x \leq 1, \quad t \geq 0,$$

where $\alpha, \beta \geq 0$ are real constants, δ is a positive integer and $\gamma \in (0, 1)$. Equation (2.1) models the interaction between reaction mechanisms, convection effects and diffusion transports [21]. The exact solution to equation (2.1) subject to the initial condition

$$(2.2) \quad u(x, 0) = \left[\frac{\gamma}{2} + \frac{\gamma}{2} \tanh(\sigma\gamma x) \right]^{1/\delta},$$

was derived by Wang *et al.* [36] using nonlinear transformations and is given by

$$(2.3) \quad u(x, t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2} \tanh \left\{ \sigma\gamma \left(x - \left\{ \frac{\gamma\alpha}{1+\delta} - \frac{(1+\delta-\gamma)(\rho-\alpha)}{2(1+\delta)} \right\} t \right) \right\} \right]^{1/\delta},$$

where $\sigma = \delta(\rho - \alpha)/4(1 + \delta)$ and $\rho = \sqrt{\alpha^2 + 4\beta(1 + \delta)}$.

Setting $\alpha = 0$ in (2.1) recovers the Huxley equation:

$$(2.4) \quad u_t - u_{xx} = \beta u(1 - u^\delta)(u^\delta - \gamma), \quad 0 \leq x \leq 1, \quad t \geq 0,$$

with the initial condition

$$(2.5) \quad u(x, 0) = \left[\frac{\gamma}{2} + \frac{\gamma}{2} \tanh(\sigma\gamma x) \right]^{1/\delta},$$

which describes nerve pulse propagation in nerve fibres and wall motion in liquid crystals. The exact solution was derived by Wang *et al.* [36] using nonlinear transformations and is given by

$$(2.6) \quad u(x, t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2} \tanh \left\{ \sigma\gamma \left(x + \left\{ \frac{(1+\delta-\gamma)\rho}{2(1+\delta)} \right\} t \right) \right\} \right]^{1/\delta},$$

where $\sigma = \delta\rho/4(1 + \delta)$ and $\rho = \sqrt{4\beta(1 + \delta)}$. Recently, Batiha *et al.* [12, 13] determined the accuracy of the VIM for the solution of (2.1).

3. Basic ideas of HAM

We consider the following differential equation,

$$N[u(x, t)] = 0,$$

where N is a nonlinear operator, x and t denotes the independent variables, $u(x, t)$ is an unknown function respectively. By means of generalizing the traditional homotopy method, Liao [27] constructs the so-called *zero-order deformation equation*.

$$(3.1) \quad (1 - q)L[\phi(x, t; q) - u_0(x, t)] = q\hbar H(x, t)N[\phi(x, t; q)],$$

where $q \in [0, 1]$ is an embedding parameter, \hbar is a nonzero auxiliary function, L is an auxiliary linear operator, $u_0(x, t)$ is an initial guess of $u(x, t)$ and $\phi(x, t; q)$ is an unknown function. It is important to note that, one has great freedom to choose auxiliary objects such as \hbar and L in HAM. Obviously, when $q = 0$ and $q = 1$, both

$$\phi(x, t; 0) = u_0(x, t) \quad \text{and} \quad \phi(x, t; 1) = u(x, t),$$

hold. Thus as q increases from 0 to 1, the solution $\phi(x, t; q)$ varies from the initial guess $u_0(x, t)$ to the solution $u(x, t)$. Expanding $\phi(x, t; q)$ in Taylor series with respect to q , one has

$$(3.2) \quad \phi(x, t; q) = u_0(x, t) + \sum_{m=1}^{+\infty} u_m(x, t)q^m,$$

where

$$(3.3) \quad u_m(x, t) = \frac{1}{m!} \left. \frac{\partial^m \phi(x, t; q)}{\partial q^m} \right|_{q=0}.$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter \hbar , and the auxiliary function are so properly chosen, then the series (3.2) converges at $q = 1$ and

$$\phi(x, t; 1) = u_0(x, t) + \sum_{m=1}^{+\infty} u_m(x, t),$$

which must be one of solutions of the original nonlinear equation, as proved by Liao [27]. As $\hbar = -1$ and $H(x, t) = 1$, equation (3.1) becomes

$$(3.4) \quad (1 - q)L[\phi(x, t; q) - u_0(x, t)] + qN[\phi(x, t; q)] = 0,$$

which is used mostly in the homotopy-perturbation method (HPM) [18].

According to (3.3), the governing equation can be deduced from the *zero-order deformation equation* (3.1). Define the vector

$$\vec{u}_n = \{u_0(x, t), u_1(x, t), \dots, u_n(x, t)\}.$$

Differentiating (3.1) m times with respect to the embedding parameter q and then setting $q = 0$ and finally dividing them by $m!$, we have the so-called *m th-order deformation equation*.

$$(3.5) \quad L[u_m(x, t) - \chi_m u_{m-1}(x, t)] = \hbar H(x, t)R_m(\vec{u}_{m-1}),$$

where

$$(3.6) \quad R_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\phi(x, t; q)]}{\partial q^{m-1}} \Big|_{q=0},$$

and

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1. \end{cases}$$

It should be emphasized that $u_m(x, t)$ ($m \geq 1$) is governed by the linear equation (3.5) with the linear boundary conditions that come from the original problem, which can be easily solved by symbolic computation softwares such as Maple and Mathematica.

4. Application

To obtain approximate solutions of equations (2.1) and (2.4) and to make comparison with VIM [12, 13], we choose the linear operator

$$L[\phi(x, t; q)] = \frac{\partial \phi(x, t; q)}{\partial t}$$

with the property

$$L[C_1] = 0,$$

where C_1 is an integration constant. Furthermore, equation (2.1) suggests that we define the nonlinear operator as

$$N[\phi(x, t; q)] = \frac{\partial \phi(x, t; q)}{\partial t} + \alpha \phi^\delta(x, t; q) \frac{\partial \phi(x, t; q)}{\partial x} - \frac{\partial^2 \phi(x, t; q)}{\partial x^2} - \beta \phi(x, t; q)[1 - \phi^\delta(x, t; q)][\phi^\delta(x, t; q) - \gamma].$$

Using above definition, we construct the *zeroth-order deformation equations*

$$(4.1) \quad (1 - q)L[\phi(x, t; q) - u_0(t)] = q\hbar N[\phi(x, t; q)].$$

Obviously, when $q = 0$ and $q = 1$,

$$\phi(x, t; 0) = u_0(x, t), \quad \phi(x, t; 1) = u(x, t).$$

Therefore, as the embedding parameter q increases from 0 to 1, $\phi(x, t; q)$ varies from the initial guess $u_0(x, t)$ to the solution $u(x, t)$. Expanding $\phi(x, t; q)$ in Taylor series with respect to q one has

$$\phi(x, t; q) = u_0(x, t) + \sum_{m=1}^{+\infty} u_m(x, t)q^m,$$

where

$$u_m(x, t) = \frac{1}{m!} \frac{\partial^m \phi(x, t; q)}{\partial q^m} \Big|_{q=0}.$$

If the auxiliary linear operator, the initial guess and the auxiliary parameters \hbar are so properly chosen, the above series is convergent at $q = 1$, then one has

$$u(x, t) = u_0(x, t) + \sum_{m=1}^{+\infty} u_m(x, t),$$

which must be one of the solutions of the original nonlinear equation (2.1), as proved by Liao [27]. Now we define the vector

$$\vec{u}_n = \{u_0(x, t), u_1(x, t), \dots, u_n(x, t)\}.$$

The m th-order deformation equation is

$$(4.2) \quad L[u_m(x, t) - \chi_m u_{m-1}(x, t)] = \hbar R_m[\vec{u}_{m-1}],$$

with the boundary conditions

$$(4.3) \quad u_m(x, 0) = 0,$$

where

$$R_1(\vec{u}_0) = \frac{\partial u_0}{\partial t} - \frac{\partial^2 u_0}{\partial x^2} + \alpha u_0^\delta \frac{\partial u_0}{\partial x} + \beta \gamma u_0 + \beta u_0^{1+2\delta} - \beta(1 + \gamma)u_0^{1+\delta},$$

$$\vdots$$

Now, the solution of the m th-order deformation equation (4.2) for $m \geq 1$ becomes

$$u_m(x, t) = \chi_m u_{m-1}(x, t) + \hbar \int_0^t R_m(\vec{u}_{m-1}) d\tau + c_1,$$

where the integration constant c_1 is determined by the boundary conditions (4.3). Firstly, we consider the solution of equation (2.1) with the initial condition (2.2), *i.e.*

$$u_0(x, t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2} \tanh(\sigma \gamma x) \right]^{1/\delta},$$

we now successively obtain

$$u_1(x, t) = \hbar \int_0^t R_1(\vec{u}_0) d\tau.$$

Thus the 2nd-order HAM solution is

$$(4.4) \quad u(x, t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2} \tanh(\sigma \gamma x) \right]^{1/\delta} + \hbar \int_0^t R_1(\vec{u}_0) d\tau.$$

Note that the special case $\hbar = -1$ recovers the VIM solution presented in [12].

Finally, following similar procedure, the 2nd-order HAM solution of the Huxley equation (2.4) with initial condition (2.5) is

$$(4.5) \quad u(x, t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2} \tanh(\sigma \gamma x) \right]^{1/\delta} + \hbar \int_0^t R_1(\vec{u}_0) d\tau,$$

where

$$R_1(\bar{u}_0) = \frac{\partial u_0}{\partial t} - \frac{\partial^2 u_0}{\partial x^2} + \beta u_0^{1+2\delta} - \beta(1 + \gamma)u_0^{1+\delta} + \beta\gamma u_0.$$

Again, setting $\hbar = -1$ yields the VIM solution [13].

5. Numerical results

The solutions given by the HAM contain an auxiliary parameter \hbar , which can be used to control and adjust the convergence region and rate of the HAM solution series. The \hbar -curves for the problems considered in this paper are presented in Figures 1 and 2 which were obtained based on the 7th-order HAM approximations solutions. By HAM, it is easy to determine the valid region of \hbar , which corresponds to the line segments nearly parallel to the horizontal axis. In HAM it is possible to obtain a large family of solutions. For the examples considered in this work, the special case $\hbar = -1$ yields the VIM solutions [12, 13].

In Table 1 we compare the 2nd-order HAM approximations (4.4) with one-iteration VIM solution [12] for the case $\gamma = 0.001$, $\alpha = \beta = \delta = 1$. The comparisons for the case $\gamma = 0.01$, $\alpha = \beta = 1$ are shown in Table 2 ($\delta = 2$) and Table 3 ($\delta = 4$). Based on these results we can conclude that the 2nd-order HAM approximations with $\hbar = 1$ are more accurate than the one iteration VIM solution [12]. In Tables 4–6 we compare the 2nd-order HAM approximations (4.5), the one iteration of VIM [13] and the exact solution for the case $\beta = 1$, $\gamma = 0.001$ and $\delta = 1, 2$ and 3. The results clearly show that HAM is more efficient and accurate than the one iteration VIM [13].

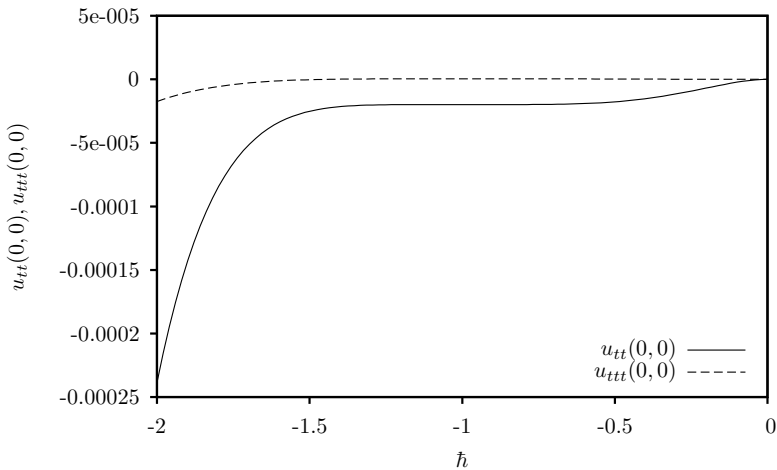


Figure 1. The \hbar -curves obtained from the 7th-order HAM approximation solutions of equation (2.1) in the case $\gamma = 0.001$, $\delta = 4$, and $\alpha = \beta = 1$.

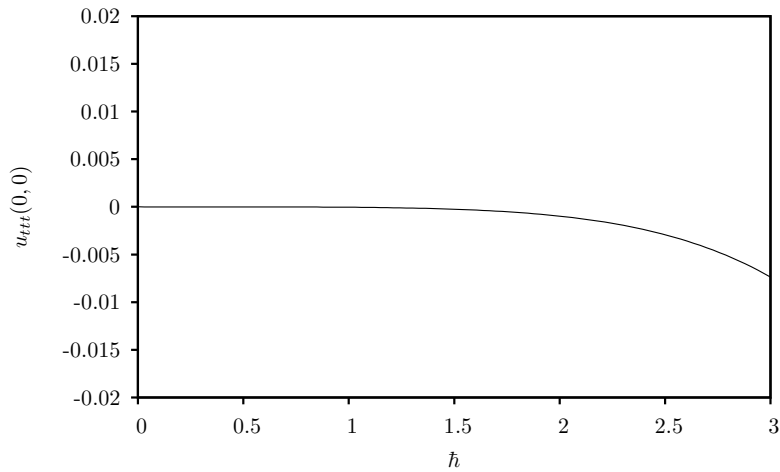


Figure 2. The \hbar -curves obtained from the 7th-order HAM approximation solutions of equation (2.1) in the case $\gamma = 0.001$, $\delta = 4$, and $\alpha = \beta = 1$.

Table 1. Absolute errors when $\gamma = 0.001$, $\alpha = \beta = \delta = 1$ using one iteration of VIM and the solution given by HAM (4.4).

x	t	VIM [12]	HAM	
			$\hbar = -0.5$	$\hbar = 1$
0.1	0.05	1.87405E-08	1.24921E-08	6.25312E-09
	0.1	3.74813E-08	2.49844E-08	1.25063E-08
	1	3.74812E-07	2.49844E-07	1.25063E-07
0.5	0.05	1.87405E-08	1.24921E-08	6.25312E-09
	0.1	3.74812E-08	2.49844E-08	1.25063E-08
	1	3.74813E-07	2.49844E-07	1.25063E-07
0.9	0.05	1.87405E-08	1.24921E-08	6.25312E-09
	0.1	3.74813E-08	2.49844E-08	1.25063E-08
	1	3.74813E-07	2.49844E-07	1.25063E-07

Table 2. Absolute errors when $\gamma = 0.01$, $\alpha = \beta = 1$ and $\delta = 2$, using one iteration of VIM and the solution given by HAM (4.4).

x	t	VIM [12]	HAM	
			$\hbar = -0.5$	$\hbar = 1$
0.1	0.1	5.51580E-05	3.75175E-05	1.54040E-05
	0.2	1.10310E-04	7.50294E-05	3.08136E-05
	0.3	1.65457E-04	1.12536E-04	4.62288E-05
	0.4	2.20598E-04	1.50036E-04	6.16496E-05
	0.5	2.75734E-04	1.87531E-04	7.70761E-05
0.3	0.1	5.51340E-05	3.75011E-05	1.53973E-05
	0.2	1.10262E-04	7.49968E-05	3.08002E-05
	0.3	1.65385E-04	1.12487E-04	4.62087E-05
	0.4	2.20502E-04	1.49971E-04	6.16229E-05
	0.5	2.75614E-04	1.87450E-04	7.70427E-05
0.5	0.1	5.51099E-05	3.74848E-05	1.53906E-05
	0.2	1.10214E-04	7.49640E-05	3.07868E-05
	0.3	1.65313E-04	1.12438E-04	4.61886E-05
	0.4	2.20406E-04	1.49905E-04	6.15962E-05
	0.5	2.75493E-04	1.87368E-04	7.70093E-05

Table 3. Absolute errors when $\gamma = 0.01$, $\alpha = \beta = 1$ and $\delta = 4$, using one iteration of VIM and the solution given by HAM (4.4).

x	t	VIM [12]	HAM	
			$\hbar = -0.5$	$\hbar = 1$
0.1	0.1	2.17687E-04	1.51330E-04	4.77435E-05
	0.2	4.35293E-04	3.02577E-04	9.55687E-05
	0.3	6.52817E-04	4.53743E-04	1.43476E-04
	0.4	8.70258E-04	6.04827E-04	1.91465E-04
	0.5	1.08762E-03	7.55829E-04	2.39536E-04
0.3	0.1	2.17453E-04	1.51167E-04	4.76921E-05
	0.2	4.34824E-04	3.02251E-04	9.54662E-05
	0.3	6.52113E-04	4.53254E-04	1.43322E-04
	0.4	8.69320E-04	6.04175E-04	1.91260E-04
	0.5	1.08644E-03	7.55013E-04	2.39281E-04
0.5	0.1	2.17218E-04	1.51003E-04	4.76408E-05
	0.2	4.34354E-04	3.01925E-04	9.53636E-05
	0.3	6.51408E-04	4.52764E-04	1.43168E-04
	0.4	8.68380E-04	6.03521E-04	1.91055E-04
	0.5	1.08527E-03	7.54196E-04	2.39025E-04

Table 4. Numerical solutions for $\beta = 1$, $\gamma = 0.001$ and $\delta = 1$.

x	t	Exact	VIM [13]	HAM	
				$\hbar = -0.5$	$\hbar = 1$
0.1	0.05	5.00030171E-04	5.00005184E-04	5.00011431E-04	5.00030171E-04
	0.1	5.00042665E-04	4.99992690E-04	5.00005184E-04	5.00042665E-04
	1	5.00267553E-04	4.99767803E-04	4.99892740E-04	5.00267553E-04
0.5	0.05	5.00100882E-04	5.00075895E-04	5.00082141E-04	5.00100882E-04
	0.1	5.00113376E-04	5.00063401E-04	5.00075895E-04	5.00113376E-04
	1	5.00338263E-04	4.99838513E-04	4.99963451E-04	5.00338263E-04
0.9	0.05	5.00171593E-04	5.00146605E-04	5.00152852E-04	5.00171593E-04
	0.1	5.00184087E-04	5.00134111E-04	5.00146605E-04	5.00184087E-04
	1	5.00408974E-04	4.99909224E-04	5.00034162E-04	5.00408974E-04

Table 5. Numerical solutions for $\beta = 1$, $\gamma = 0.001$ and $\delta = 2$.

x	t	Exact	VIM [13]	HAM	
				$\hbar = -0.5$	$\hbar = 1$
0.1	0.05	2.23618841E-02	2.23607664E-02	2.23610459E-02	2.23618841E-02
	0.1	2.23624429E-02	2.23602076E-02	2.23607664E-02	2.23624429E-02
	1	2.23724988E-02	2.23501490E-02	2.23557371E-02	2.23725016E-02
0.5	0.05	2.23644658E-02	2.23633483E-02	2.23636277E-02	2.23644658E-02
	0.1	2.23650245E-02	2.23627895E-02	2.23633483E-02	2.23650245E-02
	1	2.23750792E-02	2.23527320E-02	2.23557371E-02	2.23750820E-02
0.9	0.05	2.23670472E-02	2.23659298E-02	2.23662092E-02	2.23670472E-02
	0.1	2.23676058E-02	2.23653711E-02	2.23659298E-02	2.23676059E-02
	1	2.23776594E-02	2.23553148E-02	2.23609016E-02	2.23776622E-02

Table 6. Numerical solutions for $\beta = 1$, $\gamma = 0.001$ and $\delta = 3$.

x	t	Exact	VIM [13]	HAM	
				$\hbar = -0.5$	$\hbar = 1$
0.1	0.05	7.93740204E-02	7.93700531E-02	7.93710405E-02	7.93740201E-02
	0.1	7.93760039E-02	7.93680695E-02	7.93700531E-02	7.93760041E-02
	1	7.94116901E-02	7.93323637E-02	7.93522002E-02	7.94117099E-02
0.5	0.05	7.93819558E-02	7.93779894E-02	7.93789810E-02	7.93819559E-02
	0.1	7.93839389E-02	7.93760061E-02	7.93779894E-02	7.93839391E-02
	1	7.94196179E-02	7.93403074E-02	7.93601400E-02	7.94196378E-02
0.9	0.05	7.93898897E-02	7.93859240E-02	7.93869154E-02	7.93898897E-02
	0.1	7.93918724E-02	7.93839411E-02	7.93859240E-02	7.93918726E-02
	1	7.94275442E-02	7.93482496E-02	7.93680782E-02	7.94275640E-02

6. Conclusions

In this paper, the HAM has been successfully applied to find approximate solution to the generalized Burgers-Huxley equation. The HAM contains an auxiliary parameter \hbar , which can be used to control and adjust the convergence region and rate of the HAM solution series. It is noted that the special case $\hbar = -1$ yields the VIM solutions and taking $\hbar = 1$ in HAM gives very accurate solutions. Comparisons with the variational iteration method (VIM) reveal that the HAM is very effective and convenient.

Acknowledgements. The authors would like to acknowledge the MOSTI Science-fund grant 04-01-02-SF0177 and the Academy of Sciences Malaysia SAGA grant STGL-011-2006 (P24c) for financial support.

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