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Pulsatile Flow of Couple Stress Fluid Through a Porous Medium with Periodic Body Acceleration and Magnetic Field

[1]V. P. Rathod and [2]Shakera Tanveer
[1]Department of Studies and Research in Mathematics
Gulbarga University, Gulbarga-585106, India
[2]Department of Mathematics, New Horizon College of Engineering,
Bangalore-560087, Karnataka, India
[1]rathod_vijaykumar@rediffmail.com, [2]shakera_tanveer@rediffmail.com

Abstract. Pulsatile flow of blood through a porous medium has been studied under the influence of periodic body acceleration by considering blood as a couple stress, incompressible, electrically conducting fluid in the presence of magnetic field. An exact solution of the equation of motion is obtained by applying the Laplace and finite Hankel transforms. The expressions for axial velocity, flow rate, fluid acceleration and shear stress have been obtained analytically. The effects of magnetic field, body acceleration and permeability parameter have been discussed with the help of graphs. It is found that the velocity distribution increases with an increase of both body acceleration and permeability of the porous medium, while it decreases as the magnetic parameter increases.

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1. Introduction

When a human body experiences a sudden velocity change, the blood flow is disturbed. Though human body has remarkable adoptability to changes, a prolonged exposure of body to such vibrations leads to many health problems like headache, abdominal pain, losing vision, venous pooling of blood in the extremities and increased pulse rate on account of disturbances in blood flow. The human body is quite often subjected to accelerations. In many situations like travel or driving in vehicles (car, bus, motor bicycle, truck, tractor etc.), aircraft or spacecraft, while jogging, using lathe machine or jackhammer, athletes and sports persons for their sudden movements, the human body is subjected to vibrations (body acceleration).

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Many researchers have studied blood flow in the artery by considering blood as either Newtonian or non-Newtonian fluids, since blood is a suspension of red cells in plasma; it behaves as a non-Newtonian fluid at low shear rate. Chaturani and Upadhya [4] have developed a method for the study of the pulsatile flow of couple stress fluid through circular tubes. The Poiseuille flow of couple stress fluid has been critically examined by Chaturani and Rathod [5]. Chaturani and Palanisamy [6] studied pulsatile flow of blood, as Newtonian fluid, through a rigid tube under the influence of body acceleration. Majhi and Nair [16] have given a mathematical model for pulsatile blood flow subjected to externally-imposed periodic body acceleration by considering blood as a third grade fluid. Sud and Sekhon [20, 21] studied the blood flow subjected to single cycle of body acceleration and the arterial flow under periodic body acceleration, later it was re-examined by Chaturani and Wassf [7]. Chaturani and Palanisamy [8, 9] have given a numerical study for the effect of periodic body acceleration on pulsatile flow of Casson and Powerlaw fluids.

The study of magnetic field with porous medium is very important both from theoretical as well as practical point of view; because most of natural phenomena of the fluid flow are connected with porous medium. For instance, filtration of fluids, under ground water and oil, reservoir and fluid through pipes.

The application of Magnetohydrodynamics in physiological problems is of growing interest. The flow of blood can be controlled by applying appropriate quantity of magnetic field.

Kollin [14] has coined the idea of electromagnetic field in the medical research for the first time in the year 1936. Korchevskii and Marcochnik [15] have discussed the possibility of regulating the blood movement in human system by applying magnetic field. Rao and Deshikachar [17] have investigated the effect of transverse magnetic field in physiological type of flow, through a uniform circular pipe. Vardanyan [24] showed that the application of magnetic field reduces the speed of blood flow. Rao and Deshikachar [18] studied the MHD oscillatory flow of blood through channels of variable cross section. It has been established that the biological systems in general are greatly affected by the application of external magnetic field. As per the investigations reported by Barnothy [2], the heart rate decreases by exposing biological systems to an external magnetic field. The ECG pattern taken in the presence of a magnetic field not only provides information on blood flow but also offers a new noninvasive method of studying the cardiac performance. A mathematical model for two-layer pulsatile flow of blood with microorganism in a uniform tube under low Reynolds number and magnetic effect has been studied by Rathod and Gayatri [19]. Thus, all these researchers have reported that the effect of magnetic field reduces the velocity of blood.

Ahmadi and Manvi [1] derived a general equation of motion for the flow of a viscous fluid through a porous medium. The porous material containing the fluid is in fact a non-homogenous medium. For the sake of analysis, it is possible to replace it with a homogenous fluid, which has dynamical properties equivalent to the local averages of the original non-homogenous medium. In some pathological situations, the distribution of fatty cholesterol and artery-clogging blood clots in the lumen of the coronary artery can be considered as equivalent to a fictitious porous medium [10]. An approximate solution for the pulsatile flow of blood in a porous channel in the presence of transverse magnetic field by assuming blood as a Newtonian fluid has been obtained by Bhuyan and Hazarika [3].

In recent time, a number of theoretical studies have been made to estimate the effects of body acceleration on the human circulation system. El-Shehawey *et al.* [11] studied an unsteady flow of blood as an electrically conducting, incompressible non-Newtonian and elastico-viscous fluid in the presence of a magnetic field through a circular pipe. El-Shehawey *et al.* [12] studied an unsteady flow of blood as an incompressible Newtonian fluid through a porous medium under the influence of body acceleration. El-Shahed [13] studied the pulsatile flow of Newtonian fluid through a stenosed porous medium under the influence of periodic body acceleration.

In the present analysis a mathematical model for the pulsatile blood flow through a porous medium under the influence of periodic body acceleration with magnetic field is presented by considering blood as a couple stress fluid in a circular tube. It is assumed that the magnetic field along the radius of the pipe is present, no external electric field is imposed and magnetic Reynolds number is very small. The motivation for studying this problem is to understand the blood flow in an artery under some pathological situations when the fatty plaques of cholestrol and artery-clogging blood clots are formed in the lumen of the coronary artery. The distribution of these fatty cholestrol and artery-clogging blood clots are deemed to be equivalent to a fictitious porous medium of permeability K.

The main aim of this work is to study these phenomenon, obtain analytical expressions for axial velocity, flow rate, fluid acceleration and shear stress. Also to study the effect of body acceleration (a_0), magnetic field (Hartmann number H), permeability of the porous medium (K), Womersely parameter (α) and couple stress parameter ($\overline{\alpha}$) on the velocity and the effects of permeability of the porous medium K, Hartmann number and body acceleration on the fluid acceleration. Hence, the present mathematical model gives a simple form of velocity expression for the blood flow so that it will help not only people working in the field of Physiological fluid dynamics but also to the medical practitioners.

2. Formulation of the problem

Let us consider a one-dimensional pulsatile flow of blood through a porous medium in a straight and rigid circular tube by considering blood as couple stress, non-Newtonian, incompressible and electrically conducting fluid in the presence of magnetic field. It is assumed that the induced magnetic and electric fields are negligible. The flow is considered as axially symmetric, pulsatile and fully developed. The geometry of the flow is shown in Figure 1.

The pressure gradient and body acceleration G are given by:

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \cos(\omega t), \quad t \geq 0, \tag{2.1}$$

$$G = a_0 \cos(\omega_1 t + \phi), \quad t \geq 0, \tag{2.2}$$

where A_0 is the steady-state part of the pressure gradient, A_1 is the amplitude of the oscillatory part, $\omega = 2\pi f$ and f is the heart pulse frequency, a_0 is the amplitude of body acceleration, $\omega_1 = 2\pi f_1$ and f_1 is body acceleration frequency, ϕ is phase difference, z is the axial distance and t is time. Under the above assumptions, the equation of motion for flow as discussed by Ahmadi and Manvi [1], the pulsatile couple stress equation [22] in a cylindrical polar co-ordinates through a porous medium under the periodic body acceleration in the presence of magnetic field is given by

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \rho G + \mu \nabla^2 u - \eta \nabla^2(\nabla^2 u) - \sigma B_0^2 u - \frac{\mu}{K} u$$

where

$$\nabla^2 = \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right) \tag{2.3}$$

where $u(r,t)$ is velocity in the axial direction, ρ and μ are the density and viscosity of blood, η is the couple stress parameter, σ is the electrical conductivity, B_0 is the external magnetic field, K is the permeability of the isotropic porous medium and r is the radial coordinate.

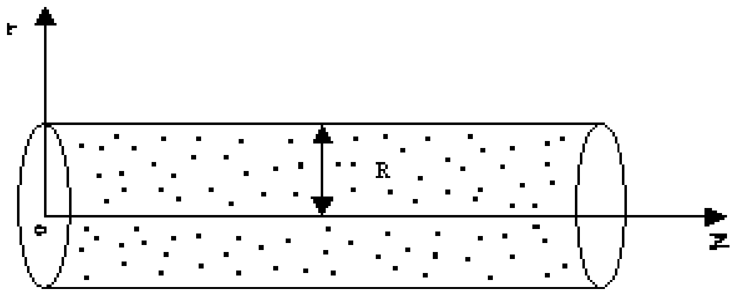

Figure 1. Schematic diagram for flow geometry.

Let us introduce the following dimensionless quantities:

(2.4)

$$u^* = \frac{u}{\omega R}, r^* = \frac{r}{R}, t^* = t\omega, A_0^* = \frac{R}{\mu\omega}A_0, A_1^* = \frac{R}{\mu\omega}A_1, a_0^* = \frac{\rho R}{\mu\omega}a_0, z^* = \frac{z}{R}, K^* = \frac{K}{R^2}$$

In terms of these variables, equation (2.3) [after dropping stars] becomes

$$\overline{\alpha}^2\alpha^2\frac{\partial u}{\partial t} = \overline{\alpha}^2(A_0 + A_1\cos t + a_0\cos(bt+\phi)) + \overline{\alpha}^2\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right)$$

$$-\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)\cdot\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right) - \overline{\alpha}^2\left(H^2 u - \frac{u}{K}\right) \tag{2.5}$$

where $\overline{\alpha}^2 = \frac{R^2\mu}{\eta}$- couple stress parameter, $\alpha = R\left(\frac{\omega\rho}{\mu}\right)^{\frac{1}{2}}$ is Womersley parameter, $H = B_0 R\left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}}$ is the Hartmann number, $b = \frac{\omega_1}{\omega}$ and R is the radius of the tube.

We assume that at $t < 0$, only the pumping action of the heart is present and at $t = 0$, the flow in the artery corresponds to the instantaneous pressure gradient, i.e. $-\frac{\partial p}{\partial z} = A_0 + A_1$. As a result, the flow velocity at $t = 0$ is given by [1]:

$$u(r,0) = \frac{(A_0 + A_1)}{h^2}\left[1 - \frac{I_0(hr)}{I_0(h)}\right], \tag{2.6}$$

where $h = \sqrt{(1/K)}$ and I_0 is a modified Bessel function of first kind of order zero. When $K \to \infty$, we obtain the velocity of the classical Hagen-poiseuille flow.

$$u(r,0) = \frac{A_0 + A_1}{4}\left(1 - r^2\right). \tag{2.7}$$

The initial and boundary conditions for this problem are

$$u(r,0) = 2\sum_{n=1}^{\infty} \frac{\overline{\alpha}^2 J_0(r\lambda_n)}{\lambda_n J_1(\lambda_n)} \frac{[A_0 + A_1 + a_0\cos\phi]}{\left[\lambda_n^4 + \overline{\alpha}^2\left(\lambda_n^2 + H^2 + h^2\right)\right]}, \tag{2.8}$$

$$u(r,t) = 0 \quad \text{at} \quad r = 1, \tag{2.9}$$

$$u(0,t) \quad \text{is finite at} \quad r = 0. \tag{2.10}$$

3. Required integral transforms

If $f(r)$ satisfies Dirichlet conditions in closed interval (0,1) and its finite Hankel transform, defined as (Sneddon [23]),

$$f^*(\lambda_n) = \int_0^1 r f(r) J_0(r\lambda_n)\, dr \tag{3.1}$$

where λ_n are the roots of $J_0(r) = 0$. Then at each point of the interval at which $f(r)$ is continuous:

$$f(r) = 2\sum_{n=1}^{\infty} f^*(\lambda_n) \frac{J_0(r\lambda_n)}{J_1^2(\lambda_n)} \tag{3.2}$$

where the sum is taken over all positive roots of $J_0(r) = 0, J_0$ and J_1 are Bessel's functions of first kind.

The Laplace transform of any function is defined as:

$$\overline{f}(s) = \int_0^{\infty} e^{-st} f(t)\, dt, \quad s > 0 \tag{3.3}$$

4. Analysis

Employing the Laplace transforms (3.3) to equation (2.5) in the light of (2.9) we get

$$\overline{\alpha}^2\alpha^2\left(s\overline{u}-u\left(r,0\right)\right)=\overline{\alpha}^2\frac{A_0}{s}+\overline{\alpha}^2\frac{A_1 s}{s^2+1}+\overline{\alpha}^2 a_0\left(\frac{s\cos\phi-b\sin\phi}{s^2+b^2}\right)+\overline{\alpha}^2\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\overline{u}}{\partial r}\right)\right)$$
$$-\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)\cdot\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\overline{u}}{\partial r}\right)\right)-\overline{\alpha}^2\left(h^2+H^2\right)\overline{u} \tag{4.1}$$

where

$$\overline{u}\left(r,s\right)=\int_0^\infty e^{-st}u\left(r,t\right)dt. \tag{4.2}$$

On applying the finite Hankel transform (3.1) to (4.1) and using (2.10) we obtain

$$\overline{u}^*\left(\lambda_n,s\right)=\frac{J_1\left(\lambda_n\right)}{\lambda_n}\left[\overline{\alpha}^2\left(\frac{A_0}{s}+\frac{A_1 s}{s^2+1}+\frac{a_0\left(s\cos\phi-b\sin\phi\right)}{s^2+b^2}\right)\right.$$
$$\left.+m\left(\frac{\left[A_0+A_1+a_0\cos\phi\right]}{\left[\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right]}\right)\right]\times\frac{1}{\left[sm+\left\{\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right\}\right]} \tag{4.3}$$

where $m=\overline{\alpha}^2\alpha^2$.

Rearranging the terms and taking the inversion of both Laplace transform and Hankels transform of (4.3) which gives the final solution as

$$u\left(r,t\right)=2\sum_{n=1}^{\infty}\frac{J_0\left(r\lambda_n\right)\overline{\alpha}^2}{\lambda_n J_1\left(\lambda_n\right)}\left[\left\{\frac{A_0}{\left[\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right]}\right.\right.$$
$$+\frac{A_1\left[\left\{\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right\}\cos t+m\sin t\right]}{\left[\left\{\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right\}^2+m^2b^2\right]}$$
$$\left.+\frac{a_0\left[\left\{\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right\}\cos\left(bt+\phi\right)+bm\sin\left(bt+\phi\right)\right]}{\left[\left\{\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right\}^2+m^2b^2\right]}\right\}$$
$$-e^{-h_1 t}\left\{\frac{A_0}{\left[\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right]}+\frac{A_1\left[\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right]}{\left[\left\{\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right\}^2+m^2\right]}\right.$$
$$+\frac{a_0\left[\left\{\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right\}\cos\phi+mb\sin\phi\right]}{\left[\left\{\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right\}^2+m^2b^2\right]}$$
$$\left.\left.-\left(\frac{\left[A_0+A_1+a_0\cos\phi\right]}{\left[\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right]}\right)\right\}\right] \tag{4.4}$$

where

$$h_1=\frac{\left[\lambda_n^4+\overline{\alpha}^2\left(\lambda_n^2+h^2+H^2\right)\right]}{m}. \tag{4.5}$$

The expression for the flow rate Q is given by

$$Q = 2\pi \int_0^1 rudr \tag{4.6}$$

then

$$Q(r,t) = 4\pi \sum_{n=1}^{\infty} \frac{\overline{\alpha}^2}{\lambda_n^2} \left[\left\{ \frac{A_0}{[\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)]} \right. \right.$$
$$+ \frac{A_1[\{\lambda_n^4 + \overline{\alpha}^2 \left(\lambda_n^2 + h^2 + H^2\right)\} \cos t + m \sin t]}{[\{\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)\}^2 + m^2 b^2]}$$
$$\left. + \frac{a_0[\{\lambda_n^4 + \overline{\alpha}^2 \left(\lambda_n^2 + h^2 + H^2\right)\} \cos (bt + \phi) + bm \sin (bt + \phi)]}{[\{\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)\}^2 + m^2 b^2]} \right\}$$
$$- e^{-h_1 t} \left\{ \frac{A_0}{[\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)]} + \frac{A_1[\lambda_n^4 + \overline{\alpha}^2 \left(\lambda_n^2 + h^2 + H^2\right)]}{[\{\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)\}^2 + m^2]} \right.$$
$$+ \frac{a_0[\{\lambda_n^4 + \overline{\alpha}^2 \left(\lambda_n^2 + h^2 + H^2\right)\} \cos \phi + mb \sin \phi]}{[\{\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)\}^2 + m^2 b^2]}$$
$$\left. \left. - \frac{[A_0 + A_1 + a_0 \cos \phi]}{[\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)]} \right\} \right]. \tag{4.7}$$

Similarly the expression for fluid acceleration F can be obtained from

$$F(r,t) = \frac{\partial u}{\partial t} \tag{4.8}$$

Then we have

$$F(r,t) = 2 \sum_{n=1}^{\infty} \frac{J_0(r\lambda_n)\, \overline{\alpha}^2}{\lambda_n J_1(\lambda_n)} \left[\left\{ \frac{A_1[m \cos t - \{\lambda_n^4 + \overline{\alpha}^2 \left(\lambda_n^2 + h^2 + H^2\right)\} \sin t]}{[\{\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)\}^2 + m^2 b^2]} \right. \right.$$
$$\left. + \frac{a_0 b[bm \cos (bt + \phi) - \{\lambda_n^4 + \overline{\alpha}^2 \left(\lambda_n^2 + h^2 + H^2\right)\} \sin (bt + \phi)]}{[\{\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)\}^2 + m^2 b^2]} \right\}$$
$$+ h_1 e^{-h_1 t} \left\{ \frac{A_0}{[\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)]} + \frac{A_1[\lambda_n^4 + \overline{\alpha}^2 \left(\lambda_n^2 + h^2 + H^2\right)]}{[\{\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)\}^2 + m^2]} \right.$$
$$+ \frac{a_0[\{\lambda_n^4 + \overline{\alpha}^2 \left(\lambda_n^2 + h^2 + H^2\right)\} \cos \phi + mb \sin \phi]}{[\{\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)\}^2 + m^2 b^2]}$$
$$\left. \left. - \left(\frac{[A_0 + A_1 + a_0 \cos \phi]}{[\lambda_n^4 + \overline{\alpha}^2 (\lambda_n^2 + h^2 + H^2)]} \right) \right\} \right] \tag{4.9}$$

Similarly the expression for the Shear Stress τ_w can be obtained from

$$\tau_w = \mu \frac{\partial u}{\partial r} \tag{4.10}$$

then

$$\tau_w(r,t) = -2\sum_{n=1}^{\infty} \frac{J_1(r\lambda_n)\overline{\alpha}^2}{\lambda_n J_1(\lambda_n)} \Bigg[\Bigg\{ \frac{A_0}{[\lambda_n^4 + \overline{\alpha}^2(\lambda_n^2 + h^2 + H^2)]}$$
$$+ \frac{A_1[\{\lambda_n^4 + \overline{\alpha}^2\left(\lambda_n^2 + h^2 + H^2\right)\}\cos t + m\sin t]}{[\{\lambda_n^4 + \overline{\alpha}^2(\lambda_n^2 + h^2 + H^2)\}^2 + m^2 b^2]}$$
$$+ \frac{a_0 b[\{\lambda_n^4 + \overline{\alpha}^2\left(\lambda_n^2 + h^2 + H^2\right)\}\cos(bt+\phi) + bm\sin(bt+\phi)]}{[\{\lambda_n^4 + \overline{\alpha}^2(\lambda_n^2 + h^2 + H^2)\}^2 + m^2 b^2]}\Bigg\}$$
$$- e^{-h_1 t}\Bigg\{ \frac{A_0}{[\lambda_n^4 + \overline{\alpha}^2(\lambda_n^2 + h^2 + H^2)]} + \frac{A_1[\lambda_n^4 + \overline{\alpha}^2\left(\lambda_n^2 + h^2 + H^2\right)]}{[\{\lambda_n^4 + \overline{\alpha}^2(\lambda_n^2 + h^2 + H^2)\}^2 + m^2]}$$
$$+ \frac{a_0[\{\lambda_n^4 + \overline{\alpha}^2\left(\lambda_n^2 + h^2 + H^2\right)\}\cos\phi + mb\sin\phi]}{[\{\lambda_n^4 + \overline{\alpha}^2(\lambda_n^2 + h^2 + H^2)\}^2 + m^2 b^2]}$$
$$- \left(\frac{[A_0 + A_1 + a_0\cos\phi]}{[\lambda_n^4 + \overline{\alpha}^2(\lambda_n^2 + h^2 + H^2)]} \right) \Bigg\}\Bigg] \tag{4.11}$$

5. Results, discussion and conclusion

The flow investigations can be carried out in two ways; either one can study the effect of individual factor like tube radius, pressure gradient (average steady pressure gradient, which has been used as amplitude A_0 for the pulsatile pressure gradient), various parameters which enter into the problem etc. or one can compute the values of flow variables at a particular site in cardiovascular system, e.g., in arteriole radius $R = 0.008$ cm, $A_0 = 2000$ dyne/cm^3 in coronary artery $R = 0.15$ cm, $A_0 = 693.65$ dyne/cm^3 and in femoral artery $R = 0.5$ cm, $A_0 = 32$ dyne/cm^3. In the present investigation, the first method has been followed.

The problem under investigation is dominated mainly by three dimensionless parameters viz, the Womersely parameter α, couple stress parameter $\overline{\alpha}$ and Hartmann number H. Our main interest is to study and investigate the role of magnetic parameter H, amplitude of body acceleration a_0 and the permeability of the porous media K on the velocity field.

The velocity profile for pulsatile flow of blood through a porous medium with periodic body acceleration in the presence of magnetic field computed by using the velocity expression (4.4) for different values of permeability of the porous medium K, Hartmann number H, amplitude of body acceleration a_0, Womersely parameter α, couple stress parameter $\overline{\alpha}$, time t, b and have been shown through Figures 2 to 10. It is observed from Figure 2 that as the permeability of the porous media K increases the velocity also increases whereas the velocity decreases as the Hartmann number H increases Figure 3. Figure 4 shows that as a_0 increases the velocity profile increases. It has been found that there is an increase in the velocity as Womerseley parameter α is increased from 3 to 11 (Table 1). Figure 5 shows that as the couple stress parameter $\overline{\alpha}$ increases the velocity first increases then decreases slightly at higher values of $\overline{\alpha}$ (at $\overline{\alpha} = 7$ and 9). As the values of time t and b increases the velocity decreases (Figures 6 and 7).

The fluid acceleration profile for pulsatile flow of blood through a porous medium with periodic body acceleration in the presence of magnetic field is computed by using the expression (4.9) for different values of permeability of the porous medium K, Hartmann number H, amplitude of body acceleration a_0. It is observed from Figure 9 that as the Hartmann number H increases the fluid acceleration decreases and a back flow is observed. Whereas the fluid acceleration increases as the permeability of the porous medium K and amplitude of body acceleration a_0 increases and a similar back flow is observed as shown in Figures 8 and 10.

In the present mathematical model the pulsatile blood flow in the presence of magnetic field with periodic body acceleration through a rigid straight circular tube has been studied. The velocity expressions have been obtained in the Bessel–Fourier series form. The corresponding expression for flow rate, fluid acceleration and Shear stress are also obtained. It is of interest to note that velocity decreases as the Hartmann number increases whereas it increases as amplitude of body acceleration increases.

The present model gives a most general form of velocity expression from which the other mathematical models can easily be obtained by proper substitutions. It is of interest to note that the velocity expression (4.4) obtained for the present model includes various velocity expressions for different mathematical models such as:

(1) The velocity expression for pulsatile flow of couple stress fluid (blood) through a porous medium with periodic body acceleration in the absence of magnetic field can be obtained by substituting $H = 0$.

(2) The velocity expression for pulsatile Newtonian fluid through a porous medium with periodic body acceleration and magnetic field can be obtained as $\overline{\alpha}$ tends to ∞.

(3) The velocity expression for pulsatile Newtonian fluid through a porous medium with periodic body acceleration can be obtained by making $\overline{\alpha} \longrightarrow \infty$ and $H = 0$, which is the result of El-Shehaway *et al.* [12].

(4) The velocity expression for pulsatile Newtonian fluid with periodic body acceleration can be obtained by making $\overline{\alpha} \longrightarrow \infty$, $K \longrightarrow \infty$ or $h = 0$ and $H = 0$, which is the result of Chaturani and Palanisamy [6].

It is possible that a proper understanding of interactions of body acceleration with blood flow may lead to a therapeutic use of controlled body acceleration. It is therefore desirable to analyze the effects of different types of vibrations on different parts of the body. Such a knowledge of body acceleration could be useful in the diagnosis and therapeutic treatment of some health problems (joint pain, vision loss and vascular disorder), to better design of protective pads and machines.

By using an appropriate magnetic field it is possible to control blood pressure and also it is effective for conditions such as poor circulation, travel sickness, pain, headaches, muscle sprains, strains and joint pain etc.

Hoping that this investigation may help for the further studies in the field of medical research, the application of magnetic field for the treatment of certain cardiovascular diseases and also the results of this analysis can be applied to the pathological situations of blood flow in coronary arteries when fatty plaques of cholesterol and artery-clogging blood clots are formed in the lumen of the coronary artery.

Table 1. Effect of Womerseley parameter α on velocity distribution. $H = 4$, $K = 2.5$, $A_0 = 2$, $A_1 = 4$, $a_0 = 3$, $\overline{\alpha} = 1$, $t = 0.5$, $b = 0.5$, $\phi = 15^0$.

Velocity $u(r,t)$			
r	$\alpha = 3.0$	$\alpha = 7.0$	$\alpha = 11.0$
0.0	0.2363245	0.2842332	0.3079754
0.2	0.2248589	0.2700626	0.2924948
0.4	0.1907518	0.2282802	0.2469693
0.6	0.1363541	0.1624923	0.1755657
0.8	0.06872854	0.08163627	0.08811477
1.0	0.0000000	0.0000000	0.0000000

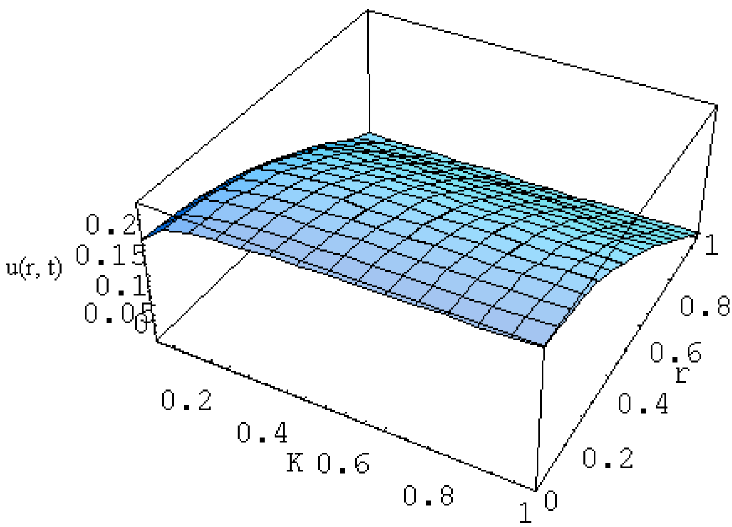

Figure 2. Variation of velocity profile for different values of permeability parameter K; $H = 4$, $A_0 = 2$, $A_1 = 4$, $a_0 = 3$, $\alpha = 1$, $\overline{\alpha} = 1$, $t = 0.5$, $b = 0.5$, $\phi = 15°$.

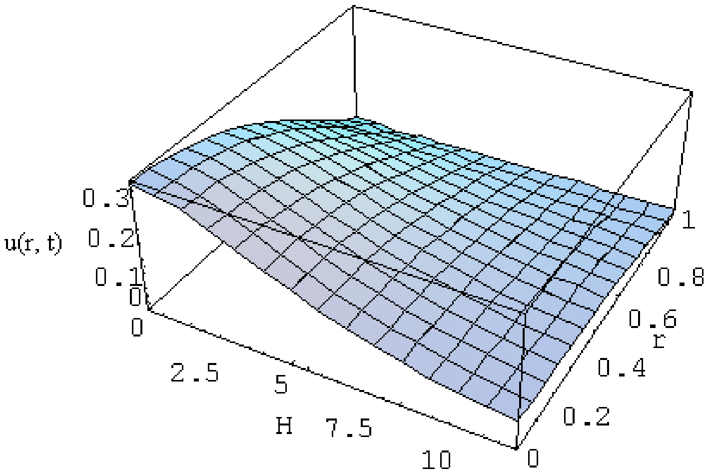

Figure 3. Variation of velocity profile for different values of Hartmann number H; $K = 2.5, A_0 = 2, A_1 = 4, a_0 = 3, \alpha = 1, \overline{\alpha} = 1, t = 0.5, b = 0.5, \phi = 15°$.

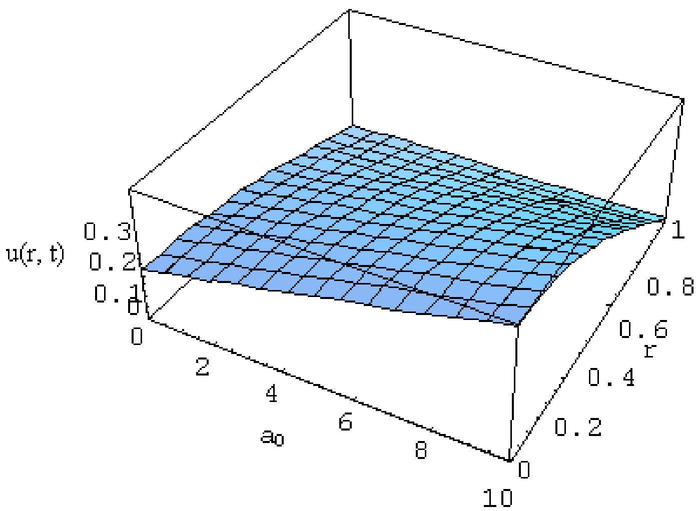

Figure 4. Variation of velocity profile with amplitude of body acceleration a_0; $H = 4, K = 2.5, A_0 = 2, A_1 = 4, \alpha = 1, \overline{\alpha} = 1, t = 0.5, b = 0.5, \phi = 15°$.

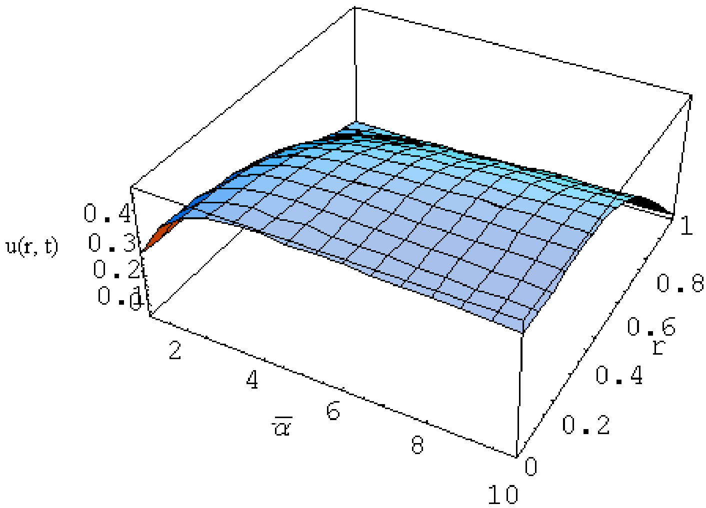

Figure 5. Variation of velocity profile for different values of couple stress parameter $\overline{\alpha}$; $H = 4, K = 2.5, A_0 = 2, A_1 = 4, a_0 = 3, \alpha = 1, t = 0.5, b = 0.5, \phi = 15°$.

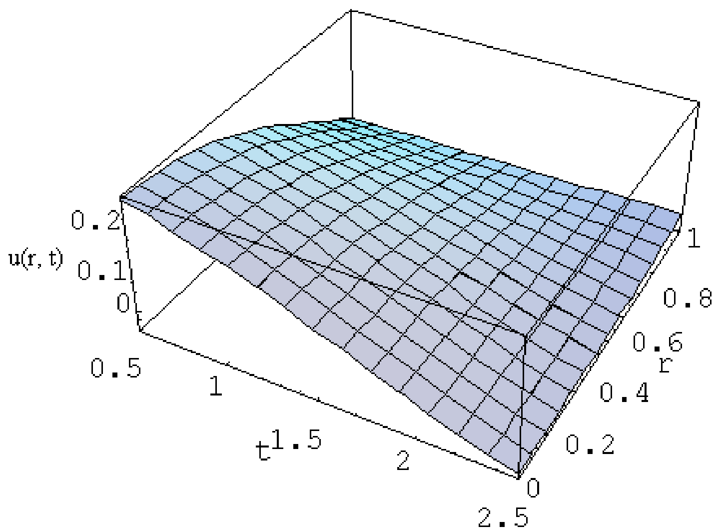

Figure 6. Variation of velocity profile for different values of time t; $H = 4, K = 2.5, A_0 = 2, A_1 = 4, a_0 = 3, \alpha = 1, \overline{\alpha} = 1, b = 0.5, \phi = 15°$.

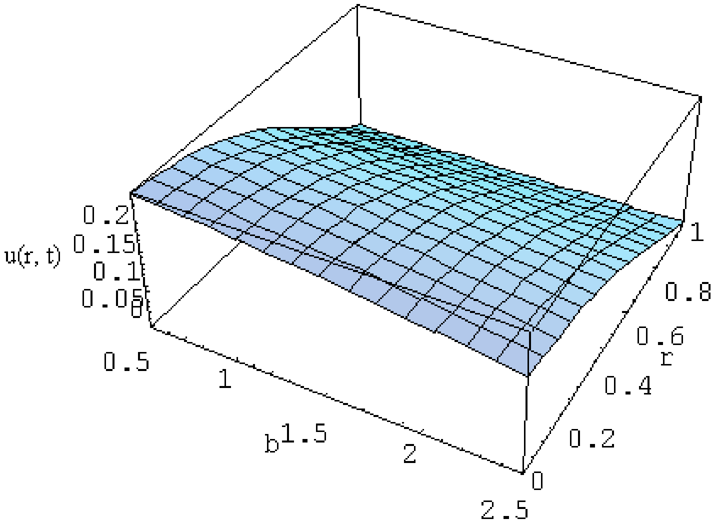

Figure 7. Variation of velocity profile for different values of b; $H = 4, K = 2.5, A_0 = 2, A_1 = 4, a_0 = 3, \alpha = 1, \overline{\alpha} = 1, t = 0.5, \phi = 15°$.

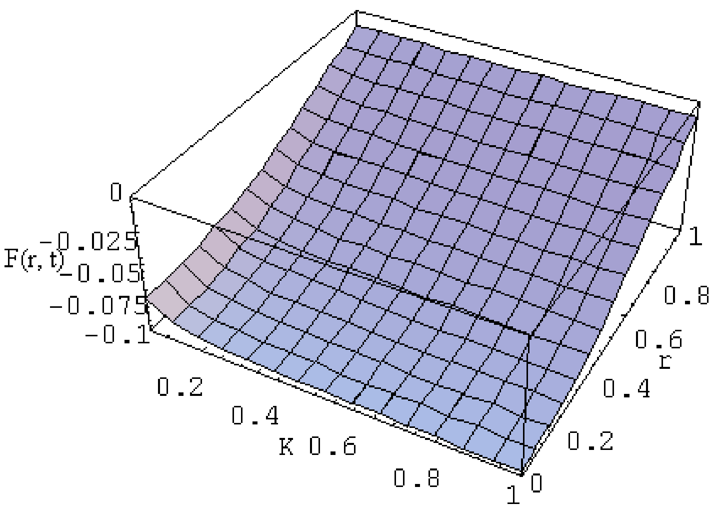

Figure 8. Variation of fluid acceleration for different values of permeability parameter K; $H = 4, A_0 = 2, A_1 = 4, a_0 = 3, \alpha = 1, \overline{\alpha} = 1, t = 0.5, b = 0.5, \phi = 15°$.

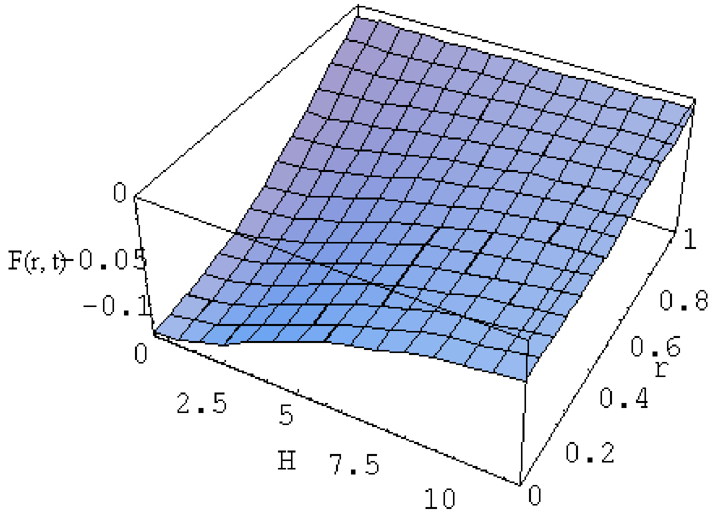

Figure 9. Variation of fluid acceleration for different values of Hartmann number H; $K = 2.5, A_0 = 2, A_1 = 4, a_0 = 3, \alpha = 1, \overline{\alpha} = 1, t = 0.5, b = 0.5, \phi = 15°$.

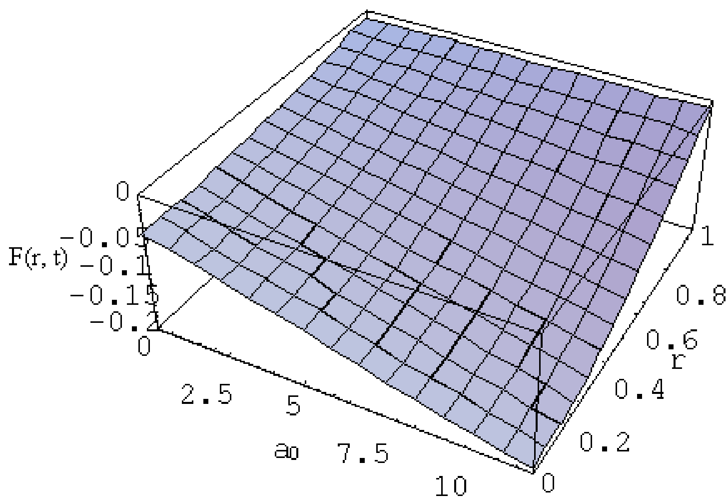

Figure 10. Variation of fluid acceleration with amplitude of body acceleration a_0; $H = 4, K = 2.5, A_0 = 2, A_1 = 4, \alpha = 1, \overline{\alpha} = 1, t = 0.5, b = 0.5, \phi = 15°$.

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