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Weighted Composition Operator from Bers-Type Space to Bloch-Type Space on the Unit Ball

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Abstract. In this paper, we characterize the boundedness and compactness of weighted composition operator from Bers-type space to Bloch-type space on the unit ball of $\mathbb{C}^n$.

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1. Introduction

Let $H(B_n)$ be the class of all holomorphic functions on B_n and $S(B_n)$ the collection of all the holomorphic self-maps of B_n, where B_n is the unit ball in the n-dimensional complex space $\mathbb{C}^n$. The Bloch space $\mathscr{B}$ (see, e.g. [22]) is defined as the space of holomorphic functions such that

$$\|f\|_{\mathscr{B}} = \sup\left\{(1-|z|^2)|\Re f(z)| : z \in B_n\right\} < \infty.$$

For each $\alpha > 0$, we define a weighted-type spaces H_α^∞ (see, e.g. [11]) as follows:

$$H_\alpha^\infty = \left\{ f \in H(B_n) : \sup_{0<r<1} (1-r^2)^\alpha M_\infty(f,r) < \infty \right\},$$

where $M_\infty(f,r) = \sup_{|z|=r} |f(z)|$. It is easy to see that $f \in H_\alpha^\infty$ if and only if $\sup_{z\in B_n}(1-|z|^2)^\alpha |f(z)| < \infty$, so we define the norm

$$\|f\|_{H_\alpha^\infty} = \sup_{z\in B_n} (1-|z|^2)^\alpha |f(z)|$$

and H_α^∞ with this norm is a Banach space. It is sometimes called Bers-type space which is a special case of the weighted-type space H_μ^∞ (see, e.g. [4]). When $\alpha = 0$, the space H_α^∞ is just H^∞ (see, e.g. [8, 9, 18]), which is defined by

$$H^\infty = \left\{ f \in H(B_n) : \|f\|_\infty = \sup_{z\in B_n} |f(z)| < \infty \right\}.$$

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A positive continuous function μ on $[0,1)$ is called normal [12], if there exist three constants a,b $(0<a<b<\infty)$, and δ $(0\le\delta<1)$, such that for $r\in[\delta,1)$

$$\frac{\mu(r)}{(1-r)^a}\downarrow 0,\quad \frac{\mu(r)}{(1-r)^b}\uparrow\infty$$

as $r\to 1$. In the rest of this paper we always assume that μ is normal on $[0,1)$, and from now on if we say that a function $\mu:\mathbb{B}\to[0,\infty)$ is normal we will also assume that it is radial on B_n, that is, $\mu(z)=\mu(|z|), z\in B_n$.

Now $f\in H(B_n)$ is said to belong to Bloch-type space $\mathscr{B}_\mu$ (see, e.g. [10, 14]), if

$$\|f\|_{\mu,1}=\sup_{z\in B_n}\mu(z)|\nabla f(z)|<\infty,$$

where $\nabla f(z)=\big(\partial f/\partial z_1(z),...,\partial f/\partial z_n(z)\big)$ is the complex gradient of f. It is clear that $\mathscr{B}_\mu$ is a Banach space with norm $\|f\|_{\mathscr{B}_\mu}=|f(0)|+\|f\|_{\mu,1}$. For $f\in H(B_n)$, we denote $\|f\|_{\mu,2}=\sup_{z\in B_n}\mu(z)|\Re f(z)|$ and $\|f\|_{\mu,3}=\sup_{z\in B_n}Q_f^\mu(z)$, where

$$\Re f(z)=\langle\nabla f(z),\bar z\rangle=\sum_{j=1}^n z_j\frac{\partial f}{\partial z_j}(z),\quad Q_f^\mu(z)=\sup_{u\in\mathbb{C}^n\setminus\{0\}}\frac{|\langle\nabla f(z),\bar u\rangle|}{\sqrt{G_z^\mu(u,u)}},$$

$$G_z^\mu(u,u)=\frac{1}{\mu^2(z)}\left\{\frac{\mu^2(z)}{\sigma_\mu^2(|z|)}|u|^2+(1-\frac{\mu^2(z)}{\sigma_\mu^2(|z|)})\frac{|\langle z,u\rangle|^2}{|z|^2}\right\}\quad(z\neq 0),$$

$$G_0^\mu(u,u)=\frac{|u|^2}{\mu^2(0)}\quad\text{and}\quad\frac{1}{\sigma_\mu(t)}=\frac{1}{\mu(0)}+\int_0^t\frac{d\tau}{(1-\tau)^{1/2}\mu(\tau)}\quad(0\le t<1).$$

It was proved that $\|f\|_{\mu,1},\|f\|_{\mu,2}$ and $\|f\|_{\mu,3}$ are equivalent for $f\in\mathscr{B}_\mu$ in [1] and [21].

Let $\varphi\in S(B_n)$ and $\psi\in H(B_n)$. The weighted composition operator $T_{\psi,\varphi}$ is defined by

$$T_{\psi,\varphi}(f)=\psi f\circ\varphi,\quad f\in H(B_n).$$

We can regard this operator as a generalization of a multiplication operator M_ψ and a composition operator C_φ (see, e.g. [1, 2, 5, 6, 15, 25, 26]). That is when $\varphi(z)\equiv z$ we obtain $T_{\psi,\varphi}f(z)=M_\psi f(z)=\psi(z)f(z)$ and when $\psi(z)\equiv 1$ we obtain $T_{\psi,\varphi}f(z)=C_\varphi f(z)=f(\varphi(z))$.

Recently, Stević characterized the boundedness and compactness of the weighted composition operators between mixed-norm spaces and H_α^∞ spaces in the unit ball in [7]. Moreover, Zhang and coauthor discussed the conditions for which the weighted composition operator is bounded or compact from Bergman space to μ-Bloch space in [19] and [21]. Zhou and Chen discussed weighted composition operators from $F(p,q,s)$ to Bloch type spaces on the unit ball in [20]. For some recent related results, see also [13, 16, 17, 23, 24] and the references therein. Now in this article, we give some necessary and sufficient conditions for the weighted composition operator $T_{\psi,\varphi}$ to be bounded and compact from weighted-type spaces H_α^∞ to Bloch-type space $\mathscr{B}_\mu$ on the unit ball of $\mathbb{C}^n$.

Throughout the remainder of this paper, C will denote a positive constant, the exact value of which may vary from one appearance to the next. The notation $A\asymp B$ means that there is a positive constant C such that $B/C\le A\le CB$. The symbol $\mathbb{N}$ stands for the set of positive integers.

2. Some lemmas

To begin the discussion, let us state a couple of lemmas which will be used in the proof of the main results. The following lemma was proved in [3].

Lemma 2.1. *Let $\alpha > 0$ and m be a positive integer. Then for every $f \in H(B_n)$ it holds*

$$\sup_{0<r<1}(1-r^2)^{\alpha}M_\infty(f,r) \asymp |f(0)| + \sup_{0<r<1}(1-r^2)^{\alpha+m}M_\infty(\Re^m f,r).$$

Lemma 2.2. *Let $\alpha > 0$. Then for every $f \in H(B_n)$ it holds*

$$|\Re f(z)| \le C\frac{\|f\|_{H^\infty_\alpha}}{(1-|z|^2)^{\alpha+1}}.$$

Proof. Using Lemma 2.1 with $m = 1$ we obtain

$$\|f\|_{H^\infty_\alpha} = \sup_{0<r<1}(1-r^2)^{\alpha}M_\infty(f,r) \ge C\sup_{0<r<1}(1-r^2)^{\alpha+1}M_\infty(\Re f,r) \ge C(1-|z|^2)^{\alpha+1}|\Re f(z)|.$$

From which the desired estimate follows. ∎

From Lemma 2.2 we can easily obtain $f \in \mathscr{B}^{\alpha+1}$ and $\|f\|_{\mathscr{B}^{\alpha+1}} \le C\|f\|_{H^\infty_\alpha}$ for $f \in H^\infty_\alpha$. For $z \in B_n$, $u \in \mathbb{C}^n$, denote

$$H_z(u,u) = \frac{(1-|z|^2)|u|^2 + |\langle z,u\rangle|^2}{(1-|z|^2)^2}.$$

It is well-known that $H_z(u,u)$ is the Bergman metric of B_n (see, e.g. [22]).

Lemma 2.3. *Let $\alpha > 0$, $v(r) = (1-r^2)^{\alpha+1}$ and $\varphi \in S(B_n)$. Then*

$$G^v_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z) \le \frac{CH_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)}{(1-|\varphi(z)|^2)^{2\alpha}}$$

for all $z \in B_n$, where $J\varphi(z)$ denotes the Jacobian matrix of $\varphi(z)$ and

$$J\varphi(z)z = \left(\sum_{k=1}^n \frac{\partial\varphi_1}{\partial z_k}z_k, \ldots, \sum_{k=1}^n \frac{\partial\varphi_n}{\partial z_k}z_k\right)^T.$$

Proof. If $\varphi(z) = 0$, the desired result is obvious. If $\varphi(z) \ne 0$, for the definition of σ_v, we have

$$\frac{1}{\sigma_v(r)} = 1 + \int_0^r \frac{dt}{(1-t)^{1/2}(1-t^2)^{\alpha+1}} \asymp \frac{(1-r^2)^{1/2}}{v(r)}, \quad 0 \le r < 1.$$

Thus,

$$\begin{aligned}
&G^v_{\varphi(z)}(J\varphi(z), J\varphi(z)z)\\
&= \frac{1}{v^2(|\varphi(z)|)}\left[\frac{v^2(|\varphi(z)|)}{\sigma_v^2(|\varphi(z)|)}|J\varphi(z)z|^2 + \left(1 - \frac{v^2(|\varphi(z)|)}{\sigma_v^2(|\varphi(z)|)}\right)\frac{|\langle\varphi(z), J\varphi(z)z\rangle|^2}{|\varphi(z)|^2}\right]\\
&= \frac{1}{v^2(|\varphi(z)|)}\left[\frac{v^2(|\varphi(z)|)}{\sigma_v^2(|\varphi(z)|)}\left(|J\varphi(z)z|^2 - \frac{|\langle\varphi(z), J\varphi(z)z\rangle|^2}{|\varphi(z)|^2}\right) + \frac{|\langle\varphi(z), J\varphi(z)z\rangle|^2}{|\varphi(z)|^2}\right]\\
&\le \frac{C}{v^2(|\varphi(z)|)}\left[(1-|\varphi(z)|^2)\left(|J\varphi(z)z|^2 - \frac{|\langle\varphi(z), J\varphi(z)z\rangle|^2}{|\varphi(z)|^2}\right) + \frac{|\langle\varphi(z), J\varphi(z)z\rangle|^2}{|\varphi(z)|^2}\right]\\
&= \frac{C}{v^2(|\varphi(z)|)}\left[(1-|\varphi(z)|^2)(|J\varphi(z)z|^2 + |\langle\varphi(z), J\varphi(z)z\rangle|^2\right]
\end{aligned}$$

$$= \frac{C(1-|\varphi(z)|^2)^2}{v^2(|\varphi(z)|)} H_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z) = C\frac{H_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)}{(1-|\varphi(z)|^2)^{2\alpha}}.$$

From which the desired result follows. ∎

Lemma 2.4. *Assume that $f \in H(B_n)$ and $\varphi \in S(B_n)$. Then*

$$\Re(f\circ\varphi)(z) = \langle \nabla f(\varphi(z)), \overline{J\varphi(z)z}\rangle.$$

Proof.

$$\Re(f\circ\varphi)(z) = \sum_{i=1}^{n} z_i \frac{\partial(f\circ\varphi)}{\partial z_i} = \sum_{i=1}^{n} z_i \sum_{j=1}^{n} \frac{\partial(f\circ\varphi)}{\partial w_j}\frac{\partial \varphi_j}{\partial z_i}$$
$$= \sum_{j=1}^{n} \frac{\partial(f\circ\varphi)}{\partial w_j} \sum_{i=1}^{n} z_i \frac{\partial \varphi_j}{\partial z_i} = \langle \nabla f(\varphi(z)), \overline{J\varphi(z)z}\rangle.$$ ∎

By Montel theorem and the definition of compact operator, the following lemma follows. The interested reader can also see the Lemma 2.1 in [5]. Hence we omit it.

Lemma 2.5. *Assume that $0<\alpha<\infty$, μ is a normal function on $[0,1)$, $\varphi \in S(B_n)$ and $\psi \in H(B_n)$. Then $T_{\psi,\varphi}: H_\alpha^\infty \to \mathscr{B}_\mu$ is compact if and only if for any bounded sequence $\{f_k\}_{k\in\mathbb{N}} \in H_\alpha^\infty$ which converges to zero uniformly on compact subsets of B_n as $k\to\infty$, we have $\|T_{\psi,\varphi} f_k\|_{\mathscr{B}_\mu} \to 0$ as $k \to \infty$.*

3. The boundedness and compactness of $T_{\psi,\varphi}: H_\alpha^\infty \to \mathscr{B}_\mu$.

In this section we characterize the boundedness and compactness of the operator $T_{\psi,\varphi}: H_\alpha^\infty \to \mathscr{B}_\mu$.

Theorem 3.1. *Suppose that $0<\alpha<\infty$, μ is a normal function on $[0,1)$, $\varphi\in S(B_n)$ and $\psi \in H(B_n)$. Then $T_{\psi,\varphi}: H_\alpha^\infty \to \mathscr{B}_\mu$ is bounded if and only if*

$$M_1 := \sup_{z\in B_n} \frac{\mu(z)|\Re\psi(z)|}{(1-|\varphi(z)|^2)^\alpha} < \infty \tag{3.1}$$

and

$$M_2 := \sup_{z\in B_n} \frac{\mu(z)|\psi(z)|}{(1-|\varphi(z)|^2)^\alpha} \left\{H_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)\right\}^{1/2} < \infty. \tag{3.2}$$

Proof. Assume that (3.1) and (3.2) hold. Then for any $f \in H_\alpha^\infty$, if $J\varphi(z)z \neq 0$, for $z\in B_n$. By Lemma 2.4, Lemma 2.3 and Lemma 2.2 we have

$$\begin{aligned}
&\mu(z)|\Re(T_{\psi,\varphi}f)(z)| \\
&\le \mu(z)|\Re\psi(z)||f(\varphi(z))| + \mu(z)|\psi(z)||\Re(f\circ\varphi)(z)| \\
&\le \frac{\mu(z)|\Re\psi(z)|\|f\|_{H_\alpha^\infty}}{(1-|\varphi(z)|^2)^\alpha} + \mu(z)|\psi(z)||\langle \nabla f(\varphi(z)), \overline{J\varphi(z)z}\rangle| \\
&\le M_1\|f\|_{H_\alpha^\infty} + \frac{C\mu(z)|\psi(z)|\{H_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)\}^{1/2}|\langle \nabla f(\varphi(z)), \overline{J\varphi(z)z}\rangle|}{(1-|\varphi(z)|^2)^\alpha\sqrt{G^v_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)}} \\
&\le M_1\|f\|_{H_\alpha^\infty} + CM_2\|f\|_{\mathscr{B}_{(1-r^2)^{\alpha+1}}} \le C\|f\|_{H_\alpha^\infty}.
\end{aligned} \tag{3.3}$$

When $z \in B_n$ and $J\varphi(z)z = 0$, from (3.1) we can easily obtain that

$$\mu(z)|\Re(T_{\psi,\varphi}(f))(z)| \leq M_1\|f\|_{H^\infty_\alpha}. \tag{3.4}$$

Combining (3.3) with (3.4) it follows that

$$\|T_{\psi,\varphi}f\|_{\mathscr{B}_\mu} = \sup_{z\in B_n}\mu(z)|\Re(T_{\psi,\varphi}f)(z)| \leq C\|f\|_{H^\infty_\alpha}.$$

From which the boundedness of $T_{\psi,\varphi} : H^\infty_\alpha \to \mathscr{B}_\mu$ follows.

For the converse direction, we suppose that $T_{\psi,\varphi} : H^\infty_\alpha \to \mathscr{B}_\mu$ is bounded. First, we assume that $w \in B_n$ and $\varphi(w) = r_w e_1$, where $r_w = |\varphi(w)|$ and e_1 is the vector $(1,0,0,...,0)$. If $\sqrt{(1-r_w^2)(|\eta_2|^2+....+|\eta_n|^2)} \leq |\eta_1|$, where $J\varphi(w)w = (\eta_1,...,\eta_n)^T$. We consider the function

$$f_w(z) = \left\{\frac{1-r_w^2}{(1-r_wz_1)^2}\right\}^\alpha \frac{z_1-r_w}{1-r_wz_1}.$$

Then

$$\sup_{z\in B_n}(1-|z|^2)^\alpha|f_w(z)| \leq \sup_{z\in B_n}\frac{(1-|z_1|^2)^\alpha}{(1-|z_1|)^\alpha}\left\{\frac{1-r_w^2}{1-r_w}\right\}^\alpha \leq 4^\alpha.$$

It shows that $f_w \in H^\infty_\alpha$ and $\|f_w\|_{H^\infty_\alpha} \leq C$. Note that $f_w(\varphi(w)) = 0$ and

$$\nabla f_w(\varphi(w)) = \left(\frac{1}{(1-r_w^2)^{\alpha+1}},0,...,0\right).$$

It follows from Lemma 2.4 that

$$\begin{aligned}\|T_{\psi,\varphi}f_w\|_{\mathscr{B}_\mu} &\geq \mu(w)|\Re(\psi f_w\circ\varphi)(w)| \\ &\geq \mu(w)|\psi(w)||\Re(f_w\circ\varphi)(w)| - \mu(w)|\Re\psi(w)||f_w(\varphi(w))| \\ &= \mu(w)|\psi(w)||\langle\nabla f_w(\varphi(w)),\overline{J\varphi(w)w}\rangle| \\ &= \frac{\mu(w)|\psi(w)||\eta_1|}{(1-r_w^2)^{\alpha+1}}. \end{aligned} \tag{3.5}$$

By the definition of $H_{\varphi(w)}(J\varphi(w)w,J\varphi(w)w)$ and (3.5), it follows that

$$\begin{aligned}&\frac{\mu(w)|\psi(w)|}{(1-|\varphi(w)|^2)^\alpha}\{H_{\varphi(w)}(J\varphi(w)w,J\varphi(w)w)\}^{1/2} \\ &= \frac{\mu(w)|\psi(w)|\{(1-|\varphi(w)|^2)|J\varphi(w)w|^2+|\langle\varphi(w),J\varphi(w)w\rangle|^2\}^{1/2}}{(1-|\varphi(w)|^2)^{\alpha+1}} \\ &= \frac{\mu(w)|\psi(w)|\{(1-r_w^2)(|\eta_2|^2+...+|\eta_n|^2)+|\eta_1|^2\}^{1/2}}{(1-|\varphi(w)|^2)^{\alpha+1}} \\ &\leq \frac{\sqrt{2}\mu(w)|\psi(w)||\eta_1|}{(1-r_w^2)^{\alpha+1}} \leq C\|T_{\psi,\varphi}f_w\|_{\mathscr{B}_\mu}. \end{aligned} \tag{3.6}$$

This shows that when $\sqrt{(1-r_w^2)(|\eta_2|^2+....+|\eta_n|^2)} \leq |\eta_1|$, the result (3.2) holds.

On the other hand, if $\sqrt{(1-r_w^2)(|\eta_2|^2+....+|\eta_n|^2)} > |\eta_1|$. For $j = 2,...,n$, let $\theta_j = \arg\eta_j$ and $a_j = e^{-i\theta_j}$, when $\eta_j \neq 0$ or $a_j = 0$ when $\eta_j = 0$. Taking

$$f_w(z) = \frac{a_2z_2+...+a_nz_n}{(1-r_wz_1)^{\alpha+1}}.$$

It is easy to check that

$$\sup_{z\in B_n}(1-|z|^2)^{\alpha}|f_w(z)| \le \sup_{z\in B_n}(1-|z|^2)^{\alpha}\frac{n-1}{(1-|z_1|)^{\alpha}(1-r_w)} \le C,$$

which implies that $f_w \in H_\alpha^\infty$ and $\|f_w\|_{H_\alpha^\infty} \le C$. Notice that $f_w(\varphi(w)) = 0$ and

$$\nabla f_w(\varphi(w)) = \left(0, \frac{a_2}{(1-r_w^2)^{\alpha+1}}, \dots, \frac{a_n}{(1-r_w^2)^{\alpha+1}}\right).$$

Similar to the proof of (3.5), we obtain that

$$\frac{\mu(w)|\psi(w)|(|\eta_2|+\dots+|\eta_n|)}{(1-r_w^2)^{\alpha+1}} \le C\|T_{\psi,\varphi}f_w\|_{\mathscr{B}\mu}. \tag{3.7}$$

It follows from (3.7) that

$$\begin{aligned}
&\frac{\mu(w)|\psi(w)|}{(1-|\varphi(w)|^2)^{\alpha}}\{H_{\varphi(w)}(J\varphi(w)w, J\varphi(w)w)\}^{1/2}\\
&= \frac{\mu(w)|\psi(w)|\{(1-|\varphi(w)|^2)|J\varphi(w)w|^2 + |\langle\varphi(w), J\varphi(w)w\rangle|^2\}^{1/2}}{(1-|\varphi(w)|^2)^{\alpha+1}}\\
&= \frac{\mu(w)|\psi(w)|\{(1-r_w^2)(|\eta_2|^2+\dots+|\eta_n|^2) + |\eta_1|^2\}^{1/2}}{(1-|\varphi(w)|^2)^{\alpha+1}}\\
&\le \frac{\mu(w)|\psi(w)|\{2(1-r_w^2)(|\eta_2|^2+\dots+|\eta_n|^2)\}^{1/2}}{(1-|\varphi(w)|^2)^{\alpha+1}}\\
&\le C\frac{\mu(w)|\psi(w)|\sqrt{2(1-r_w^2)}(|\eta_2|+\dots+|\eta_n|)}{(1-r_w^2)^{\alpha+1}} \le C\|T_{\psi,\varphi}f_w\|_{\mathscr{B}\mu}. \qquad (3.8)
\end{aligned}$$

Therefore, when $\sqrt{(1-r_w^2)(|\eta_2|^2+\dots.+|\eta_n|^2)} > |\eta_1|$, we can also obtain (3.2). Combining the two cases we know that (3.2) holds.

For the general situation, we can use the unitary transform U_w to make $\varphi(w) = r_w e_1 U_w$. In order to prove (3.2), we first give a proposition.

Proposition 3.1. *Suppose that* $0 < \alpha < \infty, \mu$ *is a normal function on* $[0,1)$, $\varphi \in S(B_n)$ *and* $\psi \in H(B_n)$. *Let* $\widetilde{\varphi}(z) = U_w\varphi(z)$, *and* $g = f \circ U_w^{-1}$ *for any* $f \in H_\alpha^\infty$. *Then*

(a) $H_{\widetilde{\varphi}(z)}(J\widetilde{\varphi}(z)z, J\widetilde{\varphi}(z)z) = H_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)$;

(b) $\|g\|_{H_\alpha^\infty} = \|f\|_{H_\alpha^\infty}$;

(c) $\|T_{\psi,\widetilde{\varphi}}g\|_{\mathscr{B}\mu} = \|T_{\psi,\varphi}f\|_{\mathscr{B}\mu}$.

Proof.

(a) Note that $J\widetilde{\varphi}(z)z = U_w J(\varphi)(z)z$ and $|\widetilde{\varphi}(z)|^2 = |\varphi(z)|^2$, we have

$$\begin{aligned}
H_{\widetilde{\varphi}(z)}(J\widetilde{\varphi}(z)z, J\widetilde{\varphi}(z)z) &= \frac{\left(1-\left|\widetilde{\varphi}(z)\right|^2\right)\left|J\widetilde{\varphi}(z)z\right|^2 + \left|\left\langle\widetilde{\varphi}(z), J\widetilde{\varphi}(z)z\right\rangle\right|^2}{(1-|\widetilde{\varphi}(z)|^2)^2}\\
&= \frac{(1-|\varphi(z)|^2)|J\varphi(z)z|^2 + |\langle\varphi(z), J\varphi(z)z\rangle|^2}{(1-|\varphi(z)|^2)^2}\\
&= H_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z).
\end{aligned}$$

(b)

$$\|g\|_{H_\alpha^\infty} = \sup_{z\in B_n} \left(1-|z|^2\right)^\alpha |g(z)| = \sup_{z\in B_n} \left(1-|z|^2\right)^\alpha |f(U_w^{-1}(z))|$$
$$= \sup_{z\in B_n} \left(1-|z|^2\right)^\alpha |f(z)| = \|f\|_{H_\alpha^\infty}.$$

In the last equality, we use the linear coordinate translation $w = U_w^{-1}z$ and $|w| = |U_w^{-1}z| = |z|$.

(c)

$$\|T_{\psi,\widetilde{\varphi}}g\|_{\mathscr{B}\mu} = \sup_{z\in B_n} \mu(z)|\psi(z)||g(\widetilde{\varphi}(z))| = \sup_{z\in B_n} \mu(z)|\psi(z)||f(\varphi(z))| = \|T_{\psi,\varphi}f\|_{\mathscr{B}\mu} ∎$$

Now we return to prove that (3.2) holds in general situation. In fact, taking the function $g_w = f_w \circ U_w^{-1}$. By Proposition 3.1, (3.6) and (3.8), it follows that

$$\frac{\mu(w)|\psi(w)|}{(1-|\varphi(w)|^2)^\alpha}\left\{H_{\varphi(w)}(J\varphi(w)w, J\varphi(w)w)\right\}^{1/2}$$
$$= \frac{\mu(w)|\psi(w)|}{(1-|\widetilde{\varphi}(w)|^2)^\alpha}\left\{H_{\widetilde{\varphi}(w)}(J\widetilde{\varphi}(w)w, J\widetilde{\varphi}(w)w)\right\}^{1/2}$$
$$\le C\|T_{\psi,\widetilde{\varphi}}g_w\|_{\mathscr{B}\mu} = C\|T_{\psi,\varphi}f_w\|_{\mathscr{B}\mu} \le C.$$

This means that (3.2) holds.

Next we prove (3.1). Set the function

$$h_w(z) = \left\{\frac{1-|\varphi(w)|^2}{(1-\langle z,\varphi(w)\rangle)^2}\right\}^\alpha, \quad w\in B_n.$$

Since

$$\sup_{z\in B_n}(1-|z|^2)^\alpha|h_w(z)| \le \sup_{z\in B_n}\frac{(1-|z|^2)^\alpha}{(1-|z|)^\alpha}\left(\frac{1-|\varphi(w)|^2}{1-|\varphi(w)|}\right)^\alpha \le 4^\alpha,$$

it follows that $h_w \in H_\alpha^\infty$ and $\|h_w\|_{H_\alpha^\infty} \le C$. Moreover $h_w(\varphi(w)) = 1/((1-|\varphi(w)|^2)^\alpha)$ and

$$\nabla h_w(\varphi(w)) = 2\alpha\left(\frac{\overline{\varphi_1(w)}}{(1-|\varphi(w)|^2)^{\alpha+1}},\ldots,\frac{\overline{\varphi_n(w)}}{(1-|\varphi(w)|^2)^{\alpha+1}}\right).$$

Thus

$$\|T_{\psi,\varphi}(h_w)\|_{\mathscr{B}_\mu} \ge \mu(w)|\Re(\psi h_w\circ\varphi)(w)|$$
$$= \mu(w)|\Re\psi(w)h_w(\varphi(w)) + \psi(w)\Re(h_w\circ\varphi)(w)|$$
$$\ge \frac{\mu(w)|\Re\psi(w)|}{(1-|\varphi(w)|^2)^\alpha} - \mu(w)|\psi(w)||\Re(h_w\circ\varphi)(w)|. \tag{3.9}$$

From (3.2) and $\nabla h_w(\varphi(w))$ we have

$$\mu(w)|\psi(w)||\Re(h_w\circ\varphi)(w)| = \mu(w)|\psi(w)||\langle\nabla h_w(\varphi(w)), \overline{J\varphi(w)w}\rangle|$$
$$= \frac{2\alpha\mu(w)|\psi(w)||\langle\varphi(w), J\varphi(w)w\rangle|}{(1-|\varphi(w)|^2)^{\alpha+1}}$$
$$\le \frac{2\alpha\mu(w)|\psi(w)|}{(1-|\varphi(w)|^2)^\alpha}\left\{H_{\varphi(w)}(J\varphi(w)w, J\varphi(w)w)\right\}^{1/2}$$
$$\le CM_2 < \infty. \tag{3.10}$$

Combining (3.9) and (3.10) we obtain (3.1). This completes the proof of Theorem 3.1. ∎

Theorem 3.2. *Suppose that* $0 < \alpha < \infty$, μ *is a normal function on* $[0,1)$, $\varphi \in S(B_n)$ *and* $\psi \in H(B_n)$. *Then* $T_{\psi,\varphi} : H_\alpha^\infty \to \mathscr{B}_\mu$ *is compact if and only if the followings are all satisfied:*

(a) $\psi \in \mathscr{B}_\mu$ *and* $\psi\varphi_l \in \mathscr{B}_\mu$ *for* $l \in \{1,...,n\}$;

(b)

$$\lim_{|\varphi(z)|\to 1} \frac{\mu(z)|\Re\psi(z)|}{(1-|\varphi(z)|^2)^\alpha} = 0; \tag{3.11}$$

(c)

$$\lim_{|\varphi(z)|\to 1} \frac{\mu(z)|\psi(z)|}{(1-|\varphi(z)|^2)^\alpha}\{H_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)\}^{1/2} = 0. \tag{3.12}$$

Proof. First suppose that (a), (b) and (c) hold. Then from (b) and (c) we have for any $\varepsilon > 0$, there is a $\delta > 0$, such that

$$\frac{\mu(z)|\Re\psi(z)|}{(1-|\varphi(z)|^2)^\alpha} < \varepsilon \tag{3.13}$$

and

$$\frac{\mu(z)|\psi(z)|}{(1-|\varphi(z)|^2)^\alpha}\{H_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)\}^{1/2} < \varepsilon, \tag{3.14}$$

when $|\varphi(z)| > \delta$. Let $\{f_k\}_{k\in\mathbb{N}}$ be any sequence which converges to 0 uniformly on compact subsets of B_n satisfying $\|f_k\|_{H_\alpha^\infty} \le 1$. Then f_k and ∇f_k converge to 0 uniformly on $K = \{w \in B_n : |w| \le \delta\}$. Since

$$\begin{aligned}\sup_{z\in B_n} \mu(z)|\Re(T_{\psi,\varphi}f_k)(z)| = &\sup_{\varphi(z)\in K} \mu(z)|\Re(T_{\psi,\varphi}f_k)(z)| \\ &+ \sup_{\varphi(z)\in B_n\setminus K} \mu(z)|\Re(T_{\psi,\varphi}f_k)(z)|.\end{aligned} \tag{3.15}$$

If $|\varphi(z)| > \delta$ and $J\varphi(z)z \neq 0$, by Lemma 2.4, Lemma 2.3 and Lemma 2.2 we have

$$\begin{aligned}\mu(z)|\Re(T_{\psi,\varphi}f_k)(z)| &\le \mu(z)|\psi(z)||\Re(f_k\circ\varphi)(z)| + \mu(z)|\Re\psi(z)||f_k(\varphi(z))| \\ &\le \frac{C\mu(z)|\psi(z)|\{H_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)\}^{1/2}|\langle\nabla f_k(\varphi(z)), \overline{J\varphi(z)z}\rangle|}{(1-|\varphi(z)|^2)^\alpha\sqrt{G^v_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)}} + \varepsilon\|f_k\|_{H_\alpha^\infty} \\ &\le C\varepsilon\|f_k\|_{\mathscr{B}_{(1-r^2)^{\alpha+1}}} + \varepsilon\|f_k\|_{H_\alpha^\infty} \le C\varepsilon.\end{aligned} \tag{3.16}$$

When $J\varphi(z)z = 0$,

$$\mu(z)|\Re(T_{\psi,\varphi}f_k)(z)| \le \varepsilon\|f_k\|_{H_\alpha^\infty} \le \varepsilon. \tag{3.17}$$

Combining (3.16) and (3.17) we obtain that

$$\sup_{\varphi(z)\in B_n\setminus K} \mu(z)|\Re(T_{\psi,\varphi}f_k)(z)| \le C\varepsilon. \tag{3.18}$$

If $|\varphi(z)| \le \delta$, it follows from (a) that

$$\begin{aligned}&\mu(z)|\Re(T_{\psi,\varphi}f_k)(z)| \\ &\le \mu(z)|\psi(z)||\Re(f_k\circ\varphi)(z)| + \mu(z)|\Re\psi(z)||f_k(\varphi(z))| \\ &\le \mu(z)|\psi(z)||\langle\nabla f_k(\varphi(z)), \overline{J\varphi(z)z}\rangle| + |f_k(\varphi(z))|\|\psi\|_{\mathscr{B}_\mu}\end{aligned}$$

$$\leq |\nabla f_k(\varphi(z))| \sum_{l=1}^{n} \Big(\mu(z)|\psi(z)||\Re\varphi_l(z)|\Big) + |f_k(\varphi(z))|\|\psi\|_{\mathscr{B}_\mu}$$
$$\leq |\nabla f_k(\varphi(z))| \sum_{l=1}^{n} \Big(\mu(z)|\psi(z)||\Re\varphi_l(z)| - \mu(z)|\Re\psi(z)||\varphi_l(z)| + \mu(z)|\Re\psi(z)|\Big)$$
$$+ |f_k(\varphi(z))|\|\psi\|_{\mathscr{B}_\mu}$$
$$\leq |\nabla f_k(\varphi(z))| \sum_{l=1}^{n} \Big(\mu(z)|\psi(z)\Re\varphi_l(z) + \Re\psi(z)\varphi_l(z)| + \mu(z)|\Re\psi(z)|\Big)$$
$$+ |f_k(\varphi(z))|\|\psi\|_{\mathscr{B}_\mu}$$
$$\leq |\nabla f_k(\varphi(z))| \sum_{l=1}^{n} \Big(\|\psi\varphi_l\|_{\mathscr{B}_\mu} + \|\psi\|_{\mathscr{B}_\mu}\Big) + |f_k(\varphi(z))|\|\psi\|_{\mathscr{B}_\mu} \to 0, \quad k \to \infty. \tag{3.19}$$

Then from (3.15), (3.18), (3.19) and Lemma 2.5 we get the compactness of $T_{\psi,\varphi} : H_\alpha^\infty \to \mathscr{B}_\mu$.

For the converse direction, we assume that $T_{\psi,\varphi} : H_\alpha^\infty \to \mathscr{B}_\mu$ is compact. Then the boundedness of $T_{\psi,\varphi} : H_\alpha^\infty \to \mathscr{B}_\mu$ is obvious. Taking $f(z) = 1 \in H_\alpha^\infty$, we obtain

$$\begin{aligned}\|T_{\psi,\varphi}f\|_{\mathscr{B}_\mu} &= \sup_{z\in B_n} \mu(z)|\Re(T_{\psi,\varphi}f)(z)| \\ &= \sup_{z\in B_n} \mu(z)|\Re\psi(z)f(\varphi(z)) + \psi(z)\Re(f\circ\varphi)(z)| = \sup_{z\in B_n} \mu(z)|\Re\psi(z)| < \infty.\end{aligned}$$

This shows that $\psi \in \mathscr{B}_\mu$.

On the other hand, for $l \in \{1,...,n\}$, taking the functions $f(z) = z_l \in H_\alpha^\infty$, we can obtain

$$\begin{aligned}\|T_{\psi,\varphi}f\|_{\mathscr{B}_\mu} &= \sup_{z\in B_n} \mu(z)|\Re\psi(z)f(\varphi(z)) + \psi(z)\Re(f\circ\varphi)(z)| \\ &= \sup_{z\in B_n} \mu(z)|\Re\psi(z)\varphi_l(z) + \psi(z)\Re\varphi_l(z)| = \sup_{z\in B_n} \mu(z)|\Re(\psi\varphi_l)(z)|.\end{aligned}$$

Then we obtain that $\psi\varphi_l \in \mathscr{B}_\mu$ for $l \in \{1,...,n\}$. The desired result (a) follows.

Next we prove (3.12). Let $\{z_k\}_{k\in\mathbb{N}}$ be a sequence in B_n such that $|\varphi(z_k)| \to 1$ as $k \to \infty$ (If such a sequence does not exist then (3.12) obviously holds). We can suppose that $\varphi(z_k) = r_k e_1$, where $r_k = |\varphi(z_k)|$, e_1 is the vector $(1,0,0,...,0)$. Thus $|r_k| \to 1$ as $k \to \infty$.

If $\sqrt{(1-r_k^2)(|\eta_2|^2 + ... + |\eta_n|^2)} \leq |\eta_1|$, where $J\varphi(z_k)z_k = (\eta_1,...,\eta_n)^T$. Defining the function

$$f_k(z) = \left\{\frac{1-r_k^2}{(1-r_k z_1)^2}\right\}^\alpha \frac{z_1 - r_k}{1 - r_k z_1}, \quad k \in \mathbb{N}.$$

From Theorem 3.1 we know that $f_k \in H_\alpha^\infty$ with $\|f_k\|_{H_\alpha^\infty} \leq C$, and notice that f_k converges to 0 uniformly on compact subsets of B_n when $k \to \infty$. From the compactness of $T_{\psi,\varphi} : H_\alpha^\infty \to \mathscr{B}_\mu$, we have that $\lim_{k\to\infty} \|T_{\psi,\varphi}f_k\|_{\mathscr{B}_\mu} = 0$. Then from the similar proof of (3.3) in Theorem 3.1 we have

$$\frac{\mu(z_k)|\psi(z_k)||\eta_1|}{(1-r_k^2)^{\alpha+1}} \leq \|T_{\psi,\varphi}f_k\|_{\mathscr{B}_\mu} \to 0, \quad k \to \infty. \tag{3.20}$$

And by the similar proofs of (3.6) and (3.20) we have

$$\frac{\mu(z_k)|\psi(z_k)|}{(1-|\varphi(z_k)|^2)^\alpha} \left\{H_{\varphi(z_k)}(J\varphi(z_k)z_k, J\varphi(z_k)z_k)\right\}^{1/2}$$

$$\le \frac{\sqrt{2}\mu(z_k)|\psi(z_k)||\eta_1|}{(1-r_k^2)^{\alpha+1}} \to 0, \quad k\to\infty. \tag{3.21}$$

On the other hand, if $\sqrt{(1-r_k^2)(|\eta_2|^2+....+|\eta_n|^2)} > |\eta_1|$. For $j=2,...,n$, let $\theta_j = \arg\eta_j$ and $a_j = e^{-i\theta_j}$, when $\eta_j \neq 0$ or $a_j = 0$ when $\eta_j = 0$. Taking

$$f_k(z) = \frac{(a_2z_2+...+a_nz_n)(1-r_k^2)}{(1-r_kz_1)^{\alpha+2}}.$$

Then from Theorem 3.1 we know $f_k \in H_\alpha^\infty, k\in N$ and f_k converges to 0 uniformly on compact subsets of B_n when $k\to\infty$. Since the compactness of $T_{\psi,\varphi}: H_\alpha^\infty \to \mathscr{B}_\mu$, we have that $\lim_{k\to\infty}\|T_{\psi,\varphi}f_k\|_{\mathscr{B}_\mu} = 0$. We notice that $f_k(\varphi(z_k)) = 0$ and

$$\nabla f_w(\varphi(z_k)) = \left(0, \frac{a_2}{(1-r_k^2)^{\alpha+1}}, ..., \frac{a_n}{(1-r_k^2)^{\alpha+1}}\right)$$

Using the similar proof of (3.7) we obtain

$$\frac{\mu(z_k)|\psi(z_k)|(|\eta_2|+...+|\eta_n|)}{(1-r_k^2)^{\alpha+1}} \le \|T_{\psi,\varphi}f_k\|_{\mathscr{B}_\mu} \to 0, \quad k\to\infty. \tag{3.22}$$

And by using (3.8) we obtain

$$\frac{\mu(z_k)|\psi(z_k)|}{(1-|\varphi(z_k)|^2)^\alpha}\left\{H_{\varphi(z_k)}(J\varphi(z_k)z_k, J\varphi(z_k)z_k)\right\}^{1/2}$$
$$\le C\frac{\mu(z_k)|\psi(z_k)|\sqrt{2(1-r_k^2)}(|\eta_2|+...+|\eta_n|)}{(1-r_k^2)^{\alpha+1}} \to 0, \quad k\to\infty. \tag{3.23}$$

Combining (3.21) and (3.23) we obtain (3.12) under two cases. For the general situation, we can use the unitary transform U_k to make $\varphi(z_k) = r_ke_1U_k$ and we can prove (3.12) by taking the function $g_k = f_k \circ U_k^{-1}$.

Next we prove (3.11). We still let $\{z_k\}_{k\in\mathbb{N}}$ be a sequence in B_n such that $|\varphi(z_k)| \to 1$ as $k\to\infty$ (If such a sequence does not exist then (3.11) obviously holds). Choosing

$$h_k(z) = \left\{\frac{1-|\varphi(z_k)|^2}{(1-\langle z, z_k\rangle)^2}\right\}^\alpha.$$

From Theorem 3.1 we know that $h_k \in H_\alpha^\infty$ and $h_k \to 0$ uniformly on the compact subsets of B_n when $k\to\infty$. By the compactness of $T_{\psi,\varphi}: H_\alpha^\infty \to \mathscr{B}_\mu$ and by the similar proof of (3.9) we obtain that

$$\|T_{\psi,\varphi}(h_k)\|_{\mathscr{B}_\mu} \ge \frac{\mu(z_k)|\Re\psi(z_k)|}{(1-|\varphi(z_k)|^2)^\alpha} - \mu(z_k)|\psi(z_k)||\Re(h_k\circ\varphi)(z_k)|. \tag{3.24}$$

Since from (3.12) we have that

$$\mu(z_k)|\psi(z_k)||\Re(h_w\circ\varphi)(z_k)|$$
$$\le \frac{2\alpha\mu(z_k)|\psi(z_k)|}{(1-|\varphi(z_k)|^2)^\alpha}\left\{H_{\varphi(z_k)}(J\varphi(z_k)z_k, J\varphi(z_k)z_k)\right\}^{1/2} \to 0, \quad k\to\infty. \tag{3.25}$$

Combining (3.24) and (3.25) we obtain (3.11). ∎

Remark 3.1. When $\psi(z)\equiv 1, T_{\psi,\varphi} = C_\varphi$, we obtain the next two Corollaries about composition operator from Theorems 3.1 and 3.2.

Corollary 3.1. *Suppose that* $0 < \alpha < \infty$*,* μ *is a normal function in* $[0,1)$ *and* $\varphi \in S(B_n)$*. Then* $C_\varphi : H_\alpha^\infty \to \mathscr{B}_\mu$ *is bounded if and only if*

$$\sup_{z \in B_n} \frac{\mu(z)\{H_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)\}^{1/2}}{(1-|\varphi(z)|^2)^\alpha} < \infty.$$

Corollary 3.2. *Suppose that* $0 < \alpha < \infty$*,* μ *is a normal function in* $[0,1)$ *and* $\varphi \in S(B_n)$*. Then* $C_\varphi : H_\alpha^\infty \to \mathscr{B}_\mu$ *is compact if and only if*

$$\lim_{|\varphi(z)| \to 1} \frac{\mu(z)\{H_{\varphi(z)}(J\varphi(z)z, J\varphi(z)z)\}^{1/2}}{(1-|\varphi(z)|^2)^\alpha} = 0.$$

and $\varphi_l \in \mathscr{B}_\mu$ *for any* $l \in \{1, ..., n\}$*.*

Remark 3.2. When $\varphi(z) \equiv z, T_{\psi,\varphi} = M_\psi$, we obtain the next two Corollaries about multiplication operator from Theorems 3.1 and 3.2.

Corollary 3.3. *Suppose that* $0 < \alpha < \infty$*,* μ *is a normal function on* $[0,1)$ *and* $\psi \in H(B_n)$*. Then* $M_\psi : H_\alpha^\infty \to \mathscr{B}_\mu$ *is bounded if and only if*

$$\sup_{z \in B_n} \frac{\mu(z)|\Re\psi(z)|}{(1-|z|^2)^\alpha} < \infty \quad \textit{and} \quad \sup_{z \in B_n} \frac{\mu(z)|\psi(z)|}{(1-|z|^2)^{\alpha+1}} < \infty.$$

Corollary 3.4. *Suppose that* $0 < \alpha < \infty$*,* μ *is a normal function on* $[0,1)$ *and* $\psi \in H(B_n)$*. Then* $M_\psi : H_\alpha^\infty \to \mathscr{B}_\mu$ *is compact if and only if*

(a) $\psi \in \mathscr{B}_\mu$ *and* $z_l\psi \in \mathscr{B}_\mu$ *for any* $l \in \{1, ..., n\}$*;*

(b)
$$\lim_{|z| \to 1} \frac{\mu(z)|\Re\psi(z)|}{(1-|z|^2)^\alpha} = 0;$$

(c)
$$\lim_{|z| \to 1} \frac{\mu(z)|\psi(z)|}{(1-|z|^2)^{\alpha+1}} = 0.$$

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References

[1] H. Chen and P. Gauthier, Boundedness from below of composition operators on α-Bloch spaces, *Canad. Math. Bull.* **51** (2008), no. 2, 195–204.

[2] C. C. Cowen and B. D. MacCluer, *Composition operators on spaces of analytic functions*, Studies in Advanced Mathematics, CRC, Boca Raton, FL, 1995.

[3] Z. Hu, Extended Cesàro operators on mixed norm spaces, *Proc. Amer. Math. Soc.* **131** (2003), no. 7, 2171–2179.

[4] X. Lü, Weighted composition operators from $F(p,q,s)$ spaces to Bers-type spaces in the unit ball, *Appl. Math. J. Chinese Univ. Ser. B* **24** (2009), no. 4, 462–472.

[5] K. Madigan and A. Matheson, Compact composition operators on the Bloch space, *Trans. Amer. Math. Soc.* **347** (1995), no. 7, 2679–2687.

[6] S. Ohno, K. Stroethoff and R. Zhao, Weighted composition operators between Bloch-type spaces, *Rocky Mountain J. Math.* **33** (2003), no. 1, 191–215.

[7] S. Stević, Weighted composition operators between mixed norm spaces and H_α^∞ spaces in the unit ball, *J. Inequal. Appl.* **2007**, Art. ID 28629, 9 pp.

[8] S. Stević, On a new operator from H^∞ to the Bloch-type space on the unit ball, *Util. Math.* **77** (2008), 257–263.
[9] S. Stević, Sibirsk. Mat. Zh. **50** (2009), no. 3, 621–624; translation in *Sib. Math. J.* **50** (2009), no. 3, 495–497.
[10] S. Stević, On an integral operator between Bloch-type spaces on the unit ball, *Bull. Sci. Math.* **134** (2010), no. 4, 329–339.
[11] S. Stević and S. Ueki, Integral-type operators acting between weighted-type spaces on the unit ball, *Appl. Math. Comput.* **215** (2009), no. 7, 2464–2471.
[12] A. L. Shields and D. L. Williams, Bonded projections, duality, and multipliers in spaces of analytic functions, *Trans. Amer. Math. Soc.* **162** (1971), 287–302.
[13] E. Wolf, A note on spectra of weighted composition operators on weighted Banach spaces of holomorphic functions, *Bull. Malays. Math. Sci. Soc.* (2) **31** (2008), no. 2, 145–152.
[14] X. Wang and T. Liu, Composition type operators from Hardy spaces to μ-Bloch spaces on the unit ball, *Acta Math. Sci. Ser. B Engl. Ed.* **29** (2009), no. 5, 1430–1438.
[15] J. Xiao, Composition operators associated with Bloch-type spaces, *Complex Variables Theory Appl.* **46** (2001), no. 2, 109–121.
[16] W. Yang, Volterra composition operators from $F(p,q,s)$ spaces to Bloch-type spaces, *Bull. Malays. Math. Sci. Soc. (2)* **34** (2011), no. 2, 267–277.
[17] C. Yuan and Z.-H. Zhou, The essential norm and spectrum of a weighted composition operator on $H^\infty(B_N)$, *Bull. Malays. Math. Sci. Soc. (2)* **33** (2010), no. 3, 499–511.
[18] X. Zhu, Extended Cesàro operators from H^∞ to Zygmund type spaces in the unit ball, *J. Comput. Anal. Appl.* **11** (2009), no. 2, 356–363.
[19] X. Zhang, Composition type operator from Bergman space to μ-Bloch type space in $\mathbf{C}^n$, *J. Math. Anal. Appl.* **298** (2004), no. 2, 710–721.
[20] Z.-H. Zhou and R.-Y. Chen, Weighted composition operators from $F(p,q,s)$ to Bloch type spaces on the unit ball, *Internat. J. Math.* **19** (2008), no. 8, 899–926.
[21] X. J. Zhang and J. C. Liu, Composition operators from weighted Bergman spaces to μ-Bloch spaces, *Chinese Ann. Math. Ser. A* **28** (2007), no. 2, 255–266.
[22] K. Zhu, *Spaces of holomorphic functions in the unit ball*, Graduate Texts in Mathematics, 226, Springer, New York, 2005.
[23] X. Zhu, An integral-type operator from H^∞ to Zygmund-type spaces, *Bull. Malays. Math. Sci. Soc. (2)* **35** (2012), no. 3, 679–686.
[24] X. Zhu, Composition operators from Zygmund spaces to Bloch spaces in the unit ball, *Bull. Malays. Math. Sci. Soc. (2)* **35** (2012), no. 4, 961–968.
[25] Z. Zhou and J. Shi, Compactness of composition operators on the Bloch space in classical bounded symmetric domains, *Michigan Math. J.* **50** (2002), no. 2, 381–405.
[26] H.-G. Zeng and Z.-H. Zhou, An estimate of the essential norm of a composition operator from $F(p,q,s)$ to β^α in the unit ball, *J. Inequal. Appl.* **2010**, Art. ID 132970, 22 pp.