

MHD Stagnation Point Flow of a Micropolar Fluid Toward a Vertical Plate with a Convective Surface Boundary Condition

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Abstract. The steady MHD mixed convection stagnation point flow towards a vertical plate immersed in a micropolar fluid with a convective surface boundary condition is investigated. The governing partial differential equations are first reduced to ordinary differential equations using a similarity transformation, before being solved numerically. The features of the flow and heat transfer characteristics for different values of the governing parameters are analyzed and discussed. Both assisting and opposing flows are considered. The results indicate that dual solutions exist for the opposing flow, whereas for the assisting flow, the solution is unique. The skin friction coefficient and the local Nusselt number increase in the presence of magnetic field and buoyancy force.

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1. Introduction

The study of stagnation point flow towards a vertical plate has received a great attention of research interest due to its wide applications in industries and practical applications. Some of the applications are cooling of electronic devices by fans, solar central receivers exposed to wind currents, cooling of nuclear reactors during emergency shutdown, and many hydrodynamic processes. Chamka [20] for instance, has studied the mixed convection flow near the stagnation point of a vertical semi-infinite permeable surface in the presence of a magnetic field. The non-magnetic effect for impermeable surface on both arbitrary wall temperature and arbitrary surface heat flux variations has been studied by Ramachandran *et al.* [51], which they found that a reverse flow developed in the buoyancy opposing flow region, and dual solutions exist for a certain range of the buoyancy parameter. This problem was then extended by Devi *et al.* [23] to the unsteady case, where they obtained the similar

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results as in Ramachandran *et al.* [51]. It is worth mentioning that the stagnation-point flows have also been studied in many flow situations, for examples in the papers by Chiam [21, 22], Bhattacharyya [12, 13], Bhattacharyya and Layek [14], Bhattacharyya and Vajravelu [15], Bhattacharyya *et al.* [16, 17, 18, 19], Ishak *et al.* [30], and Wang [55], among others.

The basic idea of micropolar fluids has risen from the need to model many engineering processes involving non-Newtonian fluids containing micro-constituents such as blood flow, lubricants, colloidal fluids, liquid crystals and suspension fluids that cannot be described by the classical Newtonian fluid. Based on this need, Eringen [24, 25] has introduced the theory of micropolar fluids that is able to describe those fluids by taking into account the microscopic effects arising from the local structure and micromotions of the fluid elements. Numerous studies of the theory and its applications have been done by many researchers. For example, the problems of MHD stagnation point flow of a micropolar fluid have been investigated by Hayat *et al.* [28, 29], using the homotopy analysis method (HAM). It is worth mentioning that this method was also employed by Alomari *et al.* [3] and Aziz *et al.* [8] in their recent papers. In addition, Takhar *et al.* [54], Yücel [59], Lok *et al.* [40], Alomari *et al.* [4], Ishak *et al.* [31] and Yacob and Ishak [57], studied the micropolar fluid in the mixed convection flow by considering some other physical aspects. Comprehensive reviews of the subject and its applications can be found in the review articles by Ariman *et al.* [6, 7] and the books by Łukaszewicz [41] and Eringen [26].

Motivated by the above mentioned investigations, we consider the problem of hydromagnetic stagnation-point flow towards a vertical plate immersed in an incompressible micropolar fluid with a convective surface boundary condition. The boundary layer flow concerning a convective boundary condition for the Blasius flow has been discussed by Aziz [9], while Magyari [42] improved this work by obtaining the exact solution for the temperature field in a compact integral form. Bataller [11] investigated the similar problem by considering radiation effects on the Blasius and Sakiadis flows. Later, the effects of radiation on the thermal boundary layer flow over a moving plate in a moving fluid have been studied by Ishak *et al.* [32]. The hydromagnetic flow over a vertical plate under a convective boundary condition was studied by Makinde [43, 44] and Makinde and Aziz [45], while Makinde and Olanrewaju [46] studied the buoyancy effects on the thermal boundary layer over a vertical plate, all under the same surface heating condition.

In the present paper, the governing equations are transformed into a system of nonlinear ordinary differential equations, which are then solved numerically. Representative results for the velocity, temperature and angular velocity profiles as well as the skin friction coefficient, local couple stress and the local Nusselt number, which represents the heat transfer rate at the surface, are presented for some values of the governing parameters.

2. Mathematical formulation

Consider a steady, two-dimensional flow of an incompressible micropolar fluid near the stagnation point on a vertical flat plate of constant ambient temperature T_∞ , as shown in Figure 1. It is assumed that the external velocity is prescribed as $U(x) = ax$ where a is a positive constant and x is the distance from the stagnation point. A uniform magnetic field of strength B_0 is assumed to be applied in the positive y -direction normal to the plate. The induced magnetic field is assumed to be small compared to the applied magnetic field, and is neglected. Further, it is assumed that the left surface of the plate is heated or cooled by convection from a hot or cool fluid of temperature $T_f(x) = T_\infty + bx$, which provides a heat

transfer coefficient h_f , where b is a constant with $b > 0$ for $T_f(x) > T_\infty$ (assisting flow) and $b < 0$ for $T_f(x) < T_\infty$ (opposing flow). Under these assumptions along with the Boussinesq and boundary layer approximations, the boundary layer equations are [28, 29, 31]:

$$(2.1) \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$(2.2) \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \left(\frac{\mu + \kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial N}{\partial y} + \frac{\sigma B_0^2}{\rho} (U - u) + g\beta (T - T_\infty),$$

$$(2.3) \quad \rho j \left(u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = \gamma \frac{\partial^2 N}{\partial y^2} - \kappa \left(2N + \frac{\partial u}{\partial y} \right),$$

$$(2.4) \quad u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2},$$

subject to the boundary conditions

$$u = 0, \quad v = 0, \quad N = -n \frac{\partial u}{\partial y}, \quad -k \frac{\partial T}{\partial y} = h_f (T_f - T_w) \quad \text{at } y = 0,$$

$$(2.5) \quad u \rightarrow U(x), \quad N \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty,$$

where u and v are the velocity components along the x and y axes, respectively. Further, μ is the dynamic viscosity, κ the vortex viscosity (or the microrotation viscosity), ρ the fluid density, γ the spin gradient viscosity, α the thermal diffusivity, β the thermal expansion coefficient, g the acceleration due to gravity, T the fluid temperature, j the microinertia density, N is the microrotation vector, k is the thermal conductivity of the fluid and n is a constant such that $0 \leq n \leq 1$. The case $n = 0$, is called strong concentration by Guram and Smith [27], which indicates $N = 0$ near the wall, represents concentrated particle flows in which the microelements close to the wall surface are unable to rotate (Jena and Mathur [38]). The case $n = 1/2$ indicates the vanishing of anti-symmetric part of the stress tensor and denotes weak concentrations (Ahmadi [1]). The case $n = 1$, as suggested by Peddieson [49], is used for the modeling of turbulent boundary layer flows, see Ishak *et al.* [33]. Further, we follow the work of many recent authors by assuming that $\gamma = (\mu + \kappa/2)j = \mu(1 + K/2)j$, where $K = \kappa/\mu$ is the constant dimensionless micropolar or material parameter. This assumption is invoked to allow the field of equations to predict the correct behavior in the limiting case when the microstructure effects become negligible and the total spin N reduces to the angular velocity (Ahmadi [1]).

The continuity equation (2.1) is satisfied by introducing a stream function ψ such that

$$(2.6) \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}.$$

The momentum, angular momentum and energy equations can be transformed into the corresponding nonlinear ordinary differential equations by the following transformation (see Ishak *et al.* [31] and Aziz [9]):

$$\eta = \left(\frac{U}{\nu x} \right)^{1/2} y, \quad f(\eta) = \frac{\psi}{(\nu x U)^{1/2}},$$

$$(2.7) \quad h(\eta) = \frac{N(vx)^{1/2}}{U^{3/2}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}.$$

The transformed ordinary differential equations are:

$$(2.8) \quad (1 + K)f''' + ff'' + 1 - f'^2 + Kh' + M(1 - f') + \lambda\theta = 0,$$

$$(2.9) \quad \left(1 + \frac{K}{2}\right) h'' + fh' - f'h - K(2h + f'') = 0,$$

$$(2.10) \quad \frac{1}{Pr}\theta'' + f\theta' - f'\theta = 0,$$

subject to the boundary conditions (2.5) which become

$$f(0) = 0, \quad f'(0) = 0, \quad h(0) = -nf''(0), \quad \theta'(0) = -c[1 - \theta(0)]$$

$$(2.11) \quad f'(\eta) \rightarrow 1, \quad h(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty,$$

where we have taken $j = v/a$ as a reference length (Nazar *et al.* [48]). In the above equations, primes denote differentiation with respect to η , $Pr = v/\alpha$ is the Prandtl number, $M = B_0^2\sigma/(\rho a)$ is the magnetic parameter and $c = (v/a)^{1/2}h_f/k$ is the convective parameter. Further, $\lambda = Gr_x/Re_x^2$ is the buoyancy or mixed convection parameter with $Gr_x = g\beta(T_f - T_\infty)x^3/v^2$ and $Re_x = Ux/v$ being the local Grashof and Reynolds numbers, respectively. We notice that λ is a constant with $\lambda < 0$ and $\lambda > 0$ correspond to the opposing and assisting flows, respectively, while $\lambda = 0$ is for pure forced convection flow. It is worth mentioning that when $K = 0$ and $M = 0$, Eqs. (2.8) reduces to the problem derived by Ramachandran *et al.* [51] for Newtonian fluid. The thermal boundary conditions occurs in a variety of real situations such as fluid flow of rarefied gas, fluid flow around micro-electromechanical (MEMS), convectonal isothermal or iso-flux boundary conditions that must be replaced with thermal slip boundary condition (Kiwani and Al-Nimr [39]). Note that the convective boundary condition in (2.11) can be written as

$$(2.12) \quad \theta(0) = 1 + \gamma\theta'(0),$$

where $\gamma = 1/c$, ($c \neq 0$) is the thermal slip parameter, so we can say that thermal slip is a special case of convective surface boundary condition. For $\gamma = 0$ this boundary condition becomes $\theta(0) = 1$, which is the prescribed surface temperature case.

The physical quantities of interest are the skin friction coefficient C_f , the local couple stress M_x and the local Nusselt number Nu_x , which are defined as (Jena and Mathur [38])

$$(2.13) \quad C_f = \frac{\tau_w}{\rho U^2}, \quad M_x = \frac{vM_w}{xU^2}, \quad Nu_x = \frac{xq_w}{k(T_f - T_\infty)},$$

where the surface shear stress τ_w , the surface couple stress M_w and the surface heat flux q_w are given by

$$(2.14) \quad \tau_w = \left[(\mu + \kappa) \frac{\partial u}{\partial y} + \kappa N \right]_{y=0}, \quad M_w = (a/v)^{-1} \left(\frac{\partial N}{\partial y} \right)_{y=0}, \quad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}.$$

Using (2.7), we obtain

$$(2.15) \quad C_f Re_x^{1/2} = [1 + (1 - n)K] f''(0), \quad M_x Re_x = h'(0), \quad Nu_x Re_x^{-1/2} = -\theta'(0).$$

3. Results and discussion

The nonlinear ordinary differential equations (2.8)-(2.10) subject to the boundary conditions (2.11) were solved numerically using the Runge-Kutta-Fehlberg method with shooting technique [34, 35, 36, 10] for several values of the magnetic parameter M , material parameter K , Prandtl number Pr , convective parameter c and buoyancy parameter λ , in which n is taken to be 0.5 (weak concentration). The descriptions of this method can be found in [14, 5].

The variations of the skin friction coefficient $C_f Re_x^{1/2}$, local couple stress $M_x Re_x$ and the local Nusselt number $Nu_x Re_x^{-1/2}$ with λ for different values of M when the other parameters are fixed to unity, are presented in Figures 2–4, respectively. In these figures, the solid lines denote the upper branch solutions, while the dash lines denote the lower branch solutions. It can be seen that dual solutions exist for the opposing flow ($\lambda < 0$), while for the assisting flow ($\lambda > 0$), the solution is unique. For the upper branch solution, the values of $C_f Re_x^{1/2}$ increases as M and λ increase, since there is a favourable pressure gradient due to the assisting bouyant flow which increases the surface shear stress and the heat transfer rate at the surface. The same behaviour is observed for $Nu_x Re_x^{-1/2}$ (for the upper branch), where the heat transfer rate at the surface increases as M and λ increase. On the other hand, for the lower branch solution, the local Nusselt number is seen to decrease as M increases (due to the fact that opposing buoyant flow induces an adverse pressure gradient, which slow down the fluid motion), while the skin friction coefficient decreases in the begining and then increases slightly, until no result could be found at certain values of λ . It should be mentioned that the computations have been performed until the point where the solution does not converge, and the computations were terminated at this location. For each selected values of M , there is indeed a critical value λ_c of λ for which the solution exists. Based on our computations, we found that $\lambda_c = -3.7644626$, -4.6517649 and -5.53465 for $M = 0.5$, 1.0 and 1.5 , respectively. Therefore, the effect of the magnetic field is to widen the range of λ for which the solutions exist.

It is worth mentioning that the existence of dual solutions in the mixed convection stagnation flow problems was also reported by Ramachandran *et al.* [51], in the MHD boundary layer flow by Ishak *et al.* [37] and in the case of stagnation-point flow of a micropolar fluid by Yacob and Ishak [58] and by Lok *et al.* [40]. Between the two solutions as presented in Figures 2–4, we expect that the first solution (upper branch solution) is stable and most physically relevance, while the second solution (lower branch solution) is not, since the first solution is the only solution for the assisting flow case, and the second solution exists only for certain range of the buoyancy parameter. However, they are still of interest as the differential equations are concerned, though such solutions are deprived of physical significant. Similar results may arise in other situations where the corresponding solutions could have more realistic meaning (Ridha [52]). For the similar problems, using a stability analysis, Weidman *et al.* [56], Merkin [47] and Postelnicu and Pop [50] have shown that the upper branch solutions are stable, while the lower branch solutions are not. Spangenberg *et al.* [53] have reported in their experimental work on turbulent boundary layer under strong adverse pressure gradient that dual solutions were obtained as a function of how the pressure gradient was realized. Another example of non-unique flow is reported by Aidun *et al.* [2] where they have observed experimentally that the primary steady state flow in a through-flow lid-driven cavity was non-unique and only one of the multiple steady-state patterns can stabilize in the cavity.

Figure 5 shows the variation of the local Nusselt number $Nu_x Re_x^{-1/2}$ with the convective parameter c for some values of M when $K = 1$, $n = 0.5$, $Pr = 0.72$ and $\lambda = -3.5$. The local Nusselt number decreases as the convective parameter increases. Therefore, the effect of the convective parameter c is to decrease the heat transfer rate at the surface. Figures 6-11 respectively present the velocity, temperature and angular velocity distributions for the selected values of magnetic parameter and material parameter for the opposing flow. It is clear that the upper branch solution displays a thinner boundary layer thickness compared to the lower branch solution. These figures also show that the far field boundary conditions (2.11) are satisfied asymptotically, hence support the validity of the numerical results obtained, besides supporting the existence of the dual solutions presented in Figures 2-4. Figures 8 and 11 show the micropolar (angular) velocity profiles for the selected values of M and K , respectively. It is observed that the effects of microrotation are more dominant near the surface for the upper branch solution than the lower branch solution.

Figures 12 and 13 present the upper branch and the lower branch streamlines for $c = 1$, $Pr = 1$ and $\lambda = -4$, respectively. It can be seen that the streamlines for the upper branch solution are quite simple and symmetric about the horizontal axis, due to the equal force of buoyant flow (assisting and opposing flow) and the pattern is almost similar to the normal stagnation point flow. On the other hand, the streamlines for the lower branch solutions are more complicated with a vertical line separate the flows into two regions.

4. Conclusions

In this paper, the steady MHD mixed convection stagnation point flow over an impermeable vertical plate in an incompressible micropolar fluid with a convective surface boundary condition was studied. The governing partial differential equations were first transformed into a system of ordinary differential equations using a similarity transformation, before being solved numerically by the Runge-Kutta-Fehlberg method with shooting technique. The effects of magnetic parameter M , material parameter K , Prandtl number Pr , convective parameter c and buoyancy parameter λ on the flow field and heat transfer characteristics were analyzed and discussed. It was found that the magnetic field increases the skin friction coefficient, the local couple stress and the heat transfer rate at the surface. Dual solutions were found to exist for the opposing flow, while for the assisting flow, the solution is unique.

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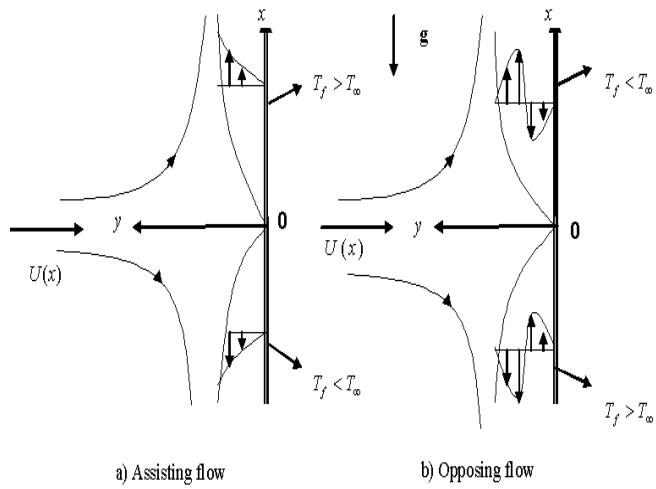


Figure 1. Physical model and coordinate system

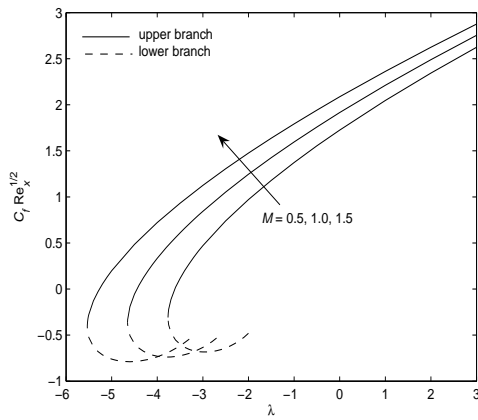


Figure 2. Variation of the skin friction coefficient $C_f Re_x^{1/2}$ with λ for some values of M when $c = 1$, $K = 1$, $n = 0.5$ and $Pr = 1$

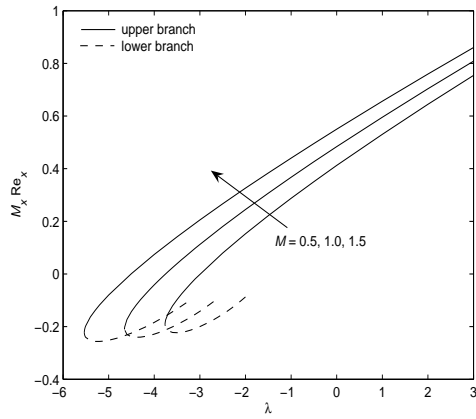


Figure 3. Variation of the local couple stress $M_x Re_x^{1/2}$ with λ for some values of M when $c = 1$, $K = 1$, $n = 0.5$ and $Pr = 1$

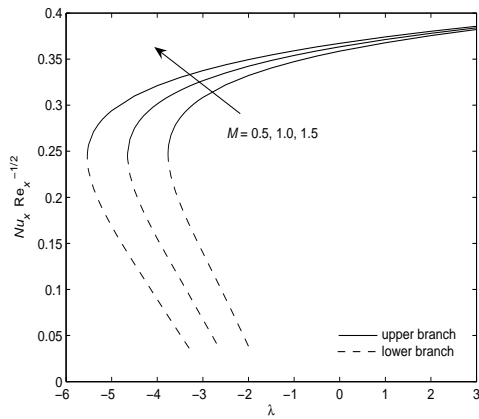


Figure 4. Variation of the local Nusselt number $Nu_x Re_x^{-1/2}$ with λ for some values of M when $c = 1$, $K = 1$, $n = 0.5$ and $Pr = 1$

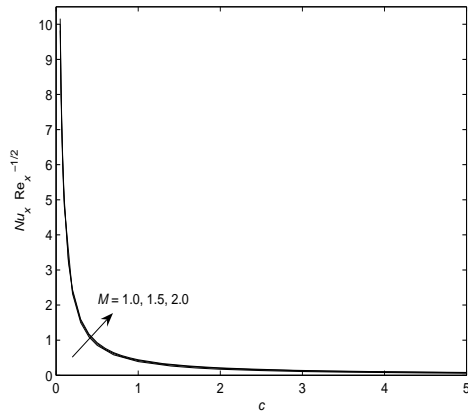


Figure 5. Variation of the local Nusselt number $Nu_x Re_x^{-1/2}$ with c for some values of M when $K = 1$, $n = 0.5$, $Pr = 0.72$ and $\lambda = -3.5$

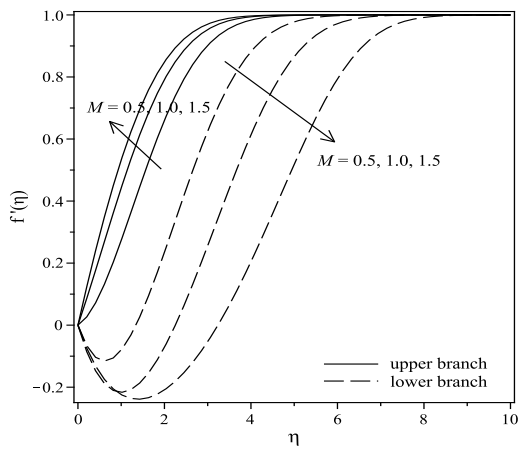


Figure 6. Velocity profiles $f'(\eta)$ for some values of M when $c = 1$, $K = 1$, $n = 0.5$, $Pr = 1$ and $\lambda = -3.5$

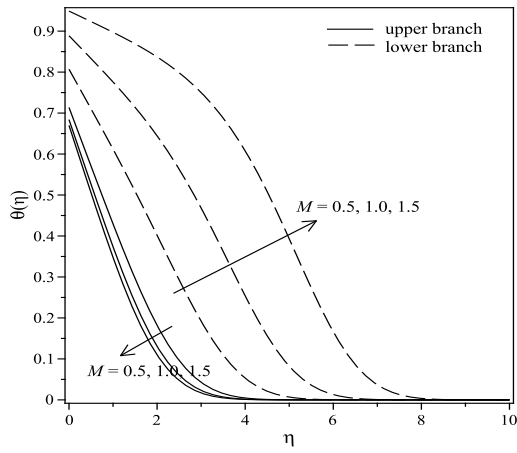


Figure 7. Temperature profiles $\theta(\eta)$ for some values of M when $c = 1, K = 1, n = 0.5, Pr = 1$ and $\lambda = -3.5$

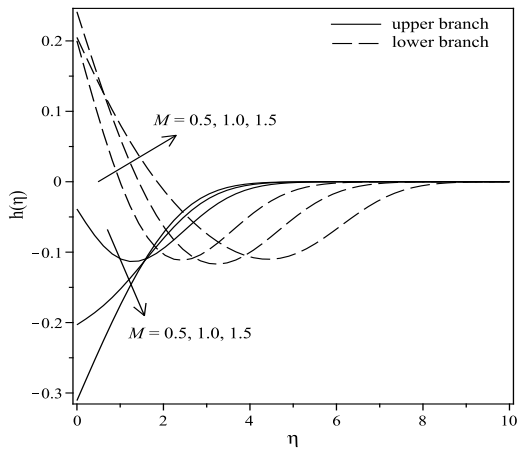


Figure 8. Angular velocity profiles $h(\eta)$ for some values M when $c = 1, K = 1, n = 0.5, Pr = 1$ and $\lambda = -3.5$

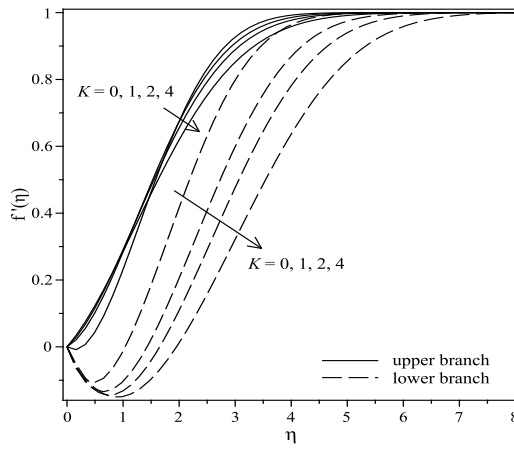


Figure 9. Velocity profiles $f'(\eta)$ for some values of K when $c = 1$, $M = 1$, $n = 0.5$, $Pr = 1$ and $\lambda = -4.3$

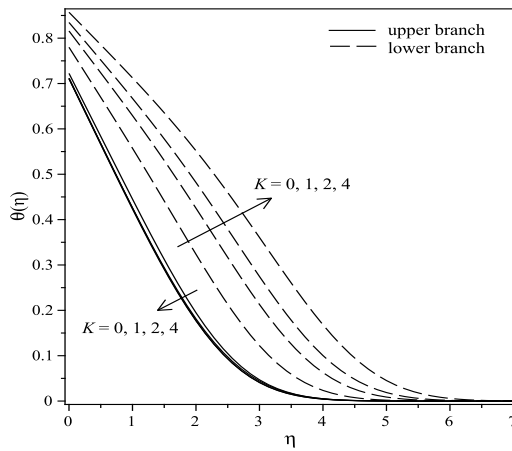


Figure 10. Temperature profiles $\theta(\eta)$ for some values of K when $c = 1$, $M = 1$, $n = 0.5$, $Pr = 1$ and $\lambda = -4.3$

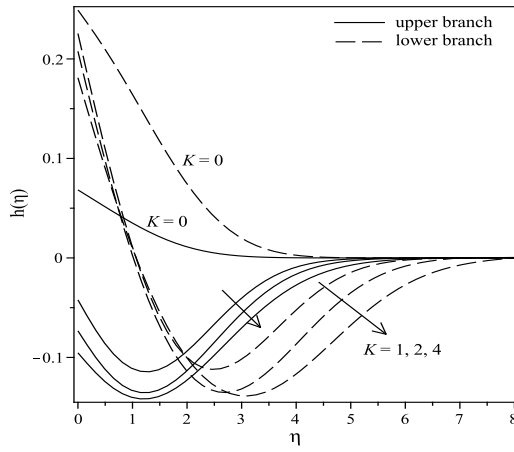


Figure 11. Angular velocity profiles $h(\eta)$ for some values K when $c = 1$, $M = 1$, $n = 0.5$, $Pr = 1$ and $\lambda = -4.3$

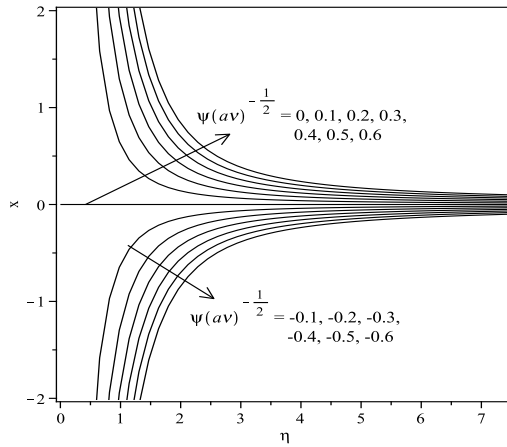


Figure 12. Upper branch streamline for two dimensional flow when $c = 1$, $K = 1$, $M = 1$, $n = 0.5$, $Pr = 1$ and $\lambda = -4$

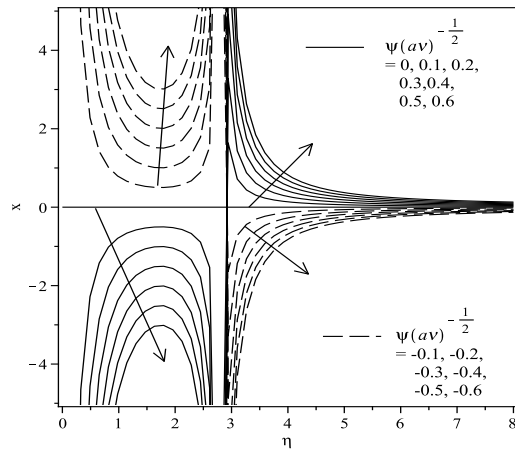


Figure 13. Lower branch streamline for two dimensional flow when $c = 1$, $K = 1$, $M = 1$, $n = 0.5$, $Pr = 1$ and $\lambda = -4$