BULLETIN *of the*
MALAYSIAN MATHEMATICAL
SCIENCES SOCIETY
http://math.usm.my/bulletin

Bull. Malays. Math. Sci. Soc. (2) **37**(1) (2014), 83–91

On the Residual Life of the *k* out of *n* System

[1]M. AHSANULLAH, [2]ABDULHAMID A. ALZAID AND [3]B. M. GOLAM KIBRIA

[1]Department of Management Sciences, Rider University, Lawrenceville, NJ, USA
[2]Department of Statistics and Operations Research, King Saud University, Riyadh, Saudi Arabia
[3]Department of Mathematics and Statistics, Florida International University, Modesto A. Maidique Campus Miami, FL USA
[1]ahsan@rider.edu, [2]alzaid@ksu.edu.sa, [3]kibriag@fiu.edu

Abstract. We consider a system with n independent and identically distributed components. The system fails if k out of the n components fails. We study various distributional properties of the mean life times of the $n-k$ remaining components after the failure of the k components. Based on these distributional properties some characterizations of distributions are given.

2010 Mathematics Subject Classification: 62N05, 62E10

Keywords and phrases: Characterization, hazard rate, mean residual life.

1. Introduction

Suppose that $X_1, X_2, ..., X_n$ are the life times of n independent and identically distributed components of a system. Let $Y_1^k, Y_2^k, ..., Y_{n-k}^k$ are the residual life times of the remaining $n-k$ components after the failure at time t of the kth component. Let $X_{1,n}, X_{2,n}, ..., X_{n,n}$ and $Y_{1,n-k}^k, Y_{2,n-k}^k, ..., Y_{n-k,n-k}^k$ denote the ordered values of $X_1, X_2, ..., X_n$ and $Y_1^k, Y_2^k, ..., Y_{n-k}^k$ respectively. We will assume that the cumulative distributive function (cdf) F of the X's is absolutely continuous with respect to Lebesgue measure and $F(0) = 0$. We denote the corresponding probability density function (pdf) by f. Let $F_{k,n}(x)$ and $f_{k.n}(x)$ be the cdf and pdf of $X_{k,.n}$.

Bairamov *et al.* [5] studied the mean residual life of $X_{k,.n} - t$ given that $X_{1,.n} > t$. Asadi and Bayramoglu [2] investigated the mean residual life of a $X_{k,.n} - t$ under the condition that $X_{k,.n} > t$. Asadi and Bairamov [3] investigated the mean residual life of $X_{k,.n} - t$ given that $X_{1,.n} > t$. Li and Zhao [9] studied some aging properties of the residual life of k-out-of-n systems given that at least $(n-i+1)$ components of the system are working. Bairamov and Arnold [4] showed that the equality of the distribution of Y_i^k and X_1 characterizes the exponential distribution. They considered in their paper some basic distributional properties

Communicated by M. Ataharul Islam.
Received: January 19, 2011; *Revised:* February 3, 2012.

of $Y_1^k, Y_2^k, ..., Y_{n-k}^k$. The joint pdf of $Y_1^k, Y_2^k, ..., Y_{n-k}^k$ can be written as

$$f_{1,2,..,n-k}^k(y_1, y_2, ..., y_{n-k}) = \int_0^\infty \prod_{i=1}^k \frac{f(y_i+t)}{\overline{F}(t)} dF_{k.n}(t), \tag{1.1}$$

where

$$dF_{k.n}(t) = k\binom{n}{k}(F(t))^{k-1}(\overline{F}(t))^{n-k} f(t). \tag{1.2}$$

Thus $Y_1^k, Y_2^k, ..., Y_{n-k}^k$ are conditionally independent with marginal pdf given by

$$f_i^k(y) = \int_0^\infty \frac{f(y+t)}{\overline{F}(t)} dF_{k.n}(t), y > 0. \tag{1.3}$$

The corresponding survival function is

$$\overline{F}_i^k(y) = \int_0^\infty \frac{\overline{F}(y+t)}{\overline{F}(t)} dF_{k.n}(t), y \geq 0. \tag{1.4}$$

The function $\overline{F}(x|t) = \frac{\overline{F}(x+t)}{\overline{F}(t)}$ is known as the lifetime residual function (LRF) of F. The corresponding mean residual life (MRL) is given by

$$m(t) = E[Y - t \mid Y > t] = \int_0^\infty \overline{F}(x \mid t) dx = \int_0^\infty \frac{\overline{F}(x+t)}{\overline{F}(t)} dx. \tag{1.5}$$

These two functions (RFL and MRL) are together with the hazard (failure) rate function of F which is given by

$$h(x) = \frac{f(x)}{1 - F(x)},\ x > 0 \tag{1.6}$$

play an important role in reliability and survival analysis. Each of the three functions determines the distribution up to scale parameter. For example, Kotz and Shanbhag [7] gave a representation of the distribution in term of MRL. The three functions have been extensively used to introduce aging concepts. A random variable X and its distribution function F are called monotone increasing (decreasing) failure rate if $h(x)$ is monotone increasing (decreasing). Note that $h(x)$ is monotone increasing (decreasing) if and only if $\overline{F}(x|t)$ is monotone decreasing (increasing) in t for every x. Another related aging concept is the new better than used. A cumulative distribution function F is said to be NBU (NWU) if $\overline{F}(x+y) \leqslant (\geqslant)\overline{F}(x)\overline{F}(y)$, for $x \geqslant 0, y \geqslant 0$. See Barlow and Prochan [6, p.95] for this and some other references for aging concepts. For more recent references on these see Lai and Xie [8] and references therein. We say the random variable $X \in C$ if F is NBU or NWU.

In this paper several distributional properties of the Y_i^k are given. Based on these distributional properties some characterizations of probability distributions are presented with special emphasis on exponential distribution. Section 2 discusses characterizations of exponential distribution among the classes C based on identities of expectations and distributions. Characterizations based on independence are also presented. In Section 3 we present a representation of the distribution of the parent distribution in terms of the mean residual function. In Section 4, it is shown that the aging properties of the parent distribution of X are reflected on partial ordering between Y_i^k and X. Some concluding remarks are given in Section 5.

2. Characterizations of the exponential distribution

In this section, we give some characterizations of the exponential distribution based on the properties of Y_i^k.

Theorem 2.1. *Suppose X is an absolutely continuous (with respect to Lebesgue measure) random variable with F(0) =0 and F(x) <1 for all* $x > 0$*. Assume that E(X) exists, then the following two conditions are equivalent*

(i) $F(x) = 1 - e^{-\lambda x},\ x \geq 0, \lambda > 0$
(ii) $E(Y_k^1) = E(X_1)$ *and* $X_1 \in C$

Proof. From (1.3), we have the pdf f of Y

$$f_k^1(x) = \int_0^\infty \frac{f(x+t)}{\overline{F}(t)} dF_{k.n}(t)$$

and

$$E(Y_1^k) = \int_0^\infty \int_0^\infty x \frac{f(x+t)}{\overline{F}(t)} dF_{k.n}(t)dx.$$

If $F(x) = 1 - e^{-\lambda x}$, then $\frac{f(x+t)}{\overline{F}(t)} = \lambda e^{-\lambda x}$ and $E(Y_1^k) = \frac{1}{\lambda} = E(X_1)$.
If $E(Y_1^k) = E(X_1)$, then

$$\int_0^\infty \int_0^\infty x \frac{f(x+t)}{\overline{F}(t)} dF_{k,n}(t)dx = \int_0^\infty x f(x)dx. \tag{2.1}$$

We can rewrite (2.1) as

$$\int_0^\infty \int_0^\infty \frac{\overline{F}(x+t)}{\overline{F}(t)} dF_{k,n}(t)dx = \int_0^\infty \overline{F}(x)dx. \tag{2.2}$$

On simplification we obtain from (2.2).

$$\int_0^\infty \int_0^\infty \left[\frac{\overline{F}(x+t)}{\overline{F}(t)} - \overline{F}(x)\right] dF_{k,n}(t)dx = 0$$

Since $X_1 \in C$, we must have

$$\frac{\overline{F}(x+t)}{\overline{F}(t)} - \overline{F}(x) = 0 \text{ for almost all } x \text{ and } t \geq 0. \tag{2.3}$$

The solution of (2.3) with the boundary condition $F(0) = 0$ and $F(\infty) = 1$ is

$$F(x) = 1 - e^{-\lambda x}$$

for all $x \geqslant 0$ and any $\lambda > 0$. ∎

Theorem 2.2. *Suppose X is an absolutely continuous (with respect to Lebesgue measure) random variable with* $F(0) = 0$ *and* $F(x) < 1$ *for all* $x > 0$*. Then the following two conditions are equivalent*

(i) $F(x) = 1 - e^{-\lambda x}$,
(ii) *The pdf of* $Y_i^k | X_{k,n} = t$ *is independent of t.*

Proof. If $F(x) = 1 - e^{-\lambda x}$, then $\frac{f(x+t)}{\overline{F}(t)} = \lambda e^{-\lambda x}$ then it follows from (3.1) that the pdf of $Y_i^k | X_{k,n}$ is independent of t. Suppose the pdf of $Y_i^k | X_{k,n} = t$ independent of t. Then we have

$$\frac{f(x+t)}{\overline{F}(t)} = g(x), \tag{2.4}$$

where $g(x)$ is independent of t for all x.

Integrating both sides of (2.4) with respect to x from x_0 to ∞, we obtain

$$\frac{\overline{F}(x_0+t)}{\overline{F}(t)} = G(x_0), \tag{2.5}$$

where $G(x_0) = \int_{x_0}^{\infty} g(x)dx$. Taking $t \to 0$, we obtain from (2.5), $G(x_0) = \overline{F}(x_0)$. Hence

$$\overline{F}(x_0+t) = \overline{F}(t)\,\overline{F}(x_0), \text{ for all } t > 0 \text{ and almost all } x_0 > 0. \tag{2.6}$$

The solution of (2.6) with the boudary conditions $F(0) = 0$ and F(∞)=1 is

$$F(x) = 1 - e^{-\lambda x}, \text{ for } x > 0 \text{ and } \lambda > 0.$$

∎

The following theorem gives a characterization of the exponential distribution using the distribution of $Y_{1,n-k}^k$.

Theorem 2.3. *Suppose the cumulative distribution function F(x) of the X's are absolutely continuous (with respect to Lebesgue measure) monotone increasing and f(x) is the corresponding pdf. Then the following two conditions are equivalent*

(i) $F(x) = 1 - e^{-\lambda x}$, $x \geqslant 0$, $\lambda > 0$

(ii) $(n-k)Y_{1,n-k}^k \stackrel{d}{=} X_1$, *where* $\stackrel{d}{=}$ *denotes equality in distribution and* $X_1 \in C$.

Proof. From (2.1), we obtain $f_{1,n-k}^k$, the pdf of $Y_{1,n-k}^k$ as follows:
If $F(x) = 1 - e^{-\lambda x}$ then

$$\begin{aligned} dF_{k.n}(t) &= \lambda \tbinom{n}{k}(1-e^{-\lambda t})^{k-1} e^{-\lambda(n-k+1)t} \\ f_{1,n-k}^k(x) &= \int_0^{\infty} (n-k)\lambda e^{-(n-k)\lambda x} dF_{k,n}(t) \\ &= (n-k)\lambda e^{-(n-k)\lambda x} \end{aligned}$$

Thus $(n-k)Y_{1,n-k}^k \stackrel{d}{=} X_1$.

Suppose $(n-k)Y_{1,n-k}^k \stackrel{d}{=} X_1$ then we have

$$\int_0^{\infty} \left(\overline{F}(\frac{x}{n-k}+t)/\overline{F}(t)\right)^{n-k} dF_{k,n}(t) = \overline{F}(x), x \geqslant 0. \tag{2.7}$$

Rewriting (2.7), we obtain

$$\int_0^{\infty} G(x_0,t) dF_{k,n}(t) = 0 \tag{2.8}$$

where

$$G(x_0,t) = \left[\frac{\overline{F}(\frac{x_0}{n-k}+t)}{\overline{F}(t)}\right]^{n-k} - \overline{F}(x_0). \tag{2.9}$$

Equation (2.8) is identical with equation (5.17) of Azlarov and Volodin [1]. Hence it follows from their proof that, under the assumption $X_1 \in C$, (2.8) implies F is exponential. ∎

Remark 2.1. Equation (2.7) can be written as

$$\int_0^\infty \left(\frac{\overline{F}(t+\frac{x}{\alpha})}{\overline{F}(t)}\right)^\alpha d\mu(t) = \overline{F}(x),\ x \geq 0 \text{ for some } \alpha > 0. \tag{2.10}$$

For $\alpha = 1$, this equation reduces to the integrated Cauchy equation. It is well known that the exponential function is the only solution for integrated Cauchy equation when μ is not a lactic measure. As the exponential function satisfies $(\overline{F}(t+\frac{x}{\alpha})/\overline{F}(t))^\alpha = \overline{F}(x)$, we conjecture that the exponential function is the only solution of (2.9). If such conjecture is true then the condition $X_1 \in C$ can be dropped from Theorem 2.3.

Let $Y^k_{1,n-k}$ be the minimum of $(Y^k_1, Y^k_2, \ldots, Y^k_{n-k}.)$. The pdf of $Y^k_{1,n-k}$ can be written as

$$f^k_{1,n-k}(x) = \int_0^\infty (n-k)\left[\frac{\overline{F}(x+t)}{\overline{F}(t)}\right]^{n-k-1} \frac{f(x+t)}{\overline{F}(t)} dF_{k.n}(t).$$

If $n-k=1$, then we get the Theorem 1 of [4].

Theorem 2.4. *Suppose X is an absolutely continuous (with respect to Lebesgue measure) random variable with F(0) =0 and F(x) <1 for all x >0. Then the following two conditions are equivalent*

(i) $F(x) = 1 - e^{-\lambda x}$,
(ii) Y^k_i *and* Y^k_j *are uncorrelated for fixed i and j.*

Proof. By definition, given $X_{k,n}$, Y^k_i and Y^k_j are conditionally independent and identically distributed. Therefore

$$E\left(Y^k_i Y^k_j\right) = E\left[E\left(Y^k_i|X_{k,n}\right)E\left(Y^k_j|X_{k,n}\right)\right] = E\left(E^2(Y^k_i|X_{k,n})\right)$$

and

$$E\left(Y^k_i\right)E\left(Y^k_j\right) = E^2\left(E\left(Y^k_i|X_{k,n}\right)\right).$$

In view of the above two identities, for Y^k_i and Y^k_j to be uncorrelated and it is equivalent to

$$Var\left(E(Y^k_i|X_{k,n})\right) = 0.$$

By the fact $X_{k,n}$ has strictly increasing distribution on $(0,\infty)$, hence $E(Y^k_i|X_{k,n}=t) = constant$ for all $t > 0$. ∎

3. Characterization based on mean residual life

The following theorem characterizes continuous distribution based on the expected value of $Y^k_{1,n-k}$. This expected value can be viewed as the mrl of the series system of $n-k$ independent and identical units.

Theorem 3.1. *Let $X_1, X_2, ..., X_n$ are the life times of n independent and identically distributed components of a system with finite first moment. Let $Y_1^k, Y_2^k, ..., Y_{n-k}^k$ are the residual life times of the n-k components after the failure at time at t of the k-th component We assume X's are absolutely continuous with cdf F such that $F(0) = 0$ and $F(x) < 1$ for all $x > 0$. If $E(Y_{1,n-k}^k | X_{k,n} = t) = g(t)$, for all $t > 0$, where g'(t) exists for all $t > 0$, then*

$$F(x) = 1 - e^{-\int_0^x \frac{1+g'(t)}{(n-k)g(t)} dt}.$$

Proof. We can write

$$E(Y_{1,n-k}^k | X_{k,n} = t) = \int_0^\infty \left[\frac{\overline{F}(x+t)}{\overline{F}(t)}\right]^{n-k} dx, \text{ for } x > 0 \text{ and } \lambda > 0. \tag{3.1}$$

Since $E(Y_{1,n-k}^k | X_{k,n-k} = t) = g(t)$, we obtain from (3.1)

$$\int_0^\infty (\overline{F}(x+t))^{n-k} dx = g(t)(\overline{F}(t))^{n-k}.$$

Substituting $x + t = u$, we obtain

$$\int_0^\infty (\overline{F}(u))^{n-k} dx = g(t)(\overline{F}(t))^{n-k}. \tag{3.2}$$

Differentiating both sides of (3.2) with respect to t, we have

$$-(\overline{F}(t))^{n-k} = g'(t)\overline{F}(t))^{n-k} - (n-k)g(t)(\overline{F}(t))^{n-k} h(t). \tag{3.3}$$

Thus

$$h(t) = \frac{1+g'(t)}{(n-k)g(t)}, \text{ for all, } t \geqslant 0. \tag{3.4}$$

Hence,

$$F(x) = 1 - e^{-\int_0^x \frac{1+g'(t)}{(n-k)g(t)} dt}. \tag{3.5}$$

∎

In the following subsections, we provide characterizations of some distributions as applications of Theorem 3.1.

3.1. Exponential distribution

As a first application we consider the exponential distribution. The pdf of the exponential distribution is given by

$$f(x) = \lambda e^{-\lambda x,}, \ x \geqslant 0, \text{ and } \lambda > 0. \tag{3.6}$$

Proposition 3.1. *Let X be a continuous random variable with $F(0) = 0$ and $F(x) < 1$ for all $x > 0$. Then X has the pdf (3.6) if and only if $E(Y_{1,n-k}^k | X_{k,n} = t) = \delta$, where δ is a constant.*

Proof. We have $E(Y_{1,n-k}^k | X_{k,n} = t) = \int_0^\infty \left[\frac{\overline{F}(x+t)}{\overline{F}(t)}\right]^{n-k} dx.$

Substituting $F(x) = 1 - e^{-\lambda x}$, we obtain

$$E(Y_{1,n-k}^k | X_{k,n-k} = t) = \frac{1}{(n-k)\lambda}.$$

Hence $E(Y_{1,n-k}^k | X_{k,n} = t) = \delta$ with $\delta = \frac{1}{(n-k)\lambda}$. This proves the first part.

Conversely, suppose that $E(Y_{1,n-k}^k | X_{k,n} = t) = \delta$, where δ is a constant. Then

$$F(x) = 1 - e^{-\int_0^x \frac{1}{(n-k)\delta}} = 1 - e^{-\frac{x}{(n-k)\delta}}.$$ ∎

3.2. Power function distribution

Let X be a continuous random variable with pdf

(3.7) $$f(x) = \alpha(1-x)^{\alpha-1}, 0 \leqslant x \leqslant 1, \alpha > 0.$$

The density (3.7) is known as power pdf.

Proposition 3.2. *Let X be a continuous random variable with $F(0) = 0$ and $F(x) < 1$ for all $x < 1$. Then X has the pdf (3.7) if and only if*

$$E(Y^k_{1,n-k}|X_{k,n} = t) = \frac{1-t}{(n-k)\alpha+1}$$

for some $\alpha > 0$.

Proof. We have $F(x) = 1-(1+x)^{\alpha}$, thus

$$E(Y^k_{1,n-k}|X_{k,n} = t) = \frac{\int_t^1 (1-x)^{(n-k)\alpha} dx}{(1-t)^{\alpha(n-k)}} = \frac{1-t}{(n-k)\alpha+1}.$$

Suppose $E(Y^k_{1,n-k}|X_{k,n} = t) = \frac{1-t}{(n-k)\alpha+1}$, $\alpha > 0, 1 \leqslant k < n$, then from (3.5) we get

$$F(x) = 1 - e^{-\int_0^x \frac{\alpha}{1-t} dt} = 1-(1-x)^{\alpha},\ 0 \leq x \leq 1,\ \alpha > 0.$$ ∎

3.3. Pareto distribution

The pdf and the cdf of Pareto distribution are given by

$$f(x) = \frac{\alpha}{(1+x)^{\alpha+1}}, x \geqslant 0, \text{ and } F(x) = 1 - \frac{1}{(1+x)^{\alpha}}$$

respectively. We assume here $\alpha > 1$.

Proposition 3.3. *Let X be a continuous random variable with $F(0) = 0$ and $F(x) < 1$ for all $x > 0$. Then X has Pareto distribution if and only if*

$$E(Y^k_{1,n-k}|X_{k,n} = t) = \frac{1+t}{(n-k)\alpha-1}$$

for some $\alpha > 1$.

Proof. Suppose X has Pareto distribution then

$$E\left(Y^k_{1,n-k}|X_{k,n} = t\right) = \frac{\int_t^\infty \frac{1}{(1+x)^{(n-k)\alpha}} dx}{\frac{1}{(1+t)^{(n-k)\alpha}}} = \frac{1+t}{(n-k)\alpha-1}.$$

Now suppose that $E(Y^k_{1,n-k}|X_{k,n} = t) = \frac{1+t}{(n-k)\alpha-1}$, then from (3.5) we get

$$F(x) = 1 - e^{-\int_0^x \frac{\alpha}{1+t} dt}$$

$$= 1 - \frac{1}{(1+x)^{\alpha}}.$$ ∎

4. A monotonicity result

We need the following definition to clarify our result.

Definition 4.1. [10] *If the ratios below are well defined, Xis said to be smaller than Y in the*

(i) *likelihood ratio order (denoted by $X \leqslant_{lr} Y$) if $g(x)/f(x)$ is increasing in x;*

(ii) *hazard rate order (denoted by $X \leqslant_{hr} Y$) if $\overline{G}(x)/\overline{F}(x)$ is increasing in x;*

(iii) *reversed hazard rate order (denoted by $X \leqslant_{rh} Y$) if $G(x)/F(x)$ is increasing in x;*

(iv) *stochastic order (denoted by $X \leqslant_{st} Y$) if $\overline{G}(x) \geqslant \overline{F}(x)$*

It is well known that $X \leqslant_{lr} Y \Rightarrow X \leqslant_{hr(rh)} Y \Rightarrow X \leqslant_{st} Y$. Bairamov and Arnold [4] *proved that if X_1 is NBU (NWU) then $Y_i^k \leqslant_{st} X_1(Y_i^k \geqslant_{st} X_1)$. This result can be extends to other ordering as the following theorem states.*

Theorem 4.1. (a) *If X_1 has log-concave (log-convex) probability density function then $Y_i^k \leqslant_{lr} X_1 \left(Y_i^k \geqslant_{lr} X_1\right)$.*

(b) *If X_1 has log-concave (log-convex) cumulative distribution function then $Y_i^k \leqslant_{hr} X_1 \left(Y_i^k \geqslant_{hr} X_1\right)$.*

(c) *If X_1has log-concave (log-convex) survival function then $Y_i^k \leqslant_{rh} X_1(Y_i^k \geqslant_{rh} X_1)$.*

Proof. Assume that f is log-concave (log-convex) then $f(x+y)/f(y)$ is decreasing (increasing) in y for every x. Now for $y_1 < y_2$ and arbitrary x, we have

$$\frac{f_1^k(x+y_2)}{f(y_2)} - \frac{f_1^k(x+y_1)}{f(y_1)} = \int_0^{\infty} \left[\frac{f(x+y_2+t)}{f(y_2)} - \frac{f(x+y_1+t)}{f(y_1)}\right] \frac{1}{\overline{F}(t)} dF_{k,n}(t) \leqslant (\geqslant) 0.$$

Hence $Y_i^k \leqslant_{lr} X_1(Y_i^k \geqslant_{lr} X_1)$.

The assertions (b) and (c) can be proven using similar arguments with replacing the probability density function by the survival function and the cumulative distribution function respectively.

Note that f is logconcave is equivalent to say f or X is strongly unimodal while $\overline{F}(x)$ is logconcave is equivalent to say X is IFR. ∎

5. Some concluding remarks

This paper studied various distributional properties of the mean life times of the $n-k$ remaining components after the failure of the k components. Some characterizations of distributions are given based on these distributional properties. A possible application of this paper would be a situation when a system is equipped with an alarm that gets activated when a certain number of its components fail. It is of interest for the engineer to know the properties of the residual lives of the remaining working components for maintenance purposes. Since we are mainly interested about the theoretical development on the residual life of the k out of n system, real life applications along with further developments on this paper are under the current investigation.

Acknowledgement. We are thankful to the anonymous referees for their valuable comments and suggestions, which certainly improved the quality and presentation of the paper. The first author is thankful to Rider University for the award of Davis Fellowship to pursue this Research. Professor Alzaid extends his appreciation to the Deanship of Scientific Research at King Saud University for funding his work through the research group project RGB – VPP – 053. This paper was completed while Dr. Kibria was on sabbatical leave (2010 – 2011). He is grateful to Florida International University for awarding him the sabbatical leave which, gave him excellent research facilities.

References

[1] T. A. Azlarov and N. A. Volodin, *Characterization Problems Associated with the Exponential Distribution*, translated from the Russian by Margaret Stein, translation edited by Ingram Olkin, Springer, New York, 1986.

[2] M. Asadi and I. Bayramoglu, A note on the mean residual life function of a parallel system, *Comm. Statist. Theory Methods* **34** (2005), no. 2, 475–484.

[3] M. Asadi and I. Bairamov, The mean residual life function of a k –out-of-n structure at the system level. *IEEE Transactions on Reliability*, **55** (2006), no. 2, 314–318.

[4] I. Bairamov and B. C. Arnold, On the residual lifelengths of the remaining components in an $n-k+1$ out of n system, *Statist. Probab. Lett.* **78** (2008), no. 8, 945–952.

[5] I. Bairamov, M. Ahsanullah and I. Akhundov, A residual life function of a system having parallel or series structures, *J. Stat. Theory Appl.* **1** (2002), no. 2, 119–131.

[6] R. E. Barlow and F. Proschan, *Statistical Theory of Reliability and Life Testing*, Holt, Rinehart and Winston, Inc., New York, 1975.

[7] S. Kotz and D. N. Shanbhag, Some new approaches to probability distributions, *Adv. in Appl. Probab.* **12** (1980), no. 4, 903–921.

[8] C.-D. Lai and M. Xie, *Stochastic Ageing and Dependence for Reliability*, Springer, New York, 2006.

[9] X. Li and P. Zhao, Some ageing properties of the residual life k-out-of-n systems, *IEEE Transaction on Reliablity*, **55** (2006), no. 3, 535–541.

[10] M. Shaked and J. G. Shanthikumar, *Stochastic Orders*, Springer Series in Statistics, Springer, New York, 2007.