

An Improved Approach for Estimating Returns to Scale in DEA

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Abstract. This article reviews the estimation of Returns To Scale (RTS) in Data Envelopment Analysis (DEA), presented by Golany and Yu and proposes a new method to do this estimation. we show that the new method does not have the shortcoming of the previous one. Furthermore, it is able to evaluate Returns To Scale (RTS) to the right and left of the given unit in all conditions. This method is elaborated by an illustrative example.

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1. Introduction

Data Envelopment Analysis (DEA) is a technique based on mathematical programming for the performance assessment and the evaluation of the efficiency of a set of homogeneous Decision Making Units (DMUs), each of which consumes multiple inputs to produce multiple outputs. The CCR (Charnes, Cooper, Rhodes) [6] and the BCC (Banker, Charnes, Cooper) [3] models are actually two basis DEA models. The latter is established by developing a variable RTS version of the first one. One of the important subjects in DEA is the concept of Returns To Scale (RTS), which is defined as the ratio of the proportional changes in outputs over the proportional changes in inputs. Nowadays, RTS has allocated a wide contribution of DEA literature to itself. First, Banker [2] introduced RTS estimation (Increasing RTS (IRS), Decreasing RTS (DRS), Constant RTS (CRS)) of the CCR model. Banker *et al.* [3] presented an approach, using the sign of the slope parameter of the BCC dual. However, both of these techniques have considered the assumption of unique optimal solution. But in the presence of alternative optimal solutions, the characteristic of RTS is not guaranteed to be unique. So far, some methods have been suggested to overcome this problem. Banker and Thrall [5] presented a method by surveying all optimal solutions of BCC and CCR models. See also Jahanshahloo and Soleimani-Damaneh [11] and Zarepisheh *et al.* [16]. It should be noted that a related issue to RTS is the Most Productive Scale Size (MPSS) that was first introduced by Banker [2]. Some methods have been suggested to estimate this notion [7,10]. There are a few review papers which elaborate different basic methods in the RTS literature. See Banker *et al.* [4] for more details. Recently, Khodabakhshi *et*

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al. [13] examined estimating RTS in both stochastic and fuzzy data envelopment analysis. Reviewing the customary methods, we could find out that: a) input and output oriented models may give different results in their RTS findings; and b) the RTS estimated by means of these methods holds only in the current position of the under-evaluation DMU. Focusing on these two points, Golany and Yu [8] discussed the estimation of RTS to the right and left neighborhood of the given DMU, and proposed a method based on solving two LP models to do this task. Hadjicostas and Soteriou [9] have recently presented a more general definition of these two concepts in RTS from scale elasticity measure point of view. Two of the suggested approaches to estimate the right and left RTS are Jahanshahloo *et al.* [12] and Zarepisheh and Soleimani-damaneh [15]. The former one suggests an enhanced method by focusing on Golany and Yu's method (GY method, hereafter), and the latter presents a dual simplex-based procedure, considering Hadjicostas and Soteriou's definition of the right and left RTS.

This paper suggests a new method based on solving two LP models, one in input orientation and the other in output orientation, by regarding the concept of the right and left RTS from GY point of view. This method, unlike GY, is feasible for all DMUs and could overcome the problem of indeterminacy of RTS for some DMUs. In other words, its objective is to provide an approach which seeks the precise classification of RTS around the DMU being evaluated.

The structure of the paper is as follows: In Section 2 the GY method is reviewed, and the new method is introduced in Section 3. A numerical example is presented in Section 4. Finally, in Section 5, some conclusions are provided.

2. GY method

Assume we have a set of n DMUs, where $DMU_j : j = 1, \dots, n$ consumes the inputs $\mathbf{x}_j = (x_{1j}, \dots, x_{mj})$ to produce the outputs $\mathbf{y}_j = (y_{1j}, \dots, y_{sj})$, with the same m inputs and the same s outputs in (possibly) different amounts. One of the purposes of DEA is evaluating the efficiency of DMUs, which can be done by the CCR model or the BCC model of DEA. We present the BCC model in the envelopment form in the input and output orientations for $DMU_o (o \in 1, \dots, n)$ as follows: (input-oriented):

$$\begin{aligned}
 z = \min & \theta - \varepsilon \left(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \\
 s.t. & \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \theta x_{i0} \quad i = 1, \dots, m \\
 & \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = y_{r0} \quad r = 1, \dots, s \\
 & \sum_{j=1}^n \lambda_j = 1 \\
 & (\lambda_j, s_i^-, s_r^+) \geq \mathbf{0} \quad \forall i, r, j.
 \end{aligned}
 \tag{2.1}$$

and
(output-oriented):

$$\begin{aligned}
 z = \max & \varphi + \varepsilon \left(\sum_{i=1}^m t_i^- + \sum_{r=1}^s t_r^+ \right) \\
 \text{s.t.} & \sum_{j=1}^n \mu_j x_{ij} + t_i^- = x_{i0} \quad i = 1, \dots, m \\
 & \sum_{j=1}^n \mu_j y_{rj} - t_r^+ = \varphi y_{r0} \quad r = 1, \dots, s \\
 & \sum_{j=1}^n \mu_j = 1 \\
 (2.2) & \quad (\mu_j, t_i^-, t_r^+) \geq \mathbf{0} \quad \forall i, r, j,
 \end{aligned}$$

where t_i^- and t_r^+ are input and output slacks and $\mathbf{0}$ is a vector with all components equal to zero. The term $\varepsilon > 0$ is also a non-Archimedean infinitesimal which is smaller than any positive real number. (To determine an assurance value for $\varepsilon > 0$ see the approach suggested in [1,14]).

DMU₀ is an efficient DMU if $z = 1$ and the slacks are zero in all optimal solutions in each of the above models.

The point which is emphasized in Golany and Yu [8] is the fact that the RTS is a local feature, while it has been left unnoticed in most of the previous studies on DEA-based RTS. So, they investigate the identification of RTS, based on the existence of solutions in the four regions determined in the neighborhood of the unit under evaluation. They proposed the two following models to do this task:

$$\begin{aligned}
 \min & \beta - \varepsilon \left(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \\
 \text{s.t.} & \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \beta x_{i0} \quad i = 1, \dots, m \quad (3a) \\
 & \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = (1 + \delta) y_{r0} \quad r = 1, \dots, s \quad (3b) \\
 & \sum_{j=1}^n \lambda_j = 1 \quad (3c) \\
 (2.3) & \quad (\lambda_j, s_i^-, s_r^+) \geq \mathbf{0} \quad \forall i, r, j.
 \end{aligned}$$

and

$$\begin{aligned}
 \max & \alpha + \varepsilon \left(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \\
 \text{s.t.} & \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = (1 - \eta) x_{i0} \quad i = 1, \dots, m \\
 & \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = \alpha y_{r0} \quad r = 1, \dots, s
 \end{aligned}$$

$$(2.4) \quad \begin{aligned} & \sum_{j=1}^n \lambda_j = 1 \\ & (\lambda_j, s_i^-, s_r^+) \geq \mathbf{0} \quad \forall i, r, j. \end{aligned}$$

where δ and η assume a positive small arbitrary value. From now on, the superscript “*” indicates the optimal value. (*GY algorithm*):

Step 1: Solve (2.3) to determine the RTS to the right of DMU_0 :

- 1i. $(1 + \delta) > \beta^* > 1 \implies$ increasing RTS.
- 1ii. $\beta^* \leq 1 \implies$ DMU_0 is BCC-inefficient.
- 1iii. $(1 + \delta) = \beta^* \implies$ constant RTS.
- 1iv. $(1 + \delta) < \beta^* \implies$ decreasing RTS.
- 1v. No feasible solution \implies there is no data to determine the RTS to the right of DMU_0 .

Step 2: Solve (2.4) to determine the RTS to the left of DMU_0 :

- 2i. $1 > \alpha^* > (1 - \eta) \implies$ decreasing RTS.
- 2ii. $\alpha^* \geq 1 \implies$ DMU_0 is BCC-inefficiency.
- 2iii. $(1 - \eta) = \alpha^* \implies$ constant RTS.
- 2iv. $\alpha^* < (1 - \eta) \implies$ increasing RTS.
- 2v. No feasible solution \implies there is no data to determine the RTS to the left of DMU_0 .

In fact, by solving Models (2.3) and (2.4), we reach some projection in the immediate neighborhood to the right and left of DMU_0 .

In order to comprehend this issue, consider a set of 6 hypothetical DMUs in Figure 1. As an example, after solving Models (2.3) and (2.4) for DMU_C we reach the points $(\beta^* x_c, (1 + \delta) y_c)$ and $((1 - \eta) x_c, \alpha^* y_c)$ in the right and left neighborhood of DMU_C , respectively (It is shown with ellipsis). As it is clear in Figures 1 and 3, the paths followed by GY models lie outside the PPS. This property causes GY method to fail for some frontier DMUs. To get the point better, the infeasibility of Models (2.3) and (2.4) have been shown in Figure 3 for DMU_D and DMU_A , respectively.

3. The new method

As noted in GY method, in order to estimate RTS more precisely, it is fair to be examined locally. For doing so, we find some certain points along the frontier in the neighboring areas around the DMU; and then, comparing the performance of these obtained points to that of the DMU, we will be able to determine the RTS on its right and left sides. In this section we try a new method, based on solving two LP models, in order to reach the above-mentioned goal.

In our proposed approach, the two following models are used to estimate the right and left RTS, respectively:

$$\begin{aligned} & \max \hat{\beta} + \varepsilon \left(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \\ & s.t. \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = (1 + \hat{\delta}) x_{i0} \quad i = 1, \dots, m \\ & \quad \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = \hat{\beta} y_{r0} \quad r = 1, \dots, s \end{aligned}$$

$$(3.1) \quad \begin{aligned} \sum_{j=1}^n \lambda_j &= 1 \\ (\lambda_j, s_i^-, s_r^+) &\geq \mathbf{0} \quad \forall i, r, j. \end{aligned}$$

and

$$(3.2) \quad \begin{aligned} \min \hat{\alpha} - \varepsilon & \left(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \\ \text{s.t.} \quad \sum_{j=1}^n \lambda_j x_{ij} + s_i^- &= \hat{\alpha} x_{i0} \quad i = 1, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ &= (1 - \hat{\eta}) y_{r0} \quad r = 1, \dots, s \\ \sum_{j=1}^n \lambda_j &= 1 \\ (\lambda_j, s_i^-, s_r^+) &\geq \mathbf{0} \quad \forall i, r, j. \end{aligned}$$

where $\hat{\delta}$ and $\hat{\eta}$ assume a positive small arbitrary value. Now, by using $\hat{\beta}^*$ and $\hat{\alpha}^*$ as the optimal values of objective function of (3.1) and (3.2) respectively, we present the following procedure to estimate RTS of all BCC-efficient DMUs.

(The procedure)

Step 1. Solve (3.1) to estimate right RTS of DMU_0 :

- 1-1). If $(1 + \hat{\delta}) > \hat{\beta}^* \geq 1 \implies$ decreasing RTS.
- 1-2). If $(1 + \hat{\delta}) < \hat{\beta}^* \implies$ increasing RTS .
- 1-3). If $(1 + \hat{\delta}) = \hat{\beta}^* \implies$ constant RTS.

Step 2. Solve (3.2) to estimate left RTS of DMU_0 :

- 2-1). If $(1 - \hat{\eta}) > \hat{\alpha}^* \implies$ decreasing RTS.
- 2-2). If $(1 - \hat{\eta}) < \hat{\alpha}^* \leq 1 \implies$ increasing RTS.
- 2-3). If $(1 - \hat{\eta}) = \hat{\alpha}^* \implies$ constant RTS.

In Figure 2 the paths taken by Models (3.1) and (3.2) are indicated on the diagram. For example by solving model (3.1), the projection of the point $((1 + \hat{\delta})X_c, Y_c)$ is obtained on the frontier, which is actually the optimal solution to this model (the point $((1 + \hat{\delta})X_c, \hat{\beta}^* Y_c)$).

Lemma 3.1. *Models (3.1) and (3.2) are always feasible.*

In what follows in this part, we prove that where GY models are feasible, the solution offered by GY is also accessible to our method; and where ever GY method fails, by solving the new method, we can reach the certain neighboring points. So the problem of indeterminacy of RTS for these DMUs is dealt with.

Theorem 3.1. *The optimal solution of Model (2.3) and Model (2.4) is an optimal solution for Model (3.1) and Model (3.2) respectively and vice versa.*

Proof. We just prove that the optimal solution of Model (2.3) is an optimal solution for Model (3.1); the other cases can be proved similarly.

Suppose, we solve Model (2.3) for DMU_0 , considering the arbitrary small positive value assigned to δ and obtain the optimal solution $(\beta^*, \delta, s_i^{-*}, s_r^{+*}, \lambda_j^*)$. By putting this solution

in (3a) and (3b) we have:

$$\sum_{j=1}^n \lambda_j^* x_{ij} + s_i^{-*} = \beta^* x_{i0} \quad i = 1, \dots, m \quad (I)$$

$$\sum_{j=1}^n \lambda_j^* y_{rj} - s_r^{+*} = (1 + \delta) y_{r0} \quad r = 1, \dots, s \quad (II)$$

Considering $\beta^* > 1$, $(1 + \delta) \geq 1$ and $(s_i^{-*}, s_r^{+*}, \lambda_j^*) \geq 0$, the point $(\beta^*, \delta, s_i^{-*}, s_r^{+*}, \lambda_j^*)$ can be a feasible solution for Model (3.1), in which β^* plays the role of $(1 + \hat{\delta})$ and $(1 + \delta)$ plays the role of $\hat{\beta}$ in this model. By contradiction, we suppose that the above-mentioned solution is not an optimal solution for Model (3.1), so:

$$\exists \tilde{\beta} \geq 1 \text{ s.t. } \tilde{\beta} > \hat{\beta} = (1 + \delta)$$

and, in (II) we have:

$$\sum_{j=1}^n \lambda_j^* y_{rj} + s_r^{+*} = (1 + \delta) y_{r0} < \tilde{\beta} y_{r0} \quad r = 1, \dots, s$$

$$\implies \sum_{j=1}^n \lambda_j^* y_{rj} + s_r^{+*} + \tilde{s}_r^+ = \tilde{\beta} y_{r0} \quad r = 1, \dots, s$$

Therefore, the solution $(\tilde{\beta}, \delta, s_i^{-*}, s_r^{+*} + \tilde{s}_r^+, \lambda_j^*)$ is a feasible solution for Model (2.3), and after putting this solution in the objective function we have:

$$\beta^* - \varepsilon(\sum_{i=1}^m s_i^{-*} + \sum_{r=1}^s s_r^{+*}) > \beta^* - \varepsilon(\sum_{i=1}^m s_i^{-*} + \sum_{r=1}^s (s_r^{+*} + \tilde{s}_r^+))$$

which contradicts the assumption. ■

Theorem 3.2. Assume that GY models are feasible for DMU_0 , then the type of RTS estimated by GY method for DMU_0 is the same as that estimated by the new method.

Proof. Here, we prove one of the cases, and the others can be similarly proved.

Suppose that by using GY method, DMU_0 has right IRS and $(\beta^*, \delta, s_i^{-*}, s_r^{+*}, \lambda_j^*)$ is the optimal solution for Model (2.3); so, considering GY algorithm, we have $(1 + \delta) > \beta^* > 1$. By contradiction, we suppose that DMU_0 has right NIRS by the new method. According to the above theorem, this point is an optimal solution for Model (3.1) and, according to the procedure in Section 4, we have $(1 + \delta) \leq \beta^*$, which contradicts the assumption and completes the proof. ■

As mentioned in GY algorithm, for a special DMUs, at least one of the GY models could be infeasible, which causes the GY method to fail, and not to be capable of providing precise information about RTS behaviour. It should be noted that, Jahanshahloo *et al.* [12], by focusing on the point that feasibility or infeasibility of GY models depend on the values assigned to δ and η , present a method to restrict the values of these two parameters. Their enhanced algorithm requires to determine an assurance interval for the feasibility of GY models, a task which is done by solving two auxiliary LP models for each DMU. Both of the new method and enhanced approach provide a remedy to the infeasibility problem of GY models for some efficient DMUs. However, the latter method is only able to determine the general RTS for these DMUs, while by using the new method not only could we overcome the disadvantages of GY method, but we will be also able to determine the type of the right and left RTS for each DMU particularly.

4. Illustrative example

In this part, we consider an example that has been presented by Jahanshahloo *et al.* [12]; but before doing so, we have the following definition.

Definition 4.1. *The general RTS of a DMU is DRS (IRS) if both RTS to the left and RTS to the right are DRS (IRS). Otherwise, its general RTS is CRS.*

Example 4.1. We consider a group of 12 DMUs, with two inputs and two outputs. The data for these DMUs are given in Table 1. The results obtained from GY method and the new method are given in Table 2 and Table 3, respectively.

Table 1. Data

DMU	A	B	C	D	E	F	G	H	I	J	K	L
Input	6	7	6	7	5	4	5	6	6	8	5	9
	4	5	5	4	5	6	8	7	6	8	7	6
Output	6	4	3	5	3	3	5	3	4	6	5	9
	2	3	3	4	2	3	6	5	5	6	5	3

Table 2. Results obtained after using the GY method with $\epsilon = \delta = \eta = 0.001$

DMU	β^*	rightRTS	α^*	leftRTS
A	1.0010	CRS	Infeas	?
B	0.8680	Ineff	1.4011	Ineff
C	0.9389	Ineff	1.3297	Ineff
D	1.0027	DRS	Infeas	?
E	1.0000	Ineff	Infeas	?
F	1.0004	IRS	Infeas	?
G	Infeas	?	0.0975	IRS
H	0.9235	Ineff	0.0993	Ineff
I	1.0002	DRS	0.9976	IRS
J	Infeas	?	0.9993	DRS
K	1.0010	CRS	0.9976	IRS
L	Infeas	?	0.9990	CRS

Table 3. Results obtained after using the new method with $\epsilon = 0.001$

DMU	$\hat{\beta}^*$	rightRTS	$\hat{\alpha}^*$	leftRTS
A	1.0010	CRS	1.0000	IRS
B	1.4028	Ineff	0.8678	Ineff
C	1.3370	Ineff	0.9387	Ineff
D	1.0004	DRS	1.0000	IRS
E	1.2613	Ineff	1.0000	IRS
F	1.0027	IRS	1.0000	IRS
G	1.0000	DRS	0.9996	IRS
H	1.1007	Ineff	0.9227	Ineff
I	1.0006	DRS	0.9996	IRS
J	1.0000	DRS	0.9985	DRS
K	1.0010	CRS	0.9996	IRS
L	1.0000	DRS	0.9990	CRS

In Table 4, we just show results (right RTS (RRTS), left RTS (LRTS), general RTS (GRTS)) obtained after using GY method, the enhanced procedure (Jahanshaloo *et al.*

[11]) and our method for efficient DMUs .

Table 4. Results obtained after using different methods for efficient DMUs

DMU	<i>GY method</i>			<i>The enhanced method</i>			<i>The new method</i>		
	<i>RRTS</i>	<i>LRTS</i>	<i>GRTS</i>	<i>RRTS</i>	<i>LRTS</i>	<i>GRTS</i>	<i>RRTS</i>	<i>LRTS</i>	<i>GRTS</i>
<i>A</i>	<i>CRS</i>	?	?	<i>CRS</i>	?	<i>CRS</i>	<i>CRS</i>	<i>IRS</i>	<i>CRS</i>
<i>D</i>	<i>DRS</i>	?	?	<i>DRS</i>	?	<i>CRS</i>	<i>DRS</i>	<i>IRS</i>	<i>CRS</i>
<i>F</i>	<i>IRS</i>	?	?	<i>IRS</i>	<i>IRS</i>	<i>IRS</i>	<i>IRS</i>	<i>IRS</i>	<i>IRS</i>
<i>G</i>	?	<i>IRS</i>	?	?	<i>IRS</i>	<i>CRS</i>	<i>DRS</i>	<i>IRS</i>	<i>CRS</i>
<i>I</i>	<i>DRS</i>	<i>IRS</i>	<i>CRS</i>	<i>DRS</i>	<i>IRS</i>	<i>CRS</i>	<i>DRS</i>	<i>IRS</i>	<i>CRS</i>
<i>J</i>	?	<i>DRS</i>	?	<i>DRS</i>	<i>DRS</i>	<i>DRS</i>	<i>DRS</i>	<i>DRS</i>	<i>DRS</i>
<i>K</i>	<i>CRS</i>	<i>IRS</i>	<i>CRS</i>	<i>CRS</i>	<i>IRS</i>	<i>CRS</i>	<i>CRS</i>	<i>IRS</i>	<i>CRS</i>
<i>L</i>	?	<i>CRS</i>	?	?	<i>CRS</i>	<i>CRS</i>	<i>DRS</i>	<i>CRS</i>	<i>CRS</i>

As it can be seen in Table 2, the Model (2.3) and (2.4) are infeasible for DMUs G, J, L and DMUs A, D, E, respectively. In Table 4, the enhanced procedure determines only the general RTS for these DMUs, while by using the new method, we will be able to eliminate the indeterminacy of RTS and determine RTS for all efficient DMUs particularly.

5. Conclusion

Determining the identification of RTS for a DMU can be used to recognize the optimal size of the unit. For this, we need precise information. So Golany and Yu introduced the concept of the right and left RTS, and suggested an algorithm to estimate these two concepts based on solving two LP models. An important advantage of their method is that RTS behavior is investigated locally. However, this method fails when at least one of the models is infeasible. Here in this paper, we present a new algorithm based on two LP models in input and output orientations. In fact GY and the new method use different processes to reach the same goal, which is to observe the RTS behavior in two specified directions around the DMU under evaluation. The superiority of the new method compared to the previous one is that its models are always feasible for all DMUs. This advantage derives from the property of paths followed by the models used in the new method.

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Figures

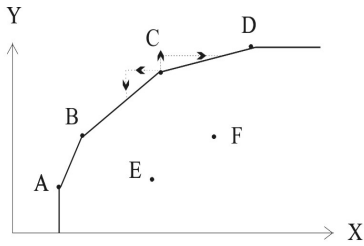


Fig.1. The paths followed by Models (2.3) and (2.4)

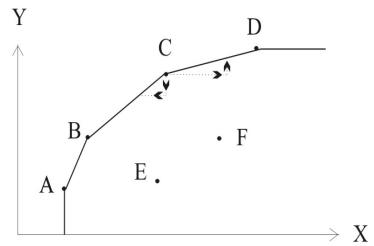


Fig.2. The paths followed by Models (3.1) and (3.2)

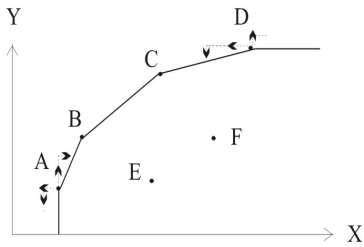


Fig.3. The infeasibility of the Models (2.3) and (2.4) for units A and D

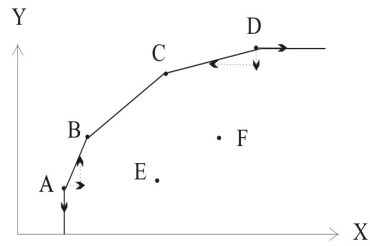


Fig.4. The feasibility of the Models (3.1) and (3.2) for units A and D