BULLETIN
of the
MALAYSIAN MATHEMATICAL SOCIETY

Bull. Malaysian Math. Soc. (Second Series) **22** (1999) 157-167

Classification and Clustering of Printed Mathematical Symbols with Improved Backpropagation and Self-Organizing Map

[1]SITI MARIYAM HJ. SHAMSUDDIN, [2]MD. NASIR SULAIMAN AND [3]MASLINA DARUS

[1]Fakulti Sains Komputer dan Sistem Maklumat, Universiti Teknologi Malaysia
[2]Jabatan Sains Komputer, Fakulti Sains Komputer dan Teknologi Maklumat, Universiti Putra Malaysia
[3]Pusat Sains Matematik, Fakulti Sains dan Teknologi, Universiti Kebangsaan Malaysia

Abstract. This paper proposes a derivation of an improved error signal for hidden layer in the backpropagation model, and its experimentation evaluation of utilizing various moments order as pattern features in recognition of printed mathematical symbols in the classification phase. The moments that have been used are geometric moment invariants in which they have been used as feature extraction for images with various orientations and scaling. In this study, we find that the recognition and the convergence rates are better using an improved backpropagation compared to standard backpropagation. In addition, we cluster these invariants on a visual map using Self-Organizing Map (SOM) whereby mathematical symbols with similar shape belong to the same cluster.

1. Introduction

Moment invariants have been proposed as pattern sensitive features in classification and recognition applications. Hu was the first to introduce the geometric moment invariants which are invariant under change of size, translation, and orientation [1]. Printed and handwritten images have been tested with different orientations using Hu's moments in many literatures [2,3,4,5]. Moments and functions of moments can provide characteristics of an object that uniquely represent its shape and have extensively employed as the invariant global features of an image in pattern recognition and image classification since 1960s.

This paper discusses a formulation of an improved backpropagation model in the classification phase of printed mathematical symbols and the clustering approach on these symbols using SOM. These symbols are extracted using geometric moment invariants. The rest of the paper is organized as follows. Section 2 gives review on feature extraction, pixel features, and geometric moments. A summary on backpropagation model and a formulation of an improved backpropagation is given in section 3. Section 4 gives an overview of self-organizing map. Section 5 explains the experimental results and section 6 is the conclusion.

2. Feature extraction

Feature extraction is the process to represent the image by a suitable set of features. In this paper, a type of feature which is presented in the following subsections is extracted to feed the backpropagation model. The image of input symbols is captured using a scanner with 300DPI resolution.

2.1. Pixel features

The collected mathematical symbols are converted into binary image using a thresholding technique. Black pixel (1) represents the foreground and white (0) pixel represents the background.

2.2. Geometric moment features

The Geometric moments [1] of order $p+q$ of a digital image are defined as:

$$m_{pq} = \sum_x \sum_y f(x, y)\ x^p y^q$$

where

$$p, q = 0, 1, 2, \cdots \infty.$$

The translation invariant central moments are obtained by placing origin at the centroid of the image,

$$\mu_{pq} = \sum_x \sum_y f(x, y)(x - x_0)^p\ (y - y_0)^q$$

where

$$x_0 = \frac{m_{10}}{m_{00}} \quad \text{and} \quad y_0 = \frac{m_{01}}{m_{00}}.$$

Then the scale invariant central moments are defined as:

$$\eta_{pq} = \frac{\mu'_{pq}}{(u')^{\frac{(p+q)}{2}+1}} = \frac{\mu_{pq}}{u^{\frac{(p+q)}{2}+1}}, \quad p + q = 2, 3, \cdots$$

Finally, the scale invariant Radial-Geometric moments are defined as [6]:

$$R_{pq} = \frac{\sum_x \sum_y f(x, y)(\bar{x}^2 + \bar{y}^2)^{\frac{1}{2}}\, \bar{x}^p \bar{y}^q}{\alpha^{\frac{(p+q+3)}{2}}}$$

3. Backpropagation model

The backpropagation model is introduced by Rumelhart *et. al* [7]. This network has served as a useful methodology to train multilayered neural networks for a wide variety of applications. The backpropagation model is a supervised learning algorithm using feedforward networks which make use of target values. Backpropagation model is basically a gradient descent method and its objective is to minimize the mean squared error between the target values and the network outputs. Thus the mean square error function (MSE) is defined as (Figure 1):

$$E = \frac{1}{2}\sum_{k}(t_{kj} - o_{kj})^2$$

where

t_{kj} is the target output from node *k* to node *j*,

o_{kj} is the network output from node *k* to node *j*.

The output-layer errors are successively propagated backwards through the network. The weights are adjusted during training using the following formula :

$$w_{kj}(t+1) = w_{kj}(t) + \frac{\alpha_k \partial E(w)}{\partial w_{kj}} + \beta \Delta w_{kj}(t)$$

where

$w_{kj}(t+1)$ is the updating weight from node *k* to node *j* at time *t*,

$w_{kj}(t)$ is the weight from node *k* to node *j* at time *t*,

$\frac{\partial E(w)}{\partial w_{kj}} = \sum \delta_k o_k$

$\delta_k = f'(\text{net}_k)(t_k - o_k)$ is the error signal for the output node,

$o_k = f'(\text{net}_k)\sum w_{kj}\delta_{kj}$ is the output for the output layer,

$\Delta w_{kj}(t) = w_{kj}(t) - w_{kj}(t-1)$, change of weights at time *t*,

β is the momentum factor,

$\alpha_k, (k = 1, 2, \cdots, n)$ is the learning rate and depend on $\left|\frac{\partial E(w)}{\partial w_{kj}}\right|$.

Since the usual training techniques for neural networks work by finding a set of weights which makes all the derivatives of the usual error zero, any set of weights which makes all output activations ± 1 (using hiperbolic tangent as an activation function) will make all the derivatives zero. If not all the errors are zero then the minimization method finds a local extremum or a saddle points[8]. The operations of mean square error function are limited by the failure of training to converge. Occasionally, such failure can be attributed to a poor starting point and the solution is merely to restart the training. More often, training finds a comfortable local minimum and refuses to move beyond it, and can cause instability of the internal structure of the network. Furthermore, if we look at the output unit of,

$$\delta_k = f'(\text{net}_k)(t_k - o_k)$$

can be zero not only when $t_0 = o_k$, but also when $f'(\text{net}_k) = 0$. This leads to $\delta_k = 0$ for internal units as well. Therefore, all the derivatives are zero, and the network loses its learning ability.

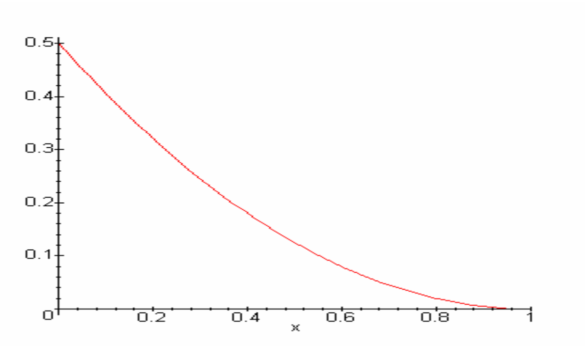

Figure 1. Mean square error of standard backpropagation

3.1. An improved backpropagation model

In this study, we consider a sigmoid function of $\frac{1}{1+e^{-\beta x}}$ as an activation function in the derivation of the proposed method for the backpropagation model, where β is a constant parameter. Therefore the modified error for standard backpropagation (*mm*) would be:

$$mm = \sum_k \rho_k$$

with

$$\rho_k = \frac{E_k^2}{2a_k(1 - a_k^2)},$$

where

$$E_K = t_k - a_k$$

and

E_k error at output unit k,
t_k target value of output unit k,
a_k an activation of unit k.

By taking partial derivatives for the updating weight using chain rule, we generate an error signal for an improved backpropagation of the output layer as,

$$\delta_k = \frac{2\left(E + \rho\left(1 - 3a_k^2\right)\right)}{1 + a_k}$$

and an error signal for an improved backpropagation of the hidden layer is the same as standard backpropagation ,

$$\delta_j = \sum \delta_k w_k f'(a_j)$$

where

w_k weight on connection between unit,
$f(a_j)$ a sigmoid function.

This expression can be illustrated geometrically as,

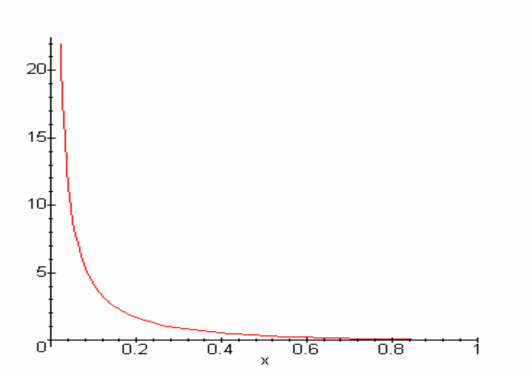

Figure 2. An improved error of backpropagation

4. Self Organizing Map (SOM)

The Kohonen self organizing map was invented by Professor Teuvo Kohonen, and is closely modeled on the way that certain parts of the brain are known to work [9]. SOM is an unsupervised artificial neural network consisting of one input layer and an output layer. Neurons of the output layer become specifically tuned to various multi dimensional input observations through a competitive learning process creating a two dimensional map consisting of regions belonging to distinct classes of input patterns.

The Kohonen algorithm is presented as follows:

Step 1. Each processing node computes an input intensity value which is a similarity measure between its weight vector and the input pattern vector.
Step 2. Select the winning node as the one with the highest intensity value i.e., its weight vector is the closest to the input pattern.
Step 3. Set the winner node's output to 1; all other units have their output set to zero.
Step 4. Modify the winner nodes's weight according to the following rule:

$$\text{Weight}_{\text{new}} = \text{Weight}_{\text{old}} + \left(\text{portion of } x - \text{Weight}_{\text{old}}\right)$$

In this study, SOM algorithm is implemented by using SOM_PAK [10] to perform the mathematic symbols clustering. The geometric invariants are placed into a 9-dimensional vectors. The steps used for using the SOM_PAK for clustering are illustrated below [11]:

i. Initialize map using mathematical symbols data with 30×20 dimension (map size) with hexagonal type for the lattice, and bubble step function.

ii. The map is trained in 2 phases. The first phase is the reference vectors of the map units are ordered. The neighborhood radius is taken almost equal to the diameter of the map and decreases to 1.0 during training. The learning rate, α, decreases to 0.03 during the training phase. In the second phase, the values of the reference vectors are fine-tuned.

iii. Evaluate the resulting quantization error over all the samples in the data file.

iv. Calibrate the map units using known input data samples.

v. Generate a list of coordinates corresponding to the best-matching unit in the map for each data sample in the data file. The best-matching units are then labeled.

vi. Generate a Sammon mapping from 9-dimensional input vectors to 2-dimensional points on a plane producing an encapsulate postcript file.

vii. Finally generate an encapsulated postcript code from one selected component plane of the map imaging the values of the components using gray levels.

5. Experimental results

We test 40 samples of printed mathematical symbols (Figure 3) with various orientations and different fonts. The moments are calculated up to the 4^{th} order for all samples. Feature moments from order 2 to 4 were used as feature patterns in our experiments. The learning rate and momentum parameter were set to 0.9 and 0.2 for the standard backpropagation and 0.2 and 0.7 for the an improved backpropagation model with sigmoid as an activation function.

We used moments of order 2, 3 and 4 for classifications using an improved backpropagation and standard backpropagation model. Table 1 shows the invariants features of some mathematical symbols. Figure 4 shows that the convergence rates of an improved backpropagation is better compared to a standard backpropagation for printed mathematical symbols. The recognition rates are 100% recognized by an improved backpropagation and 95% by a standard backpropagation model. Table 2 shows the recognition rates, processing time and iterations for geometric invariants of printed mathematical symbols using an improved backpropagation and standard backpropagation.

Figure 3. Samples of printed mathematical symbols

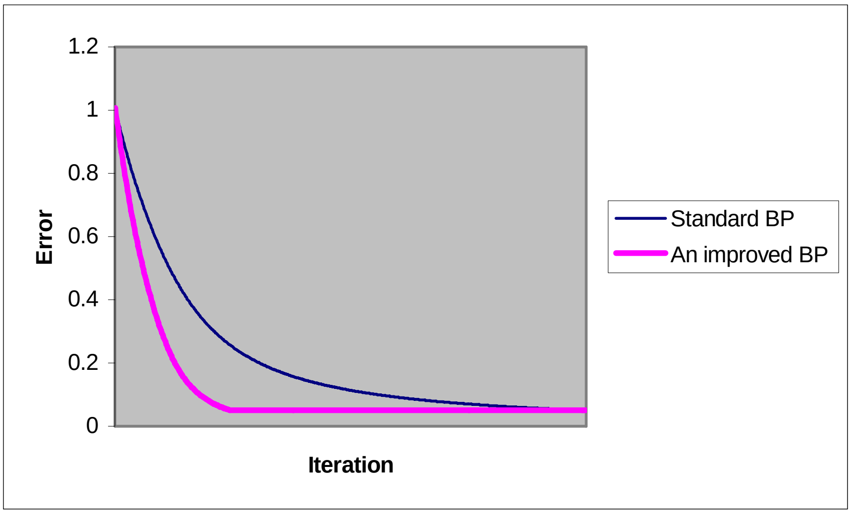

Figure 4. Convergence rates of printed mathematical symbols

Table 1. Invariants features of Mathematical symbols

Symbol	η_{02}	η_{03}	η_{11}	η_{12}	η_{13}	η_{21}	η_{22}	η_{30}	η_{31}
χ	0.86711	0.18849	0.08184	0.16839	0.12728	0.01923	0.24873	0.12638	0.04125
	0.54536	0.02198	0.02583	0.0241	0.01231	0.01844	0.1193	0.00087	0.00535
δ	0.58806	0.05518	0.08122	0.00895	0.07504	0.01626	0.18318	0.03664	0.05776
	0.61814	0.00880	0.05408	0.01927	0.05894	0.00178	0.07934	0.01363	0.02165
ε	0.88477	0.14812	0.01660	0.13137	0.06236	0.02861	0.21195	0.04551	0.00528
	0.80491	0.05006	0.03593	0.01596	0.04019	0.00195	0.12116	0.01324	0.01841
α	0.73293	0.05052	0.16291	0.05135	0.11263	0.02107	0.1385	0.00799	0.07375
	0.66253	0.08034	0.03918	0.01415	0.10883	0.01978	0.11662	0.0049	0.01161
β	0.91948	0.02059	0.01081	0.06653	0.00924	0.01543	0.15602	0.00388	0.00697
	0.82281	0.06182	0.02135	0.03221	0.03237	0.01006	0.12365	0.00398	0.00606
μ	2.213	0.71402	0.059	0.22918	0.00903	0.01181	0.63556	0.05279	0.08960
	2.15402	0.18761	0.08548	0.33771	0.81689	0.11741	0.70659	0.03468	0.13071
π	0.15565	0.00002	0.00662	0.00547	0.00182	0.00775	0.03896	0.02263	0.00017
	0.16081	0.01299	0.01091	0.00812	0.00205	0.01267	0.04902	0.04908	0.01069

Table 2. Recognition rates and processing time for printed mathematical symbols

Handprinted digits	Standard backpropagation	An improved backpropagation
Geometric moments	Time = 1540 seconds	Time = 506 seconds
Iterations	17320	5700
Recognition rates	95%	100%

Figure 5 shows that the mathematical symbols of various scaling, translating and rotating are clustered together to the same class. SOM uses Euclidean distance as a criterion to cluster the approriate input patterns. From 40 image samples, there are 10 distinct clusters (Table 3). A light shade on the map indicates the average distance of neighboring codebook vectors is small, indicating there is a small variation in the shape of the symbol. The dark shades indicate the distance of codebook vectors from the reference vectors is large. Thus it indicates a drastic variation in the shape of the associated symbols.

Table 3. Clusters and their symbols

Cluster	Symbols
1	χ
2	δ
3	ε
4	α
5	η
6	β
7	κ
8	λ
9	μ
10	π

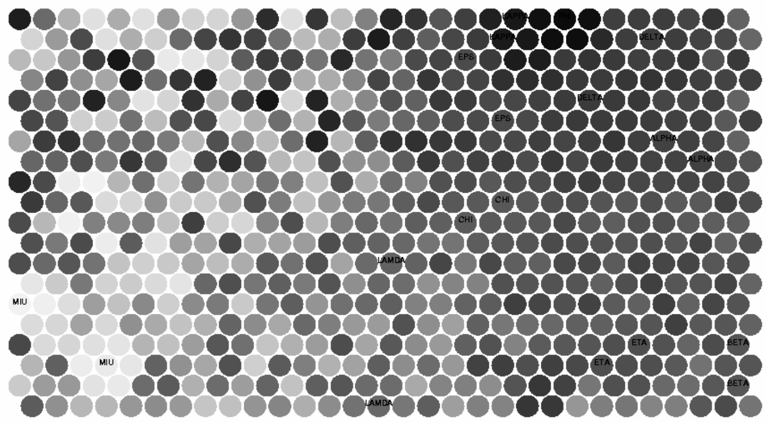

Figure 5. Clustering of mathematical symbols

6. Conclusion

In this paper, we derive a formulation for an improved backpropagation and cluster mathematical symbols using SOM for classification purposes. Geometric moments are used as a feature extraction for printed mathematical symbols with different orientations and fonts. These moments are calculated up to the 4^{th} order. The experiments have shown that the classification rates for printed mathematical symbols are better for moments of higher order, and the results are promising. The convergence and recognition rates are better using an improved backpropagation compared to a standard backpropagation. In addition, these invariants are clustered using self-organizing map for better visualization since it is a useful algorithm for clustering input pattern based on their unique features.

References

1. M.K. Hu, Visual problem recognition by moment invariant, *IRE Trans. Inform. Theory, **IT-8*** (1962), 179-187.
2. A. Khotanzad and J. Lu, Classification of invariant image representations using a neural network, *IEEE Transaction on Acoustics, Speech, and Signal Processing* **38** (1990).
3. S.X. Liao and M. Pawlak, On image analysis by moments, *IEEE Trans. On Pattern Analy. and Machine Intell* **18** (1996), 254-266.
4. O.D. Trier, A.K. Jains and T. Taxt, Feature extraction methods for character recognition - a survey, *Pattern Recognition* **29** (1996), 641-662.

5. J. Dayhoff, *Neural Network Architecture: An Introduction,* Van Nostrand Reinhold, NY, 1990.
6. M. Dehghan and K. Faez, Farsi handwritten character recognition with moment invariants, *Intern. Conf. of Digital Signal Processing* (1997), 507-510.
7. D.E. Rumelhart, J.L. Mc. Clelland and PDD Research Group, *Paralled Distributed Processing: Explorations in the Microstructure of Cognition.* Cambridge: MIT Press, 1987.
8. T. Kohonen, *Self-Organizing Map,* Second Edition, Springer-Verlaag, 1992.
9. B.L. Kalman and S.C. Kwasny, A superior error function for training neural network, *International Joint Conference of Neural Network* **2** (1991), 42-52.
10. T. Kohonen, *SOM_PAK: The Self-Organizing Map Program Package, Ver 3.1.* Helsinki University of Technology, Finland, 1995.
11. Puteh Saad, Dzulkifli Mohamed, Safaai Deris and Siti Mariyam Shamsuddin, Trademark image clustering technique using self-organizing map, *Proc. of Intern. Conf. on IT & Multimedia*, 28^{th} - 30^{th} Sept., Universiti Tenaga Malaysia (1998), sc7-1.

Keywords: Moment invariants, backpropagation, feature extraction, printed mathematical symbols, self-organizing map (S0M).