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Faα-Irresolute Mappings

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Abstract. The present note introduces a new class of mappings called $Fa\alpha$-irresolute mappings. We obtain several characterizations of this class and study its properties and investigate the relationship with the known mappings.

1. Introduction

T.H. Yalvac [10] introduced the notion of fuzzy irresolute mappings. The present author [9] introduced the notion of fuzzy semi α-irresolute (shortly *Fs*α-irresolute) mappings. The purpose of this note is to introduce and investigate the concept of *Fa*α-irresolute mappings and give several characterization and its properties. Relation between this class and other classes of functions are obtained. The class of fuzzy almost α-irresolute mapping, which is stronger than *Fβ*-continuity, is a generalization of *both Fa*-irresolute mappings and $Fs\alpha$-irresolute mappings.

2. Preliminaries

Throughout this note, spaces always mean fuzzy topological spaces and $f : X \to Y$ denotes a mapping from a space (X, τ) into a space (Y, σ). The closure and the interior of *A* are denoted by $C1(A)$ and $Int(A)$, respectively.

Definition 2.1. *A fuzzy subset A of space X is said to be fuzzy α-open* [3], *(resp . fuzzy semi open* [1], *fuzzy pre open* [3], *fuzzy β-open* [4]*) if* $A \le Int(C1(Int(A)))$ *(resp.* $A \le C1(Int(A))$, $A \le Int(C1(A))$, $A \le C1(Int(C1(A)))$*)*.

The family of all $F\alpha$-open (resp. *Fs*-open *Fp*-open, *Fβ*-open) sets in a space *X* is denoted by $F\alpha O(X)$ (resp. $FSO(X)$, $FPO(X)$ and $F\beta O(X)$). The complement of a $F\alpha$-open (resp. *Fβ*-open) set is said to be $F\alpha$-closed (resp. *Fβ*-closed). The intersection of all $F\alpha$-closed sets containing A is called α-closure of A and is denoted by $\alpha C1(A)$; the union of all $F\alpha$-open set contained in A is called the α-interior of A and is denoted by $\alpha\ Int(A)$. [6]

Definition 2.2. *A mapping* $f : X \to Y$ *is said to be fuzzy* α*-irresolute* [6] *(resp. fuzzy semi* α*-irresolute* [9] *) if* $f^{-1}(A)$ *is* $F\alpha$*-open (resp . Fs-open) in* X *for every* $F\alpha$*-open set* A *of* Y.

Definition 2.3. *A mapping* $f : X \to Y$ *is said to be fuzzy strongly* α*-continuous* [9] *(resp. fuzzy irresolute* [10], *Fa-irresolute [8]) if* $f^{-1}(A)$ *is* $F\alpha$*-open (resp. Fs-open,* $F\beta$*-open) in X for every Fs-open set A of* Y.

Definition 2.4. *A mapping* $f : X \to Y$ *is said to be* $F\beta$*-continuous* [4] *(resp.* $F\beta$*-irresolute) if* $f^{-1}(A)$ *is* $F\beta$*-open in X for every fuzzy open (resp.* $F\beta$*-open) set* A *of* Y.

Definition 2.5. *A fuzzy point* x_t *is said to be quasi-coincident with a fuzzy set A in X if* $t + A(x) > 1$. *A fuzzy set A in X is said to be quasi- coincident with a fuzzy set B in X, denoted by* $A\, q\, B$, *if there exists a point x in X , such that,* $A(x) + B(x) > 1$. [7]

Lemma 2.1. *Let* $f : X \to Y$ *be a mapping and* x_t *be a fuzzy point of X. Then ,*

(a) $f(x_t)\, q\, B \Rightarrow x_t\, qf^{-1}(B)$, *for every fuzzy set B of Y.*

(b) $x_t\, q\, A \Rightarrow f(x_t)\, q\, f(A)$, *for every fuzzy set A of* X. [10]

3. *Faα*–irresolute mappings

Definition 3.1. *A mapping* $f : X \to Y$ *is said to be* $Fa\alpha$*-irresolute if* $f^{-1}(A)$ *is fuzzy* β*-open (shortly* $F\beta$*-open) in X for every* $F\alpha$*-open set A of* Y.

From the definition , we obtain the following diagram:

$$\begin{tikzcd}
F\alpha\text{-irresolute} \arrow[r] & Fs\alpha\text{-irresolute} \arrow[r] \arrow[d] & Fa\alpha\text{ -irresolute} \arrow[r] & F\beta\text{-continuity} \\
\text{fuzzy strongly } \alpha\text{-continuity} \arrow[u] \arrow[r] & \text{fuzzy irresolute} \arrow[r] & Fa\text{-irresolute} & F\beta\text{-irresolute} \arrow[l] \arrow[u]
\end{tikzcd}$$

The examples given below shows that the converse of these implications are not true in general. For,

Example 3.1. Let $X = \{x, y\}, Y = \{a, b\}$. The fuzzy set A, B, H, E, G are defined as: $A(a) = 0.5$, $= A(b) = 0.6$; $B(x) = 0$, $B(y) = 0.6$; $H(a) = 0$, $H(b) = 0.8$; $E(x) = 0.6$, $E(y) = 0.3$; $G(a) = 0.2$, $G(b) = 0.6$. Let $\tau = \{0,1\}, \sigma = \{0, A, 1\}$. $\tau_1 = \{0, B, 1\}$, $\sigma_1 = \{0, H, 1\}$, $\tau_2 = \{0, E, 1\}$, $\sigma_2 = \{0, g, 1\}$. Then the mapping $f : (x, \tau) \to (Y, \sigma)$ defined by $f(x) = a$ and $f(y) = b$ is $F\beta$-irresolute and hence $Fa\alpha$-irresolute but not $Fs\alpha$-irresolute; the mapping $g : (X, \tau_1) \to (Y, \sigma_1)$ defined by $f(x) = a$, $f(y) = b$ is $F\alpha$-irresolute but not Fa-irresolute. The mapping $h: (X, \tau_2) \to (Y, \sigma_2)$ defined by $f(x) = a$, $f(y) = b$ is $F\beta$-continuous but not $Fa\alpha$-continuous.

Example 3.2. Let $X = \{x, y\}, Y = \{a, b\}$, a fuzzy sets A is defined as: $A(x) = 0.5$, $A(y) = 0.3$. Let $\tau = \{0, A, 1\}$ and $\sigma = \{0,1\}$ then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(x) = a$, $f(y) = b$ is fuzzy strongly α-continuous but not $F\beta$-irresolute .

Other examples can be seen in [6].

Theorem 3.1. *The following are equivalent for a mapp*ing $f : X \to Y$,

(a) *f is $Fa\alpha$-irresolute;*

(b) *for every fuzzy point x_t in X and every $F\alpha$-open set V of Y containing $f(x_t)$, there exists an $F\beta$-open set U of X containing x_t such that $f(U) \leq V$;*

(c) *for every fuzzy point x_t of X and for every $F\alpha$-open set V of Y containing $f(x_t)$, there exists an $F\beta$-open set U in X such that $x_t \in U \leq f^{-1}(V)$;*

(d) *for every fuzzy point x_t in X, the inverse image of each α-neighbourhood* [7] *of $f(x_t)$ is a β-neighbourhood* [6] *of x_t.*

(e) *for every fuzzy point x_t in X and each α-neighbourhood B of $f(x_t)$ there exists a β-neighbourhood A of x_t such that $f(A) \leq B$;*

(f) *$f^{-1}(V) \leq C1(Int(C1(f^{-1}(V))))$, for every $F\alpha$-open set V of Y;*

(g) *$f^{-1}(H)$ is $F\beta$-closed in X, for every $F\alpha$-closed set H of Y;*

(h) *$Int(C1(Int(f^{-1}(B))) \leq f^{-1}(\alpha C1(B))$ for every fuzzy subset B of Y;*

(i) *$f(Int(C1(Int(A))) \leq \alpha C1(f(A))$ for every fuzzy subset A of X.*

Proof. **(a) ⇔ (b) ⇔ (c); (d) ⇒ (e):** obvious

(b) ⇒ (f): Let V be any $F\alpha$-open set of Y and $x_t \in f^{-1}(V)$. By (b), there exists a $F\beta$-open set U of X containing x_t such that $f(U) \leq V$. Thus we have $x_t \in U \leq C1(Int(C1(U))) \leq C1(Int(C1(f^{-1}(V)))$ and hence $f^{-1}(V) \leq C1(Int(C1(f^{-1}(V))))$.

(f) $\Rightarrow$ (g): Let H be any $F\alpha$-closed set of Y. Set $V = Y - H$, then V is $F\alpha$-open in Y. By (*f*), we obtain $f^{-1}(V) \le C1(Int(C1(f^{-1}(V))))$ and hence, $f^{-1}(H) = X - f^{-1}(Y - H) = X - f^{-1}(V)$ is $F\beta$-closed in X.

(g) $\Rightarrow$ (h): Let B be any fuzzy sub set of Y. Since $\alpha C1(B)$ is an $F\alpha$-closed subset of Y, $f^{-1}(\alpha(C1(B))$ is $F\beta$-closed in X, and hence $Int(C1(f^{-1}(\alpha C1)(B))))) \le f^{-1}(\alpha C1(B))$. Therefore, we obtain $Int(C1(Int(f^{-1}(B)))) \le f^{-1}(\alpha C1(B))$.

(h) $\Rightarrow$ (i): Let A be any fuzzy subset of X by (*h*), we have $IntC1(Int(A)))$ $\le Int(C1(Int(f^{-1}(f(A))))) \le f^{-1}(\alpha C1(f(A))$ and hence $f(Int(C1(Int(A))))$ $\le \alpha C1(f(A))$.

(i) $\Rightarrow$ (a): Let V be any $F\alpha$-open subset of Y. Since $f^{-1}(Y - V)$ $= X - f^{-1}(V)$ is a fuzzy subset of X, by (*f*), we obtain $f(Int(C1(Int(f^{-1}(Y - V))))) \le \alpha C1(f(f^{-1}(Y - V))) = Y - \alpha Int(V) = Y - V$, and hence, $X - C1(Int(C1(f^{-1}(V)))) = (Int(C1(Int(X - f^{-1}(V)))) = (Int(C1(Int\ f^{-1}(Y - V)))))$ $\le f^{-1}(f(Int(C1(Int(f^{-1}(Y - V)))))) \le f^{-1}(Y - V) = X - f^{-1}(V)$. Therefore, we have $f^{-1}(V) \le C1(Int(C1(f^{-1}(V))))$ and hence $f^{-1}(V)$ is $F\beta$-open in X. Thus f is $Fa\alpha$-irresolute.

(a) $\Rightarrow$ (d): Let x_t be a fuzzy point in X and V be α-neighbourhood of $f(x_t)$, then there exist a $F\alpha$-open set G of, Y such that, $f(x_t) \in G \le V$. Now $f^{-1}(G)$ is $F\beta$-open in X and $x_t \in f^{-1}(G) \le f^{-1}(V)$. Thus $f^{-1}(V)$ is a β-neighbourhood of x_t in X.

(e) $\Rightarrow$ (b): Let x_t be a fuzzy point in X and V is $F\alpha$-open set of Y such that $f(x_t) \in V$. Then V is α-neighbourhood of $f(x_t)$, so there is a β-neighbourhood A of x_t, such that $x_t \in A$, and $f(A) \le V$. Hence there exists a $F\beta$-open set U in X such that $x_t \in U \le A$ and so $f(U) \le f(A) \le V$.

Theorem 3.2. *The following are equivalent for a mapping* $f : X \to Y$ *:*

(a) f *is* $Fa\alpha$*-irresolute ;*

(b) *for each point* x_t *of X and every* $F\alpha$*-open set B of Y, such that* $f(x_t)\ q\ B$, *there exists, a* $F\beta$*-open set A in X such that* $x_t\ q\ A$ *and* $f(A) \le B$.

(c) *For every fuzzy point* x_t *of X and every* $F\alpha$*-open set B of Y such that* $f(x_t)\ q\ B$, *there exists, a* $F\beta$*-open set A of X such that* $x_t\ q\ A$ *and* $A \le f^{-1}(B)$.

Proof. **(a) $\Rightarrow$ (b):** Let x_t be a fuzzy point of *X* and *B* be a $F\alpha$-open set of *Y* such that $f(x_t)\ q\ B$. Then $f^{-1}(B)$ is *Fβ*-open in *X*, and $x_t\ q\ f^{-1}(B)$ by Lemma 2.1. If we take $A = f^{-1}(B)$ then $x_t\ q\ A$ and $f(A) = f(f^{-1}(B)) \le B$.

(b) $\Rightarrow$ (c): Let x_t be a fuzzy point and *B* be a $F\alpha$-open set of *Y* such that $f(x_t)\ q\ B$. Then by (b), there exists a *Fβ*-open set *A* of *X* such that $x_t\ q\ A$ and $f(A) \le B$. Hence we have $x_t\ q\ A$ and $A \le f^{-1}(f(A)) \le f^{-1}(B)$.

(c) $\Rightarrow$ (a): Let *B* be a $F\alpha$-open set of *Y* and x_t be a fuzy point of *X* such that $x_t \in f^{-1}(B)$. Then $f(x_t) \in B$. Choose the fuzzy point $x_t^c(x) = 1 - x_t(x)$. Then $f(x_t^c)\ q\ B$. And so by (c), there exists a *Fβ*-open set *A* of *X* such that $x_t^c\ q\ A$ and $f(A) \le B$. Now $x_t^c\ q\ A$ implies $x_t^c\ (x) + A(x) = 1 - x_t(x) + A(x) > 1$. It follows that $x_t \in A$. Thus $x_t \in A \le f^{-1}(B)$. Hence $f^{-1}(B)$ is *Fβ*-open in *X*.

Theorem 3.3. *A mapping* $f : X \to Y$ *is* $Fa\alpha$*-irresolute if the graph mapping* $g : X \to X \times Y$, *defined by* $g(x) = (x, f(x))$ *for each* $x \in X$, *is* $Fa\alpha$*-irresolute.*

Proof. Let $x_t \in X$ and *V* be any $F\alpha$-open set of *Y* containing $f(x_t)$. Then, $X \times V$ is $F\alpha$-open is $X \times Y$ containing $g(x_t)$. Since *g* is $Fa\alpha$-irresolute there exists , *Fβ*-open set *U* of *X* containing x_t such that $g(U) \le X \times V$ and hence $f(U) \le V$. Thus, f is $Fa\alpha$-irresolute.

Theorem 3.4. *If* $f : X \to Y$ *is* $Fa\alpha$*-irresolute and A is* $F\alpha$*-open subset of X, then the restriction* $f \mid A : A \to Y$ *is* $Fa\alpha$*-irresolute.*

Proof. Let *V* be any $F\alpha$-open set of *Y*. Since *f* is $Fa\alpha$-irresolute, then $f^{-1}(V)$ is *Fβ*-open in *X*. Since *A* is $F\alpha$-open in *X*, then $(f \mid A)^{-1}(V) = A \cap f^{-1}(V)$ is *Fβ*-open in *A* and hence $f \mid A$ is $Fa\alpha$-irresolute.

Theorem 3.5. *Let* $f : X \to Y$ *be a mapping and* $\{A_i : i \in \wedge\}$ *be a cover of* *X* *by* *Fβ-open sets of X. Then f is* $Fa\alpha$*-irresolute if* $f \mid A_i : A_i \to Y$ *is* $Fa\alpha$*-irresolute for each* $i \in \wedge$.

Proof. Let *V* be any $F\alpha$-open set of *Y*. Since $f \mid A_i$ is $Fa\alpha$-irresolute, then $(f \mid A_i)^{-1}(V)$ is *Fβ*-open in A_i and since $A_i \in F\beta O(X)$, then $(f \mid A_i)^{-1}(V)$ is *Fβ*-open in *X* for each $i \in \wedge$. Therefore $f^{-1}(V) = X \cap f^{-1}(V) = \cup\{A_i \cap f^{-1}(V) : i \in \wedge\}$ $= \cup\{f \mid A_i)^{-1}\ (V) : i \in \wedge\}$ is *Fβ*-open in X. Hence *f* is $Fa\alpha$-irresolute.

Theorem 3.6. *A mapping $f : X \to Y$ is Faα-irresolute, then $f^{-1}(B)$ is Fβ-closed in X for any nowhere dense set B of Y.*

Proof. Let *B* be any nowhere dense subset of *Y*, then $Y - B$ is $F\alpha$-open in *Y*. Since *f* is $Fa\alpha$-irresolue, then $f^{-1}(Y - B) = X - f^{-1}(B)$ is *Fβ*-open in *X* and hence $f^{-1}(B)$ is *Fβ*-closed in *X*.

Theorem 3.7. *A mapping $f : X \to Y$ is Faα-irresolute iff, for each $p \in Y$ and each fuzzy open set V of Y such that $p \in Y$ and each fuzzy open set V of Y such that $p \in Int(C1(V))$, the inverse image of $V \cup p$ is Fβ-open in X.*

Proof. **Necessity.** Since $V \le V \cup p \le Int(C1(V))$, then $V \cup p$ is a $F\alpha$-open set of *Y*. Since *f* is $Fa\alpha$-irresolute, then $f^{-1}(V \cup p)$ is *Fβ*-open in *X*.

Sufficiency. Since *V* be a $F\alpha$-open set of *Y*. Then, there exists an fuzzy open set *B* of *Y* such that $B \le V < Int(C1(B))$. By hypothesis, $f^{-1}(B \cup p)$ is *Fβ*-open in *X* for each $p \in V$. This shows that $f^{-1}(V) = \cup\{f^{-1}(B \cup p) : p \in V\}$ is *Fβ*-open in *X* and hence f is $Fa\alpha$-irresolute .

Theorem 3.8. *Let $f : X \to Y$ and $g : Y \to Z$ be mapping. Then the composition $g \circ f : X \to Z$ is Faα-irresolute if f and g satisfy one of the following condition:*

(a) f is Faα-irresolute and g is Fα-irresolute,
(b) f is Fβ-irresolute and g is Faα-irresolute,
(c) f is Fa-irresolute and g is Fsα-irresolute.

Proof. Let *W* be any *fα*-open subset of *Z*. Since *g* is $F\alpha$-irresolute, then $g^{-1}(W)$ is *Fβ*-open in *Y*. Since f is $Fa\alpha$-irresolute, $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is *Fβ*-open in *X* and hence $g \circ f$ is $Fa\alpha$-irresolute.

The proof of the conditions (b) and (c) is analogous to that of (a). It follows from the definitions.

We recall that a space *X* is said to be fuzzy submaximal if every fuzzy dense subset of *X* is fuzzy open in *X* and fuzzy extremely disconnected if the closure of each fuzzy open set of *X* is fuzzy open in *X*.

Theorem 3.9. *Let X be a fuzzy submaximal and fuzzy extremely disconnected space. Then the following are equivalent for a mapping $f : X \to Y$:*

(a) f is Fα-irresolute;
(b) f is Fsα-irresolute;
(c) f is Faα-irresolute;

Proof. This follows from the fact that if (X,τ) is fuzzy submaximal and extremely disconnected then $\tau = F\alpha O(X) = FSO(X) = F\beta O(X)$.

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