BULLETIN *of the*
MALAYSIAN
MATHEMATICAL
SCIENCES
SOCIETY

Bull. Malaysian Math. Sc. Soc. (Second Series) **26** (2003) 189–199

Results on Some Neutrix Convolutions of Functions and Distributions

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Abstract. The neutrix convolution of two locally summable functions or distributions f and g is defined to be the limit of the sequence $\{f_n * g\}$, where $f_n(x) = f(x)\tau_n(x)$ and $\tau_n(x)$ is a certain function with compact support and the sequence $\{\tau_n\}$ converges to the identity function on the real line. The neutrix convolution of the functions $x_+^r \ln x_+$ and $e^{\lambda x}$ is evaluated for $r = 0, 1, 2, \cdots$ and all $\lambda \neq 0$. Further neutrix convolutions are then deduced.

The *exponential integral* $ei(\lambda x)$, see Sneddon [7], can be defined on the real line for $\lambda \neq 0$ by

$$ei(\lambda x)\int_{\lambda x}^{\infty} t^{-1}\left[e^{-t} - H(1-t)\right] dt - H(1-\lambda x)\ln|\lambda x|,$$

where H denotes Heaviside's function.

In particular, if $\lambda < 0$, it was shown in [4] that

$$ei(\lambda x) = -\gamma - \ln|\lambda| - \int_0^x t^{-1}\left(e^{-\lambda t} - 1\right) dt - \ln x, \tag{1}$$

for $x > 0$, where

$$\gamma = -\int_0^{\infty} t^{-1}\left[e^{-t} - H(1-t)\right] dt \tag{2}$$

is Euler's constant.

Before proving our results, we need the following two lemmas, easily proved by induction:

Lemma 1. *If* $\lambda \neq 0$, *then*

$$a_r(x,\lambda) = \int_0^x t^r e^{-\lambda t} dt = -\sum_{i=0}^{r} \frac{r!}{i!\,\lambda^{r-i+1}} x^i e^{-\lambda x} + \frac{r!}{\lambda^{r+1}} \tag{3}$$

for $r = 0, 1, 2, \cdots$.

Lemma 2. *If* $\lambda \neq 0$, *then*

$$\begin{aligned} b_r(x,\lambda) &= \int_0^x t^r \ln t \; e^{-\lambda t} dt \\ &= \sum_{i=1}^{r} \frac{r!}{i!\,\lambda^{r-i+1}} \left[a_{i-1}(x,\lambda) - x^i \ln x \, e^{-\lambda x}\right] + \frac{r!}{\lambda^r} b_0(x,\lambda) \end{aligned} \tag{4}$$

for $r = 0, 1, 2, \cdots$.

The classical definition for the convolution $f * g$ of two locally summable functions *f* and *g* is as follows:

Definition 1. *Let f and g be locally summable functions. Then the convolution* $f * g$ *is defined by*

$$(f * g) = \int_{-\infty}^{\infty} f(t) g(x - t)\, dt$$

for all points x for which the integral exists.

It follows easily from the definition that if $f * g$ exists, then $g * f$ exists and $f * g = g * f$. Further, if $(f * g)'$ and $f * g'$ (or $f' * g$) exist, then

$$(f * g)' = f * g' \ \left(\text{or } f' * g\right). \tag{5}$$

Definition 1 can be extended to define the convolution $f * g$ of two distributions f and g in D', the space of distributions defined on D, the space of infinitely differentiable functions with compact support, see Gel'fand and Shilov [6].

Definition 2. *Let f and g be distributions in D'. Then the convolution $f * g$ is defined by*

$$\langle (f * g)(x), \varphi(x) \rangle = \langle f(y), \langle g(x), \varphi(x+y) \rangle \rangle$$

for all φ in D, provided f and g satisfy either of the conditions

(a) either f or g has bounded support,
(b) the supports of f and g are bounded on the same side, see Gel'fand and Shilov [6].

Note that if f and g are locally summable functions satisfying either of the above conditions and the classical convolution $f * g$ exists, then it is in agreement with Definition 1.

The following results were proved in [5].

$$(x^s e^{\lambda x}) * (x_+^r \ln x_+) = \sum_{i=0}^{s} \binom{s}{i} \frac{(-1)^i (r+i)! \left[\phi(r) - \gamma - \ln|\lambda| \right]}{\lambda^{r+i+1}} x^{s-i} e^{\lambda x}$$
$$- \sum_{i=0}^{s-1} \sum_{j=0}^{i} \binom{s}{i} \binom{i}{j} \frac{(-1)^{s+i+j} (r+j)! (s-i-1)!}{\lambda^{r+s-i+j+1}} x^{i-j} e^{\lambda x},$$

for $r, s = 0, 1, 2, \cdots$, if $\lambda > 0$ and

$$(x^s e^{\lambda x}) * (x_-^r \ln x_-) = \sum_{i=0}^{s} \binom{s}{i} \frac{(-1)^{r+i+1} (r+i)! \left[\phi(r) - \gamma - \ln|\lambda| \right]}{\lambda^{r+i+1}} x^{s-i} e^{\lambda x}$$
$$- \sum_{i=0}^{s-1} \sum_{j=0}^{i} \binom{s}{i} \binom{i}{j} \frac{(-1)^{r+s+i+j} (r+j)! (s-i-1)!}{\lambda^{r+s-i+j+1}} x^{i-j} e^{\lambda x},$$

for $r, s = 0, 1, 2, \cdots$, if $\lambda < 0$, where

$$\phi(r) = \begin{cases} \sum_{i=1}^{r} i^{-1} & , \quad r \geq 1, \\ 0 & , \quad r = 0. \end{cases}$$

The above definition of the convolution is rather restrictive and so the non commutative neutrix convolution was introduced in [2]. In order to define the neutrix convolution product we first of all let τ be a function in D satisfying the following properties:

(i) $\tau(x) = \tau(-x)$,

(ii) $0 \le \tau(x) \le 1$,

(iii) $\tau(x) = 1$ for $|x| \le \frac{1}{2}$,

(iv) $\tau(x) = 0$ for $|x| \ge 1$.

The function τ_n is now defined by

$$\tau_n(x) = \begin{cases} 1 & , \quad |x| \le n\,, \\ \tau(n^n x - n^{n+1}) & , \quad x > n\,, \\ \tau(n^n x + n^{n+1}) & , \quad x < -n\,, \end{cases}$$

for $n = 1, 2, \cdots$.

Definition 3. *Let f and g be locally summable functions or distributions in D' and let $f_n = f\tau_n$ for $n = 1, 2, \cdots$. Then the neutrix convolution $f \circledast g$ is defined as the neutrix limit of the sequence $\{f_n * g\}$, provided that the limit h exists in the sense that*

$$N - \lim_{n\to\infty} \langle f_n * g, \varphi \rangle = \langle h, \varphi \rangle,$$

for all φ in D, where N is the neutrix, see van der Corput [1], *having domain $N' = \{1, 2, \cdots, n, \cdots\}$ and range N'' the real numbers, with negligible functions finite linear sums of the functions*

$$n^\alpha \ln^{r-1} n,\ \ln^r n \quad (\alpha > 0,\ r = 1, 2, \cdots)$$

and all functions which converge to zero in the usual sense as n tends to infinity.

Note that in this definition the convolution $f_n * g$ is defined in Gel'fand and Shilov's sense, the distribution f_n having bounded support. Note also that because of the lack of symmetry in the definition of $f \circledast g$, the neutrix convolution is in general non-commutative.

The following two theorems were proved in [2], showing that the neutrix convolution is a generalization of the convolution.

Theorem 1. *Let f and g be distributions in D' satisfying either condition (a) or condition (b) of Gel'fand and Shilov's definition. Then the neutrix Convolution $f \circledast g$ exists and*

$$f \circledast g = f * g.$$

Theorem 2. *Let f and g be distributions in* D' *and suppose that* $f \circledast g$ *exists, then the neutrix convolution* $f \circledast g'$ *exists and*

$$(f \circledast g)' = f \circledast g'. \tag{6}$$

Note however that $(f \circledast g)'$ is not necessarily equal to $f' \circledast g$. We do however have the following lemma which was proved in [3].

Lemma 3. *Let* f *and* g *be distributions in* D' *and suppose that* $f \circledast g$ *exists. If* $N - \lim_{n\to\infty} \langle (f\tau_n') * g,\ \varphi \rangle$ *exists and equals* $\langle h,\ \varphi \rangle$ *for all* φ in D', *then the neutrix convolution* $f' \circledast g$ *exists and*

$$(f \circledast g)' = f \circledast g + h. \tag{7}$$

In order to define further neutrix convolution products, we increase our set of negligible functions given in Definition 3 to also include finite linear sums of the functions

$$n^i \ln n\, e^{-\lambda n}, \quad ei(\lambda n) \qquad (i = 0, 1, 2, \cdots;\ \lambda < 0).$$

The following results were proved in [5].

$$\left(x^s e^{\lambda x}\right) \circledast \left(x_+^r \ln x_+\right) = \sum_{i=0}^{s} \binom{s}{i} \frac{(-1)^i (r+i)!\left[\phi(r) - \gamma - \ln|\lambda|\right]}{\lambda^{r+i+1}} x^{s-i} e^{\lambda x}$$
$$+ \sum_{i=0}^{s-1} \sum_{j=0}^{i} \binom{s}{i}\binom{i}{j} \frac{(-1)^{s+i+j}(r+j)!(s-i-1)!}{\lambda^{r+s-i+j+1}} x^{i-j} e^{\lambda x},$$

$$\left(x^s e^{\lambda x}\right) \circledast \left(x_-^r \ln x_-\right) = \sum_{i=0}^{s} \binom{s}{i} \frac{(-1)^{r+i+1} (r+i)!\left[\phi(r) - \gamma - \ln|\lambda|\right]}{\lambda^{r+i+1}} x^{s-i} e^{\lambda x}$$
$$- \sum_{i=0}^{s-1} \sum_{j=0}^{i} \binom{s}{i}\binom{i}{j} \frac{(-1)^{r+s+i+j}(r+j)!(s-i-1)!}{\lambda^{r+s-i+j+1}} x^{i-j} e^{\lambda x},$$

$$\left(x^s e^{\lambda x}\right) \circledast \left(x' \ln|x|\right) = 0,$$

for $\lambda \neq 0$ and $r, s = 0, 1, 2, \cdots$.

We now prove

Theorem 3. *If* $\lambda < 0$, *then the neutrix convolution* $(x_+^r \ln x_+) \circledast e^{\lambda x}$ *exists and*

$$\left(x_+^r \ln x_+\right) \circledast e^{\lambda x} = \frac{r!\left[\phi(r) - \gamma - \ln|\lambda|\right]}{\lambda^{r+1}} e^{\lambda x} \tag{8}$$

for $r = 0, 1, 2, \cdots$.

Proof. We put $(x_+^r \ln x_+)_n = (x_+^r \ln x_+)\tau_n(x)$ for $n = 1, 2, \cdots$. Since $(x_+^r \ln x_+)_n$ has compact support, the classical convolution $(x_+^r \ln x_+)_n * e^{\lambda x}$ exists and

$$\begin{aligned}\left(x_+^r \ln x_+\right)_n * e^{\lambda x} &= \int_0^n t^r \ln t\, e^{\lambda(x-t)} dt + \int_n^{n+n^{-n}} t^r \ln t\, \tau_n(t)\, e^{\lambda(x-t)}\, dt \\ &= I_1 + I_2.\end{aligned} \tag{9}$$

It is easily seen that

$$\lim_{n\to\infty} I_2 = 0 \tag{10}$$

Further,

$$\begin{aligned}I_1 &= b_r(n,\lambda) e^{\lambda x} \\ &= -\sum_{i=1}^{r}\sum_{j=0}^{i-1} \frac{r!}{ij!\,\lambda^{r-j+1}} n^j e^{\lambda(x-n)} - \sum_{i=1}^{r} n^i \ln n\, e^{\lambda(x-n)} \\ &\quad + \frac{r!\phi(r)}{\lambda^{r+1}} e^{\lambda x} + \frac{r!}{\lambda^r} b_0(n,\lambda) e^{\lambda x}\end{aligned} \tag{11}$$

and

$$\begin{aligned}b_0(n,\lambda) &= \int_0^n \ln t\, e^{-\lambda t}\, dt \\ &= -\lambda^{-1} \ln n\,(e^{-\lambda n} - 1) + \lambda^{-1}\int_0^n t^{-1}(e^{-\lambda t} - 1)\, dt\end{aligned} \tag{12}$$

Further, we have from equation (1)

$$\int_0^n t^{-1}(e^{-\lambda t} - 1)\, dt = -ei(\lambda n) - \ln n - \gamma - \ln|\lambda|$$

and it follows from equation (12) that

$$N-\lim_{n\to\infty} b_0(n,\lambda) = -\lambda^{-1}\left(\gamma + \ln|\lambda|\right). \tag{13}$$

It now follows from equations (11) and (13) that

$$N - \lim_{n\to\infty} I_1 = \frac{r![\phi(r) - \gamma - \ln|\lambda|]}{\lambda^{r+1}} e^{\lambda x}. \tag{14}$$

Equation (8) now follows from equations (9), (10) and (14).

Corollary 3.1. *If* $\lambda > 0$, *then the neutrix convolution* $(x_-^r \ln x_-) \circledast e^{\lambda x}$ *exists and*

$$\left(x_-^r \ln x_-\right) \circledast e^{\lambda x} = (-1)^{r+1} \frac{r![\phi(r) - \gamma - \ln|\lambda|]}{\lambda^{r+1}} e^{\lambda x} \tag{15}$$

for $r = 0, 1, 2, \cdots$.

Proof. Equation (15) follows on replacing λ by $-\lambda$ and x by $-x$ in equation (8).

Corollary 3.2. *If* $\lambda \neq 0$, *then the neutrix convolution* $(x^r \ln|x|) \circledast e^{\lambda x}$ *exists and*

$$\left(x^r \ln|x|\right) \circledast e^{\lambda x} = 0 \tag{16}$$

for $r = 0, 1, 2, \cdots$.

Proof. Equation (16) follows from (8) and (15) on noting that

$$x^r \ln|x| = x_+^r \ln x_+ + (-1)^r x_-^r \ln x_-.$$

Theorem 4. *If* $\lambda \neq 0$, *then the neutrix convolutions* $(x_+^r \ln x_+) \circledast (x^s e^{\lambda x})$ and $(x_-^r \ln x_-) \circledast (x^s e^{\lambda x})$ *exist and*

$$\left(x_+^r \ln x_+\right) \circledast \left(x^s e^{\lambda x}\right) = \sum_{i=0}^{s} \binom{s}{i} \frac{(-1)^i (r+i)![\phi(r) - \gamma - \ln|\lambda|]}{\lambda^{r+i+1}} x^{s-i} e^{\lambda x}$$

$$+ \sum_{i=0}^{s-1} \sum_{j=0}^{i} \binom{s}{i} \binom{i}{j} \frac{(-1)^{s+i+j} (r+j)!(s-i-1)!}{\lambda^{r+s-i+j+1}} x^{i-j} e^{\lambda x}, \tag{17}$$

$$\left(x_-^r \ln x_-\right) \circledast \left(x^s e^{\lambda x}\right) = \sum_{i=0}^{s} \binom{s}{i} \frac{(-1)^{r+i+1} (r+i)![\phi(r) - \gamma - \ln|\lambda|]}{\lambda^{r+i+1}} x^{s-i} e^{\lambda x}$$

$$- \sum_{i=0}^{s-1} \sum_{j=0}^{i} \binom{s}{i} \binom{i}{j} \frac{(-1)^{r+s+i+j} (r+j)!(s-i-1)!}{\lambda^{r+s-i+j+1}} x^{i-j} e^{\lambda x} \tag{18}$$

for $r, s = 0, 1, 2, \cdots$.

Proof. Equation (17) follows on differentiating equation (8) partially s times with respect to λ and equation (18) follows on replacing λ by $-\lambda$ and x by $-x$ in equation (17).

Corollary 4.1. *If* $\lambda \neq 0$, *then the neutrix convolution* $(x^r \ln|x|) \circledast (x^s e^{\lambda x})$ *exists and*

$$\left(x^r \ln|x|\right) \circledast \left(x^s e^{\lambda x}\right) = 0 \tag{19}$$

for $r, s = 0,1,2,\cdots$.

Proof. Equation (19) follows from equations (17) and (18) on noting that

$$\left(x^r \ln|x|\right) \circledast \left(x^s e^{\lambda x}\right) = \left(x_+^r \ln x_+\right) \circledast \left(x^s e^{\lambda x}\right) + (-1)^r \left(x_-^r \ln x_-\right) \circledast \left(x^s e^{\lambda x}\right).$$

In the following, the distributions x_+^{-r} and x_-^{-r} are defined by

$$x_+^{-r} = \frac{(-1)^{r-1}}{(r-1)!}\frac{d^r}{dx^r}\ln x_+ \quad , \quad x_-^{-r} = -\frac{1}{(r-1)!}\frac{d^r}{dx^r}\ln x_-$$

for $r = 1,2,\cdots$ and not as in Gel'fand and Shilov [6].

The following results were also proved in [5].

$$\left(x^s e^{\lambda x}\right) \circledast x_+^{-r} = \sum_{i=0}^{s}\binom{s}{i}\frac{(-1)^r\left(\gamma + \ln|\lambda|\right)\lambda^{r-i-1}}{(r-i-1)!}x^{s-i}e^{\lambda x}$$
$$-\sum_{i=0}^{s-1}\sum_{j=0}^{i}\binom{s}{i}\binom{i}{j}\frac{(-1)^{r+s-i}(s-i-1)!\,\lambda^{r-s+i-j-1}}{(r-j-1)!}x^{i-j}e^{\lambda x},$$

$$\left(x^s e^{\lambda x}\right) \circledast x_-^{-r} = -\sum_{i=0}^{s}\binom{s}{i}\frac{\left(\gamma + \ln|\lambda|\right)\lambda^{r-i-1}}{(r-i-1)!}x^{s-i}e^{\lambda x}$$
$$+\sum_{i=0}^{s-1}\sum_{j=0}^{i}\binom{s}{i}\binom{i}{j}\frac{(-1)^{s-i}(s-i-1)!\,\lambda^{r-s+i-j-1}}{(r-j-1)!}x^{i-j}e^{\lambda x},$$

$$\left(x^s e^{\lambda x}\right) \circledast x^{-r} = 0$$

for $\lambda \neq 0$, $r = 1,2,\cdots$ and $s = 0,1,2,\cdots$.

We now prove

Theorem 5. *If* $\lambda \neq 0$, *then the neutrix convolutions* $x_+^{-r} \circledast e^{\lambda x}$ *and* $x_-^{-r} \circledast e^{\lambda x}$ *exist and*

$$x_+^{-r} \circledast e^{\lambda x} = -\frac{\left(\gamma + \ln|\lambda|\right)(-\lambda)^{r-1}}{(r-1)!} e^{\lambda x}, \tag{20}$$

$$x_+^{-r} \circledast e^{\lambda x} = -\frac{\left(\gamma + \ln|\lambda|\right)\lambda^{r-1}}{(r-1)!} e^{\lambda x} \tag{21}$$

for $r = 1, 2, \cdots$.

Proof. We have

$$\begin{aligned}\left[\ln x_+ \, \tau_n'(x)\right] * e^{\lambda x} &= \int_n^{n+n^{-n}} \ln t \, e^{\lambda(x-t)} d\tau_n(t) \\ &= -\ln n \, e^{\lambda(x-n)} - e^{\lambda x} \int_n^{n+n^{-n}} t^{-1} e^{-\lambda t} \tau_n(t)\, dt \\ &\quad + \lambda e^{\lambda x} \int_n^{n+n^{-n}} \ln t \, e^{-\lambda t} \tau_n(t)\, dt\end{aligned}$$

and it follows easily that

$$N-\lim_{n\to\infty} \left[\ln x_+ \tau_n'(x)\right] * e^{\lambda x} = 0 \tag{22}$$

Equation (20) for the case $r = 1$ now follows on differentiating equation (8) with $r = 0$, using Lemma 3 and equation (22).

Now assume that equation (20) holds for some *r*. We have

$$\begin{aligned}\left[x_+^{-r} \, \tau_n'(x)\right] * e^{\lambda x} &= \int_n^{n+n^{-n}} t^{-r} \, e^{\lambda(x-t)} \, d\tau_n(t) \\ &= n^{-r} \, e^{\lambda(x-n)} + r e^{\lambda x} \int_n^{n+n^{-n}} t^{r+1} e^{-\lambda t} \tau_n(t)\, dt \\ &\quad + \lambda e^{\lambda x} \int_n^{n+n^{-n}} t^{-r} \, e^{-\lambda t} \tau_n(t)\, dt\end{aligned}$$

and it follows easily that

$$N-\lim_{n\to\infty} \left[x_+^{-r} \tau_n'(x)\right] * e^{\lambda x} = 0 \tag{23}$$

Equation (20) for the case $r+1$ now follows on differentiating equation (20), using Lemma 3 and equation (23), proving equation (20) by induction.

Equation (21) follows on replacing λ by $-\lambda$ and *x* by $-x$ in equation (20).

Corollary 5.1. *If* $\lambda \neq 0$, *then the neutrix convolution* $x^{-r} \circledast e^{\lambda x}$ *exists and*

$$x^{-r} \circledast e^{\lambda x} = 0 \tag{24}$$

for $r = 1, 2, \cdots$.

Proof. Equation (24) follows from equations (20) and (21) on noting that

$$x^{-r} = x_+^{-r} + (-1)^r x_-^{-r}.$$

Theorem 6. *If* $\lambda \neq 0$, *then the neutrix convolutions* $x_+^{-r} \circledast (x^s e^{\lambda x})$ *and* $x_-^{-r} \circledast (x^s e^{\lambda x})$ *exist and*

$$x_+^{-r} \circledast \left(x^s e^{\lambda x}\right) = \sum_{i=0}^{s} \binom{s}{i} \frac{(-1)^r \left(\gamma + \ln|\lambda|\right)\lambda^{r-i-1}}{(r-i-1)!} x^{s-i} e^{\lambda x}$$
$$- \sum_{i=0}^{s-1} \sum_{j=0}^{i} \binom{s}{i}\binom{i}{j} \frac{(-1)^{r+s-i}(s-i-1)!\,\lambda^{r-s+i-j-1}}{(r-j-1)!} x^{i-j} e^{\lambda x}, \tag{25}$$

$$x_-^{-r} \circledast \left(x^s e^{\lambda x}\right) = -\sum_{i=0}^{s} \binom{s}{i} \frac{\left(\gamma + \ln|\lambda|\right)\lambda^{r-i-1}}{(r-i-1)!} x^{s-i} e^{\lambda x}$$
$$+ \sum_{i=0}^{s-1} \sum_{j=0}^{i} \binom{s}{i}\binom{i}{j} \frac{(-1)^{s-i}(s-i-1)!\,\lambda^{r-s+i-j-1}}{(r-j-1)!} x^{i-j} e^{\lambda x} \tag{26}$$

for $r, s = 1, 2, \cdots$ *where* $[(r-i)!]^{-1}$ *is interpreted as being zero when* $r < i$.

Proof. Equation (25) follows on differentiating equation (20) s times partially with respect to λ and equation (26) follows on replacing λ by $-\lambda$ and x by $-x$ in equation (25).

Corollary 6.1. *If* $\lambda \neq 0$, *then the neutrix convolution* $x^{-r} \circledast (x^s e^{\lambda x})$ *exists and*

$$x^{-r} \circledast \left(x^s e^{\lambda x}\right) = 0 \tag{27}$$

for $r, s = 1, 2, \cdots$.

Proof. Equation (27) follows from equations (25) and (26) on noting that

$$x^{-r} ⊛ \left(x^s e^{\lambda x}\right) = x_+^{-r} ⊛ \left(x^s e^{\lambda x}\right) + (-1)^r x_-^{-r} ⊛ \left(x^s e^{\lambda x}\right)$$

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Keywords and phrases: distribution, exponential integral, neutrix, neutrix limit, convolution, neutrix convolution.

2000 AMS Subject Classification: 46F10