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Linear Transformations of *N*-connections in OSC^2M (II)

[1]IRENA ČOMIĆ AND [2]MONICA PURCARU

[1]Faculty of Technical Sciences, University of Novi Sad, Trg Dositeja Obradovića 6, 21000 Novi Sad, Yugoslavia

[2]"Transilvania" University of Braşov, Department of Geometry, Iuliu Maniu 50, 2200 Braşov, Romania

[1]e-mail: comirena@uns.ns.ac.yu and [2]e-mail: mpurcaru@unitbv.ro

Abstract. In the tangent space $T(E)$ and in its dual $T^*(E)$, two different adapted basis was introduced, in another paper with the same title [10]. In the present paper are given the connections between $\hat{K}_{\alpha a}{}^{\gamma c}{}_{\beta b}$ and $K_{\alpha a}{}^{\gamma c}{}_{\beta b}$, between the torsions: $\hat{T}_{\alpha a}{}^{\gamma c}{}_{\beta b}$ and $T_{\alpha a}{}^{\gamma c}{}_{\beta b}$ and the components of the metric tensor $\hat{g}_{\alpha a\ \beta b}$ and $g_{\alpha a\ \beta b}$.

1. Introduction

We define $E = Osc^2 M$ as a $3-n$ dimensional C^∞ real manifold, in which the transformation of form (1.1) are allowed. It is formed as a tangent space of order two of the n-dimensional base manifold M. In some local chart (U, φ) some point $u \in E$ has coordinates

$$\left(x^a, y^{1a}, y^{2a}\right) = \left(y^{0a}, y^{1a}, y^{2a}\right) = \left(y^{\alpha a}\right),$$

where $x^a = y^{0a}$ and $a, b, c, d, e, \cdots = \overline{1, n}$, $\alpha, \beta, \gamma, \delta, \cdots = \overline{0, 2}$.

The following abbreviations will be used

$$\partial_{\alpha a} = \frac{\partial}{\partial y^{\alpha a}}, \quad \alpha = \overline{0, 2}.$$

If in some other chart (U', φ') the point $u \in E$ has coordinates $(x^{a'}, y^{1a'}, y^{2a'})$, then in $U \cap U'$ the allowable coordinates transformations are given by

$$\begin{cases} x^{a'} = x^{a'}\left(x^1, x^2, \cdots, x^n\right), \\ y^{1a'} = \left(\partial_a x^{a'}\right) y^{1a} = \left(\partial_{0a} y^{0a'}\right) y^{1a}, \\ y^{2a'} = \left(\partial_{0a} y^{1a'}\right) y^{1a} + \left(\partial_{1a} y^{1a'}\right) y^{2a}. \end{cases} \tag{1.1}$$

The natural base of $T(E)$ and of $T^*(E)$ are respectively

$$\overline{\mathsf{B}} = \left\{\partial_{0a}, \partial_{1a}, \partial_{2a}\right\} \tag{1.2}$$

and

$$\overline{\mathsf{B}}^* = \left\{dy^{0a}, dy^{1a}, dy^{2a}\right\}. \tag{1.3}$$

The elements of $\overline{\mathsf{B}}$ and $\overline{\mathsf{B}}^*$ are dual to each other, i.e.,

$$\left\langle dy^{\alpha a}, \partial_{\beta b}\right\rangle = \delta^\alpha_\beta \, \delta^a_b, \tag{1.4}$$

but with respect to (1.1) they have not a tensorial character.

The adapted basis B^* of $T^*(E)$ is given by

$$\mathsf{B}^* = \left\{\delta y^{0a}, \delta y^{1a}, \delta y^{2a}\right\} \tag{1.5}$$

where

$$\begin{cases} \delta y^{0a} = dx^a = dy^{0a}, \\ \delta y^{1a} = dy^{1a} + M^{1}_{0}{}^{a}_{b}\, \mathrm{d}y^{0b}, \\ \delta y^{2a} = dy^{2a} + M^{2}_{1}{}^{a}_{b}\, \mathrm{d}y^{1b} + M^{2}_{0}{}^{a}_{b}\, \mathrm{d}y^{0b}. \end{cases} \tag{1.6}$$

Theorem 1.1. [10] *The necessary and sufficient conditions that $\delta y^{\alpha a}$ are transformed as d-tensor fields are that the following equations are satisfied*

$$\begin{cases} M^{1}_{0}{}^{a}_{b}\left(\partial_a x^{b'}\right) = M^{1}_{0}{}^{b'}_{c'}\left(\partial_b x^{c'}\right) + \partial_b y^{1b'}, \\ M^{2}_{1}{}^{a}_{b}\left(\partial_a x^{b'}\right) = M^{2}_{1}{}^{b'}_{c'}\left(\partial_{1b} y^{1c'}\right) + \partial_{1b} y^{2b'}, \\ M^{2}_{0}{}^{a}_{b}\left(\partial_a x^{b'}\right) = M^{2}_{0}{}^{b'}_{c'}\left(\partial_b x^{c'}\right) + M^{2}_{1}{}^{b'}_{c'}\left(\partial_b y^{1c'}\right) + \partial_b y^{2b'}. \end{cases} \tag{1.7}$$

Let us denote the adapted basis of $T(E)$ by $\mathcal{B}$, where

$$\mathcal{B} = \{\delta_{0a},\ \delta_{1a},\ \delta_{2a}\} = \{\delta_{\alpha a}\},\ \left(\alpha = \overline{0,2}\right), \tag{1.8}$$

and

$$\begin{cases} \delta_{0a} = \partial_{0a} - N_0{}^1{}_a^b\,\partial_{1b} - N_0{}^2{}_a^b\,\partial_{2b}, \\ \delta_{1a} = \partial_{1a} - N_1{}^2{}_a^b\,\partial_{2b}, \\ \delta_{2a} = \partial_{2a}. \end{cases} \tag{1.9}$$

Theorem 1.2. [10] $\mathcal{B}$ *is dual to* $\mathcal{B}^*$, *if and only if the following relations hold*

$$\begin{cases} N_0{}^1{}_a^b = M_0{}^1{}_a^b, \\ N_0{}^2{}_a^b = M_0{}^2{}_a^b - M_1{}^2{}_c^b\, N_0{}^1{}_a^c, \\ N_1{}^2{}_a^b = M_1{}^2{}_a^b. \end{cases} \tag{1.10}$$

Definition 1.1. *The generalized connection*

$$\nabla : T(E)\otimes T(E) \to T(E),\ \nabla :(X,Y) \to \nabla_X Y,$$

or equivalently

$$\nabla_X : T(E)\to T(E),\ \nabla_X : Y \to \nabla_X Y,$$

is a linear connection determined by

$$\nabla_{\delta_{\beta b}}\delta_{\alpha a} = \Gamma_{\alpha a}{}^{\gamma c}{}_{\beta b}\ \delta_{\gamma c}, \tag{1.11}$$

where the summation is going over γ and *c*. If in (1.11) we set $\gamma = \alpha$ this provides the so called *d*-connection:

$$\nabla_{\delta_{\beta b}}\delta_{\alpha a} = \Gamma_{\alpha a}{}^{\alpha c}{}_{\beta b}\,\delta_{\alpha c}, \tag{1.12}$$

(with no summation over α).

The explicite from of (1.11) is given by

$$\begin{aligned}
\nabla_{\delta_{0b}}\delta_{0a} &= \underline{\Gamma_{0a}{}^{0c}{}_{0b}\,\delta_{0c}} + \Gamma_{0a}{}^{1c}{}_{0b}\,\delta_{1c} + \Gamma_{0a}{}^{2c}{}_{0b}\,\delta_{2c}\,,\\
\nabla_{\delta_{0b}}\delta_{1a} &= \Gamma_{1a}{}^{0c}{}_{0b}\,\delta_{0c} + \underline{\Gamma_{1a}{}^{1c}{}_{0b}\,\delta_{1c}} + \Gamma_{1a}{}^{2c}{}_{0b}\,\delta_{2c}\,,\\
\nabla_{\delta_{0b}}\delta_{2a} &= \Gamma_{2a}{}^{0c}{}_{0b}\,\delta_{0c} + \Gamma_{2a}{}^{1c}{}_{0b}\,\delta_{1c} + \underline{\Gamma_{2a}{}^{2c}{}_{0b}\,\delta_{2c}}\,.
\end{aligned} \tag{1.13}$$

If in the above formulaes we substitute $0b$ with $1b$, and then $0b$ with $2b$, we obtain the complete list of 9 formulaes. The underlined terms are $\Gamma_{0a}{}^{0c}{}_{X}$, $\Gamma_{1a}{}^{1c}{}_{X}$, $\Gamma_{2a}{}^{2c}{}_{X}$, where instead of X stays $1b$ or $2b$.

Comparing (1.13) with (1.12) we see that if in (1.13) all terms are zero except the underlined ones, we obtain the explicite from of the so called d-connection defined by (1.12).

Assume that $\hat{B} = \{\hat{\delta}_{0a}, \hat{\delta}_{1a}, \hat{\delta}_{2a}\}$ is another adapted basis of $T(E)$, which is formed as B (1.9) but with N replaced with $\hat{N}$. Another adapted basis of $T^*(E)$ is $\hat{B}^* = \{\hat{\delta}y^{0a}, \hat{\delta}y^{1a}, \hat{\delta}y^{2a}\}$ which is formed as B^* (1.6) but with M replaced with $\hat{M}$.

2. The connection between the torsions in the new adapted basis

The torsion tensor $T(X,Y)$ is defined as usual by

$$T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y].$$

Theorem 2.1. [10] *The torsion tensor of generalized connection has the form*

$$T(X,Y) = T_{\beta b}{}^{\gamma c}{}_{\alpha a} X^{\alpha a} Y^{\beta b} \delta_{\gamma c}\,, \tag{2.1}$$

where

$$T_{\beta b}{}^{\gamma c}{}_{\alpha a} = \Gamma_{\beta b}{}^{\gamma c}{}_{\alpha a} - \Gamma_{\alpha a}{}^{\gamma c}{}_{\beta b} - K_{\alpha a}{}^{\gamma c}{}_{\beta b}\,,$$

$$\left[\delta_{\alpha a}, \delta_{\beta b}\right] = K_{\alpha a}{}^{\gamma c}{}_{\beta b}\delta_{\gamma c}\,,$$

with

$$
\begin{aligned}
K_{0a}{}^{1c}{}_{0b} &= \delta_{0b}\, N^{1}_{0}{}^{c}_{a} - \delta_{0a}\, N^{1}_{0}{}^{c}_{b},\\
K_{0a}{}^{2c}{}_{0b} &= \left(\delta_{0b} N^{2}_{0}{}^{c}_{a} - \delta_{0a} N^{2}_{0}{}^{c}_{b}\right) + M^{2}_{1}{}^{c}_{e}\, K_{0a}{}^{1e}{}_{0b},\\
K_{0a}{}^{1c}{}_{1b} &= \delta_{1b}\, N^{1}_{0}{}^{c}_{a},\\
K_{0a}{}^{2c}{}_{1b} &= \left(\delta_{1b}\, N^{2}_{0}{}^{c}_{a} - \delta_{0a}\, N^{2}_{1}{}^{c}_{b}\right) + M^{2}_{1}{}^{c}_{d}\, K_{0a}{}^{1d}{}_{1b},\\
K_{0a}{}^{1c}{}_{2b} &= \delta_{2b}\, N^{1}_{0}{}^{c}_{a},\\
K_{0a}{}^{2c}{}_{2b} &= \delta_{2b} N^{2}_{0}{}^{c}_{a} + M^{2}_{1}{}^{c}_{d}\, K_{0a}{}^{1d}{}_{2b},\\
K_{0a}{}^{2c}{}_{1b} &= \delta_{1b}\, N^{2}_{1}{}^{c}_{a} - \delta_{1a}\, N^{2}_{1}{}^{c}_{b},\\
K_{1a}{}^{2c}{}_{2b} &= \delta_{2b}\, N^{2}_{1}{}^{c}_{a}.
\end{aligned}
$$

Using the relations (4.10), [10] we prove

Theorem 2.2. *Between* $\hat{K}_{\alpha a}{}^{\gamma c}{}_{\beta b}$ *and* $K_{\alpha a}{}^{\gamma c}{}_{\beta b}$ *there are the following connections*

$$
\begin{cases}
\hat{K}_{0a}{}^{0e}{}_{0b} = 0,\\
\hat{K}_{0a}{}^{1e}{}_{0b} = K_{0b}{}^{1e}{}_{0a} + \mathsf{A}_{ab}\left\{\overline{K}_{0a}{}^{1e}{}_{0b} + \hat{\delta}_{0a} A^{1}_{0}{}^{e}_{b}\right\},\\
A^{2}_{1}{}^{e}_{c}\, \hat{K}_{0a}{}^{1c}{}_{0b} + \hat{K}_{0a}{}^{2e}{}_{0b} = K_{0b}{}^{2e}{}_{0a} + A^{1}_{0}{}^{c}_{a}\, A^{1}_{0}{}^{d}_{b}\, K_{1c}{}^{2e}{}_{1d} + \mathsf{A}_{ab}\\
\qquad \left\{\overline{K}_{0a}{}^{2e}{}_{0b} + \hat{\delta}_{0a}\overline{A}^{2}_{0}{}^{e}_{b} + A^{1}_{0}{}^{c}_{a}\, \overline{A}^{2}_{0}{}^{d}_{b}\, K_{1c}{}^{2e}{}_{2d}\right\}.
\end{cases}
\tag{2.2}
$$

$$
\begin{cases}
\hat{K}_{0a}{}^{0e}{}_{1b} = 0,\\
\hat{K}_{0a}{}^{1e}{}_{1b} = \overline{K}_{0b}{}^{1e}{}_{1b} - \hat{\delta}_{1b} A^{1}_{0}{}^{e}_{a},\\
A^{2}_{1}{}^{e}_{c}\, \hat{K}_{0a}{}^{1c}{}_{1b} + \hat{K}_{0a}{}^{2e}{}_{1b} = \overline{K}_{0a}{}^{2e}{}_{1b} + A^{1}_{0}{}^{c}_{a}\, \overline{K}_{1c}{}^{2e}{}_{1b} + \hat{\delta}_{0a} A^{2}_{1}{}^{e}_{b} - \hat{\delta}_{1b}\overline{A}^{2}_{0}{}^{e}_{a} -\\
\qquad - A^{2}_{0}{}^{c}_{a}\, K_{1b}{}^{2e}{}_{2c}.
\end{cases}
\tag{2.3}
$$

$$
\begin{cases}
\hat{K}_{0a}{}^{0e}{}_{2b} = 0,\\
\hat{K}_{0a}{}^{1e}{}_{2b} = K_{0a}{}^{1e}{}_{2b} - \delta_{2b} A^{1}_{0}{}^{e}_{a},\\
A^{2}_{1}{}^{e}_{c}\, \hat{K}_{0a}{}^{1c}{}_{2b} + \hat{K}_{0a}{}^{2e}{}_{2b} = K_{0a}{}^{2e}{}_{2b} + A^{1}_{0}{}^{c}_{a}\, K_{1c}{}^{2e}{}_{2b} - \delta_{2b}\overline{A}^{2}_{0}{}^{e}_{a}.
\end{cases}
\tag{2.4}
$$

$$
\begin{cases}
\hat{K}_{1a}{}^{0e}{}_{1b} = 0,\\
\hat{K}_{1a}{}^{1e}{}_{1b} = 0,\\
\hat{K}_{1a}{}^{2e}{}_{1b} = K_{1b}{}^{2e}{}_{1a} + \mathsf{A}_{ab}\left\{\overline{K}_{1a}{}^{2e}{}_{1b} + \hat{\delta}_{1a} A^{2}_{1}{}^{e}_{b}\right\}.
\end{cases}
\tag{2.5}
$$

$$\begin{cases}\hat{K}_{1a}{}^{0e}{}_{2b} = 0, \\ \hat{K}_{1a}{}^{1e}{}_{2b} = 0, \\ \hat{K}_{1a}{}^{2e}{}_{2b} = K_{1a}{}^{2e}{}_{2b} - \delta_{2b} A_1^2{}^e_a,\end{cases} \qquad \begin{cases}\hat{K}_{2a}{}^{0e}{}_{2b} = K_{2a}{}^{0e}{}_{2b} = 0, \\ \hat{K}_{2a}{}^{1e}{}_{2b} = K_{2a}{}^{1e}{}_{2b} = 0, \\ \hat{K}_{2a}{}^{2e}{}_{2b} = K_{2a}{}^{2e}{}_{2b} = 0,\end{cases} \tag{2.6}$$

where

$$\begin{cases}\overline{K}_{\delta d}{}^{\gamma e}{}_{0b} = K_{\delta d}{}^{\gamma e}{}_{0b} + A_0^1{}^c_b K_{\delta d}{}^{\gamma e}{}_{1c} + \overline{A}_0^2{}^c_b K_{\delta d}{}^{\gamma e}{}_{2c}, \\ \overline{K}_{\delta d}{}^{\gamma e}{}_{1b} = K_{\delta d}{}^{\gamma e}{}_{1b} + A_1^2{}^c_b K_{\delta d}{}^{\gamma e}{}_{2c}.\end{cases} \tag{2.7}$$

The connections between $\hat{T}_{\alpha a}{}^{\gamma c}{}_{\beta b}$ and $T_{\alpha a}{}^{\gamma c}{}_{\beta b}$ are given in the following theorem.

Theorem 2.3. *The components of the same torsion tensor $T(X,Y)$, of the generalized connection ∇ in the basis B and $\hat{B}$ are connected by*

$$\begin{cases}\hat{T}_{0b}{}^{0e}{}_{0a} = \overline{T}_{0b}{}^{0e}{}_{0a} + A_0^1{}^d_b \overline{T}_{1d}{}^{0e}{}_{0a} + \overline{A}_0^2{}^d_b \overline{T}_{2d}{}^{0e}{}_{0a}, \\ \hat{T}_{0b}{}^{1e}{}_{0a} + A_0^1{}^e_d \hat{T}_{0b}{}^{0d}{}_{0a} = \overline{T}_{0b}{}^{1e}{}_{0a} + A_0^1{}^d_b \overline{T}_{1d}{}^{1e}{}_{0a} + \overline{A}_0^2{}^d_b \overline{T}_{2d}{}^{1e}{}_{0a}, \\ \hat{T}_{0b}{}^{2e}{}_{0a} + A_1^2{}^e_d \hat{T}_{0b}{}^{1d}{}_{0a} + \overline{A}_0^2{}^e_d \hat{T}_{0b}{}^{0d}{}_{0a} = \overline{T}_{0b}{}^{2e}{}_{0a} + A_0^1{}^d_b \overline{T}_{1d}{}^{2e}{}_{0a} + \overline{A}_0^2{}^d_b \overline{T}_{2d}{}^{2e}{}_{0a}.\end{cases} \tag{2.8}$$

$$\begin{cases}\hat{T}_{0b}{}^{0e}{}_{1a} = \overline{T}_{0b}{}^{0e}{}_{1a} + A_0^1{}^d_b \overline{T}_{1d}{}^{0e}{}_{1a} + \overline{A}_0^2{}^d_b \overline{T}_{2d}{}^{0e}{}_{1a}, \\ \hat{T}_{0b}{}^{1e}{}_{1a} + A_0^1{}^e_d \hat{T}_{0b}{}^{0d}{}_{1a} = \overline{T}_{0b}{}^{1e}{}_{1a} + A_0^1{}^d_b \overline{T}_{1d}{}^{1e}{}_{1a} + \overline{A}_0^2{}^d_b \overline{T}_{2d}{}^{1e}{}_{1a}, \\ \hat{T}_{0b}{}^{2e}{}_{1a} + A_1^2{}^e_d \hat{T}_{0b}{}^{1d}{}_{1a} + \overline{A}_0^2{}^e_d \hat{T}_{0b}{}^{0d}{}_{1a} = \overline{T}_{0b}{}^{2e}{}_{1a} + A_0^1{}^d_b \overline{T}_{1d}{}^{2e}{}_{1a} + \overline{A}_0^2{}^d_b \overline{T}_{2d}{}^{2e}{}_{1a}.\end{cases} \tag{2.9}$$

$$\begin{cases}\hat{T}_{0b}{}^{0e}{}_{2a} = T_{0b}{}^{0e}{}_{2a} + A_0^1{}^d_b T_{1d}{}^{0e}{}_{2a} + \overline{A}_0^2{}^d_b T_{2d}{}^{0e}{}_{2a}, \\ \hat{T}_{0b}{}^{1e}{}_{2a} + A_0^1{}^e_d \hat{T}_{0b}{}^{0d}{}_{2a} = T_{0b}{}^{1e}{}_{2a} + A_0^1{}^d_b T_{1d}{}^{1e}{}_{2a} + \overline{A}_0^2{}^d_b T_{2d}{}^{1e}{}_{2a}, \\ \hat{T}_{0b}{}^{2e}{}_{2a} + A_1^2{}^e_d \hat{T}_{0b}{}^{1d}{}_{2a} + \overline{A}_0^2{}^e_d \hat{T}_{0b}{}^{0d}{}_{2a} = T_{0b}{}^{2e}{}_{2a} + A_0^1{}^d_b T_{1d}{}^{2e}{}_{2a} + \overline{A}_0^2{}^d_b T_{2d}{}^{2e}{}_{2a}.\end{cases} \tag{2.10}$$

$$\begin{cases}\hat{T}_{1b}{}^{0e}{}_{1a} = \overline{T}_{1b}{}^{0e}{}_{1a} + A_1^2{}^d_b \overline{T}_{2d}{}^{0e}{}_{1a}, \\ \hat{T}_{1b}{}^{1e}{}_{1a} + A_0^1{}^e_d \hat{T}_{1b}{}^{0d}{}_{1a} = \overline{T}_{1b}{}^{1e}{}_{1a} + A_1^2{}^d_b \overline{T}_{2d}{}^{1e}{}_{1a}, \\ \hat{T}_{1b}{}^{2e}{}_{1a} + A_1^2{}^e_d \hat{T}_{1b}{}^{1d}{}_{1a} + \overline{A}_0^2{}^e_d \hat{T}_{1b}{}^{0d}{}_{1a} = \overline{T}_{1b}{}^{2e}{}_{1a} + A_1^2{}^d_b \overline{T}_{2d}{}^{2e}{}_{1a}.\end{cases} \tag{2.11}$$

$$\begin{cases} \hat{T}_{2b\ 1a}^{\ 0e} = \overline{T}_{2b\ 1a}^{\ 0e}, \\ \hat{T}_{2b\ 1a}^{\ 1e} + A_{0\ d}^{1\ e}\ \hat{T}_{2b\ 1a}^{\ 0d} = \overline{T}_{2b\ 1a}^{\ 1e}, \\ \hat{T}_{2b\ 1a}^{\ 2e} + A_{1\ d}^{2\ e}\ \hat{T}_{2b\ 1a}^{\ 1d} + \overline{A}_{0\ d}^{2\ e}\ \hat{T}_{2b\ 1a}^{\ 0d} = \overline{T}_{2b\ 1a}^{\ 2e}. \end{cases} \tag{2.12}$$

$$\hat{T}_{2b\ 2a}^{\ \gamma e} = T_{2b\ 2a}^{\ \gamma e}, \ \ \gamma = \overline{0, 2}, \tag{2.13}$$

where

$$\begin{cases} \overline{T}_{\delta d\ 0b}^{\ \gamma e} = T_{\delta d\ 0b}^{\ \gamma e} + A_{0\ b}^{1\ c}\ T_{\delta d\ 1c}^{\ \gamma e} + \overline{A}_{0\ b}^{2\ c}\ T_{\delta d\ 2c}^{\ \gamma e}, \\ \hat{T}_{\delta d\ 1b}^{\ \gamma e} = T_{\delta d\ 1b}^{\ \gamma e} + A_{1\ b}^{2\ c}\ T_{\delta d\ 2c}^{\ \gamma e}, \ \ \delta = \overline{0, 2}. \end{cases} \tag{2.14}$$

3. The connections between the components of the metric tensor in the new adapted basis

In the space $T^*(E) \otimes T^*(E)$ the metric tensor *G* can be given by

$$G = \begin{bmatrix} \delta y^{0a} & \delta y^{1a} & \delta y^{2a} \end{bmatrix} \begin{bmatrix} g_{0a\ 0b} & g_{0a\ 1b} & g_{0a\ 2b} \\ g_{1a\ 0b} & g_{1a\ 1b} & g_{1a\ 2b} \\ g_{2a\ 0b} & g_{2a\ 1b} & g_{2a\ 2b} \end{bmatrix} \begin{bmatrix} \delta y^{0b} \\ \delta y^{1b} \\ \delta y^{2b} \end{bmatrix}$$

$$= g_{\alpha a\ \beta b} \delta y^{\alpha a} \otimes \delta y^{\beta b}, \ \ \alpha, \beta = \overline{0, 2}.$$

For the components of the metric tensor we have:

$$g_{\alpha a\ \beta b} = g_{\alpha a'\ \beta b'} \left(\partial_a x^{a'}\right) \left(\partial_b x^{b'}\right), \ \ \alpha, \beta = \overline{0, 2}.$$

Theorem 3.1. *The connections between the components of the metric tensor, with respect to the generalized connection* ∇, *in the basis* B *and* $\hat{\mathsf{B}}$ *are given by*

$$\hat{g}_{0a\ 0b} = \overline{g}_{0a\ 0b} + A_{0\ b}^{1\ d}\ \overline{g}_{0a\ 1d} + \overline{A}_{0\ b}^{2\ d}\ \overline{g}_{0a\ 2d}, \tag{3.1}$$

$$\hat{g}_{1a\ 0b} = \overline{g}_{1a\ 0b} + A_{1\ a}^{2\ c}\ \overline{g}_{2c\ 0b}, \tag{3.2}$$

$$\hat{g}_{2a\ 0b} = \overline{g}_{2a\ 0b}, \tag{3.3}$$

$$\hat{g}_{0a\ 1b} = \overline{g}_{0a\ 1b} + A_{1\ b}^{2\ d}\ \overline{g}_{0a\ 2d}, \tag{3.4}$$

$$\hat{g}_{1a\ 1b} = \overline{g}_{1a\ 1b} + A_{1\ b}^{2\ d}\ \overline{g}_{1a\ 2d}, \tag{3.5}$$

$$\hat{g}_{2a\ 1b} = \bar{g}_{2a\ 1b}\,, \tag{3.6}$$

$$\hat{g}_{0a\ 2b} = \bar{g}_{0a\ 2b}\,, \tag{3.7}$$

$$\hat{g}_{1a\ 2b} = \bar{g}_{1a\ 2b}\,, \tag{3.8}$$

$$\hat{g}_{2a\ 2b} = g_{2a\ 2b} \tag{3.9}$$

where

$$\bar{g}_{0a\ \gamma b} = g_{0a\ \gamma b} + A^{1}_{0}{}^{c}_{a}\, g_{1c\ \gamma b} + \bar{A}^{2}_{0}{}^{c}_{a}\, g_{2c\ \gamma b}\,,\ \ \gamma = \overline{0,2}\,, \tag{3.10}$$

$$\bar{g}_{\gamma a\ 0b} = g_{\gamma a\ 0b} + A^{1}_{0}{}^{d}_{b}\, g_{\gamma a\ 1d} + \bar{A}^{2}_{0}{}^{d}_{b}\, g_{\gamma a\ 2d}\,,\ \ \gamma = \overline{1,2}\,, \tag{3.11}$$

$$\bar{g}_{1a\ \gamma b} = g_{1a\ \gamma b} + A^{2}_{1}{}^{c}_{a}\, g_{2c\ \gamma b}\,, \tag{3.12}$$

$$\bar{g}_{2a\ 1b} = g_{2a\ 1b} + A^{2}_{1}{}^{d}_{b}\, g_{2a\ 2d}\,. \tag{3.13}$$

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