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On Fuzzy Quotient Mappings

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Abstract. Azad [1] introduced the concepts of fuzzy semi-open sets and fuzzy semi-continuous mappings. Fuzzy α-open sets and fuzzy α-continuous functions were introduced by Mashhour *et. al.* [4] and Singal *et. al.* [6] respectively. The concepts of fuzzy pre-open sets and fuzzy pre-continuous mappings were introduced by Bin Sahana [2]. In this paper the concepts of fuzzy α-quotient maps, fuzzy semi quotient maps, fuzzy pre quotient maps, fuzzy strongly α-quotient maps, fuzzy strongly semi quotient maps, fuzzy strongly pre quotient maps, fuzzy α^*-quotient maps, fuzzy semi*-quotient maps and fuzzy pre*-quotient maps are introduced and some of their properties are studied. Also the interconnections between the new mappings and the fuzzy quotient mappings are investigated. Some examples are given to illustrate the results.

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1. Preliminaries

Throughout this paper by a fuzzy topological space(fts) (X, τ), we mean a fuzzy topological space in the sense of Chang [3]. Interior and closure of a fuzzy subset A of X is denoted by $\mathrm{int}(A)$ *and* $\mathrm{cl}(A)$ respectively.

Definition 1.1. *A fuzzy subset S of a fts (X, τ) is called a*

(1) *fuzzy α-open set* [2] *if* $S \subset \mathrm{int}(\mathrm{cl}(intS))$
(2) *fuzzy semi-open set* [1] *if* $S \subset \mathrm{cl}(\mathrm{int}(S))$
(3) *fuzzy pre-open set* [2] *if* $S \subset \mathrm{int}(\mathrm{cl}(S))$.

Note 1.1. We denote the family of all fuzzy α-open sets of fts (X, τ) by τ^α and of all fuzzy semi-open sets and of all fuzzy pre-open sets of (X, τ) by $SO(X)$ and $PO(X)$ respectively.

Definition 1.2. *Let (X, τ) and (Y, δ) be fuzzy topological spaces. A function* $f : (X, \tau) \to (Y, \delta)$ *is called*

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(1) *fuzzy α-continuous* [2] *(respectively, fuzzy semi-continuous* [1], *fuzzy pre-continuous* [2]*) if the inverse image of each fuzzy open set in Y is a fuzzy α-open set (respectively, fuzzy semi-open set, fuzzy pre-open set) in Y.*
(2) *fuzzy α-open mapping* [2]*(resp. fuzzy semi-open mapping* [1], *fuzzy pre-open mapping* [2]*) if the image of each fuzzy open set in X is a fuzzy α-open set (resp. fuzzy semi-open set, fuzzy pre-open set) in Y.*

Theorem 1.1. [2] *A subset S of a fts (X, τ) is a fuzzy α-open set iff S is fuzzy semi-open and fuzzy pre-open.*

Corollary 1.1. *Let (X, τ) and (Y, δ) be fuzzy topological spaces. A function $f : X \to Y$ is*

(1) *fuzzy α-continuous iff it is fuzzy semi-continuous and fuzzy pre-continuous*
(2) *fuzzy α-open map iff it is fuzzy semi-open and fuzzy pre-open.*

Definition 1.3. *Let (X, τ) and (Y, δ) be fuzzy topological spaces. A function $f : X \to Y$ is called fuzzy α-irresolute* [2]*(resp. fuzzy irresolute* [5], *fuzzy pre-irresolute* [2]*) if the inverse image of every fuzzy α-open set(resp. fuzzy semi-open set, fuzzy pre-open set) in Y is a fuzzy α-open set (resp. fuzzy semi-open set, fuzzy pre-open set) in X.*

Definition 1.4. *Let (X, τ) and (Y, δ) be fuzzy topological spaces. A mapping $f : (X, \tau) \to (Y, \delta)$ is called a*

(1) *fuzzy continuous mapping* [3] *if $f^{-1}(A) \in \tau$ for each $A \in \delta$.*
(2) *fuzzy open mapping* [7] *if $f(A) \in \delta$ for each $A \in \tau$.*

2. Fuzzy α-quotient mappings

Definition 2.1. *Let (X, τ) and (Y, δ) be fuzzy topological spaces. Let $f : X \to Y$ be an onto map. Then f is said to be a*

(1) *fuzzy α-quotient map if f is fuzzy α-continuous and $f^{-1}(V)$ is fuzzy open in X implies V is a fuzzy α-open set in Y.*
(2) *fuzzy semi-quotient map if f is fuzzy semi-continuous and $f^{-1}(V)$ is fuzzy open in X implies V is a fuzzy semi-open set in Y.*
(3) *fuzzy pre-quotient map if f is fuzzy pre-continuous and $f^{-1}(V)$ is fuzzy open in X implies V is a fuzzy pre-open set in Y.*

Example 2.1. Fuzzy α-quotient map. Let $X = \{a, b, c\}, Y = \{p, q\}, \tau = \{0, 1, A_{a,b,c}^{1,\frac{1}{2},\frac{1}{2}}, B_{a,b,c}^{1,\frac{3}{4},\frac{3}{4}}\}, \delta = \{0, 1, P_{p,q}^{1,\frac{1}{2}}\}$ where A is a fuzzy set given by $A(a) = 1, A(b) = \frac{1}{2}, A(c) = \frac{1}{2}$ etc. Clearly (X, τ) and (Y, δ) are fuzzy topological spaces. Also $\delta^\alpha = \{0, 1, Q_{p,q}^{1,\beta} / \beta \in [\frac{1}{2}, 1]\}$. Define $f : X \to Y$ by $f(a) = p, f(b) = f(c) = q$. Clearly f is a fuzzy continuous map and hence a fuzzy α-continuous map. Also it is clear that when $f^{-1}(V)$ is fuzzy open in X, then V is a fuzzy α-open set in Y. So, f is a fuzzy α-quotient map.

Theorem 2.1. *Let (X,τ) and (Y,δ) be fuzzy topological spaces. If $f:(X,\tau)\to(Y,\delta)$ is an onto fuzzy α-continuous and fuzzy α-open map, then f is a fuzzy α-quotient map.*

Proof. Obvious. □

Theorem 2.2. *Let $(X,\tau),(Y,\delta)$ and (Z,μ) be fuzzy topological spaces. Let $f:(X,\tau)\to(Y,\delta)$ be an onto fuzzy open and fuzzy α-irresolute map. Let $g:(Y,\delta)\to(Z,\mu)$ be a fuzzy α-quotient map. Then $g\circ f$ is a fuzzy α-quotient map.*

Proof. Let V be any fuzzy open set in Z. Then $g^{-1}(V)$ is a fuzzy α-open set as g is a fuzzy α-quotient map. Since f is fuzzy α-irresolute, $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is a fuzzy α-open set in X. So, $g\circ f$ is a fuzzy α-continuous map. Suppose $(g\circ f)^{-1}(V)$ is fuzzy open in X. Then $f^{-1}(g^{-1}(V))$ is fuzzy open in X. Since f is fuzzy open and onto, $f(f^{-1}(g^{-1}(V))=g^{-1}(V)$ is fuzzy open in Y. Since g is a fuzzy α-quotient map, V is a fuzzy α-open set in Z. Hence $g\circ f$ is a fuzzy α-quotient map. □

Corollary 2.1. *Let $f:(X,\tau)\to(Y,\delta)$ be an onto fuzzy open fuzzy irresolute (respectively, fuzzy pre-irresolute) map and $g:(Y,\delta)\to(Z,\mu)$ be a fuzzy semi-quotient (respectively, fuzzy pre-quotient) map. Then $g\circ f$ is a fuzzy semi-quotient (respectively, fuzzy pre-quotient) map.*

Theorem 2.3. *Let $(X,\tau),(Y,\delta)$ and (Z,μ) be fuzzy topological spaces. If $p:(X,\tau)\to(Y,\delta)$ is a fuzzy α-quotient map and $g:(X,\tau)\to(Z,\mu)$ is a fuzzy continuous map such that it is constant on each set $p^{-1}(\{y\})$ for $y\in Y$. Then g induces a fuzzy α-continuous map $f:(Y,\delta)\to(Z,\mu)$ such that $f\circ p=g$.*

Proof. Since map g is constant on $p^{-1}(\{y\})$ for each $y\in Y$, the set $g(p^{-1}(\{y\})$ is a one point set in Z. If we let $f(y)$ to denote this point, then it is clear that map f is well defined and for each $x\in X, f(p(x))=g(x)$. Now we claim that f is fuzzy α-continuous. Let V be any fuzzy open set in Z. Then $g^{-1}(V)$ is a fuzzy open set as g is fuzzy continuous. That is $g^{-1}(V)=(f\circ p)^{-1}(V)=p^{-1}(f^{-1}(V))$ is fuzzy open in X. Since p is a fuzzy α-quotient map, $f^{-1}(V)$ is a fuzzy α-open set in Y. □

Theorem 2.4. *Let (X,τ) and (Y,δ) be fuzzy topological spaces. A function $f:(X,\tau)\to(Y,\delta)$ is a fuzzy α-quotient map iff it is a fuzzy semi quotient map and a fuzzy pre-quotient map.*

Proof. Let f be a fuzzy α-quotient map. So, $f^{-1}(V)\in\tau^{\alpha}$ whenever V is a fuzzy open set in Y. By Theorem 1.1 it follows that $f^{-1}(V)$ is both fuzzy semi-open and fuzzy pre-open. Hence f is both fuzzy semi-continuous and fuzzy pre-continuous. Let $f^{-1}(V)$ be a fuzzy open set in X. Since f is fuzzy α-quotient, $V\in\delta^{\alpha}$ where $\delta^{\alpha}=SO(Y)\cap PO(Y)$. So $V\in SO(Y)$ and $PO(Y)$. This shows that f is both fuzzy semi-quotient and fuzzy pre-quotient. Conversely let f be a fuzzy semi-quotient and a fuzzy pre-quotient map. We claim that f is a fuzzy α-quotient map. Let V be any fuzzy open set in Y. Since f is both fuzzy semi-quotient and fuzzy pre-quotient, $f^{-1}(V)\in SO(X)\cap PO(X)$ so that $f^{-1}(V)\in\tau^{\alpha}$. Hence f is fuzzy α-continuous. Let $f^{-1}(V)$ be fuzzy open in X. So $V\in SO(Y)\cap PO(Y)$ so that $V\in\delta^{\alpha}$. Hence f is a fuzzy α-quotient map. □

3. Fuzzy strongly α-quotient mappings

Definition 3.1. *Let (X,τ) and (Y,δ) be fuzzy topological spaces. Let $f:(X,\tau)\to(Y,\delta)$ be an onto map. Then f is called a fuzzy strongly α-quotient (resp. fuzzy strongly semi-quotient, fuzzy strongly pre-quotient) map provided a fuzzy subset A of Y is fuzzy open in Y iff $f^{-1}(A)$ is a fuzzy α-open set (resp. fuzzy semi-open set, fuzzy pre-open set) in X.*

Theorem 3.1. *A fuzzy strongly α-quotient map is a fuzzy α-quotient map.*

Proof. Let $f:(X,\tau)\to(Y,\delta)$ be a fuzzy strongly α-quotient map where (X,τ) and (Y,δ) are fuzzy topological spaces. Let V be a fuzzy open set in Y. So, $f^{-1}(V)$ is a fuzzy α-open set in X. Let $f^{-1}(V)$ be fuzzy open in X and so it is a fuzzy α-open set in X. Hence V is a fuzzy open set in Y and hence also a fuzzy α-open set in Y. $\square$

Remark 3.1. Converse of the above theorem is false as can be seen from the following example.

Example 3.1. In Example 2.1, we note that $f:X\to Y$ is a fuzzy α-quotient map. Also $f^{-1}(Q_{p.q}^{1,\frac{3}{4}})=B_{a,b,c}^{1,\frac{3}{4},\frac{3}{4}}\in\tau$, but $Q_{p,q}^{1,\frac{3}{4}}\notin\delta$.

Theorem 3.2. *Let (X,τ) and (Y,δ) be fuzzy topological spaces. If a function $f:(X,\tau)\to(Y,\delta)$ is fuzzy strongly semi-quotient and fuzzy strongly pre-quotient, then f is fuzzy strongly α-quotient.*

Proof. Let V be a fuzzy open set in Y. Since f is fuzzy strongly semi-quotient and fuzzy strongly pre-quotient, $f^{-1}(V)$ is fuzzy semi-open as well as fuzzy pre-open. So, $f^{-1}(V)$ is fuzzy α-open. Let $f^{-1}(V)$ be a fuzzy α-open set in X. So, $f^{-1}(V)\in SO(X)$. Since f is fuzzy strongly semi-quotient, V is fuzzy open in Y. Hence it follows that V is fuzzy open in Y iff $f^{-1}(V)$ is fuzzy α-open in X. So f is a fuzzy strongly α-quotient map. $\square$

Remark 3.2. Converse of the above theorem is false as can be seen from the following example.

Example 3.2. Let $X=\{a,b,c,d\}$ and $Y=\{p,q,r\}$, $\tau=\{0,1,A_{a,b,c,d}^{\frac{3}{4},\frac{3}{4},\frac{3}{4},0}\}$, $\delta=\{0,1,P_{p,\ q,\ r}^{u_1,u_2,s}: s\in[0,1],\frac{3}{4}\leq u_i\leq 1, i=1,2\}=\delta^{\alpha}$, $\tau^{\alpha}=\{0,1,B_{a,\ b,\ c,\ d}^{v_1,v_2,v_3,s}: s\in[0,1],\frac{3}{4}\leq v_i\leq 1, i=1,2,3\}$. Clearly (X,τ) and (Y,δ) are fuzzy topological spaces. Define a map $f:X\to Y$ by $f(a)=p,\ f(b)=f(c)=q,\ f(d)=r$. We claim that f is fuzzy strongly α-quotient but not fuzzy strongly pre-quotient. We note that $f^{-1}(P)=B_{a,\ b,\ c,\ d}^{u_1,u_2,u_2,s}$, $f^{-1}(0)=0$, $f^{-1}(1)=1$. Clearly $B,1,0\in\tau^{\alpha}$. Also it is clear that whenever $f^{-1}(U)$ is fuzzy α-open in X then U is fuzzy open in Y. So f is a fuzzy strongly α-quotient map. Now consider $R_{p,q,r}^{\frac{1}{2},\frac{1}{2},\frac{1}{2}}$. We note that $f^{-1}(R)_{a,b,c,d}^{\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}}$ is a fuzzy pre-open set. For, $\text{int}(\text{cl}(f^{-1}(R))=1$. So $f^{-1}(R)\subset\text{int}(clf^{-1}(R)$. But $R\notin\delta$. Hence f is not a fuzzy strongly pre-quotient map.

4. Fuzzy α^*-quotient Mappings

Definition 4.1. *Let (X,τ) and (Y,δ) be fuzzy topological spaces. Let $f:(X,\tau)\to(Y,\delta)$ be an onto map. Then f is called a*

(1) *fuzzy α^*-quotient map if f is fuzzy α-irresolute and $f^{-1}(U)$ is a fuzzy α-open set in X implies U is fuzzy open in Y.*
(2) *fuzzy semi*-quotient map if f is fuzzy irresolute and $f^{-1}(U)$ is fuzzy semi-open in X implies U is fuzzy open in Y.*
(3) *fuzzy pre*-quotient map if f is fuzzy pre-irresolute and $f^{-1}(U)$ is fuzzy pre-open in X implies U is fuzzy open in Y.*

Example 4.1 (Fuzzy α^*-quotient map)**.** In Example 3.2, f is clearly a fuzzy α-irresolute map. Also $f^{-1}(U)$ is a fuzzy α-open set in X implies U is a fuzzy open set in Y. So f is a fuzzy α^*-quotient map.

Definition 4.2. [2] *Let (X,τ) and (Y,δ) be fuzzy topological spaces. A function $f:(X,\tau)\to(Y,\delta)$ is called a fuzzy strongly α-open map if the image of every fuzzy α-open set in X is a fuzzy α-open set in Y.*

Theorem 4.1. *Let $(X,\tau),(Y,\delta)$ and (Z,μ) be fuzzy topological spaces. Let $f:(X,\tau)\to(Y,\delta)$ be an onto fuzzy strongly α-open and a fuzzy α-irresolute map. Let $g:(Y,\delta)\to(Z,\mu)$ be a fuzzy α^*-quotient map. Then $g\circ f$ is a fuzzy α^*-quotient map.*

Proof. We claim that $g\circ f$ is fuzzy α-irresolute. Let V be a fuzzy α-open set in Z. Then $g^{-1}(V)$ is a fuzzy α-open set in Y as g is a fuzzy α^*-quotient map. Since f is fuzzy α-irresolute $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is a fuzzy α-open set in X. So, $g\circ f$ is a fuzzy α-irresolute map. Suppose $(g\circ f)^{-1}(V)=f^{-1}(g^{-1}(V))$ is a fuzzy α-open set in X. Since f is fuzzy strongly α-open, $f(f^{-1}(g^{-1}(V)))$ is a fuzzy α-open set in Y. Since f is an onto map $f(f^{-1}(g^{-1}(V)))=g^{-1}(V)$. So $g^{-1}(V)$ is a fuzzy α-open set in Y. This implies that V is a fuzzy open set in Z as g is a fuzzy α^*-quotient map. Hence $g\circ f$ is a fuzzy α^*- quotient map. □

Theorem 4.2. *Let (X,τ) and (Y,δ) be fuzzy topological spaces. If a function $f:(X,\tau)\to(Y,\delta)$ is fuzzy semi*-quotient and fuzzy pre*-quotient then f is fuzzy α^*-quotient.*

Proof. Let V be a fuzzy α-open set in Y. Since f is fuzzy *semi**-quotient and fuzzy *pre**-quotient, $f^{-1}(V)\in SO(X)\cap PO(X)$. So $f^{-1}(V)\in\tau^\alpha$. Hence f is a fuzzy α-irresolute map. Let $f^{-1}(V)$ be a fuzzy α-open set in X. So $f^{-1}(V)\in SO(X)\cap PO(X)$. Since f is fuzzy *semi**-quotient and fuzzy *pre**-quotient, V is a fuzzy open set in Y. This shows that f is a fuzzy α^*-quotient map. □

Remark 4.1. Converse of the above theorem is false as can be seen from the following example.

Example 4.2. Let $X=\{a,b,c\}$ and $Y=\{p,q\}$, $\tau=\{0,1,A_{a,b,c}^{\frac{1}{2},1,1},B_{a,b,c}^{\frac{1}{2},0,0}\}=\tau^\alpha$, $\delta=\{0,1,P_{p,q}^{\frac{1}{2},1},Q_{p,q}^{\frac{1}{2},0}\}=\delta^\alpha$. Define $f:X\to Y$ by $f(a)=p,f(b)=f(c)=q$. Clearly f is a fuzzy α^*-quotient map. For, when $U\in\delta^\alpha,f^{-1}(U)\in\tau^\alpha$. Also when

$f^{-1}(U) \in \tau^{\alpha}$, clearly $U \in \delta$. We note that f is not a fuzzy *pre**-quotient map. For, $R_{p,q}^{\frac{1}{2},\frac{1}{2}} \notin \delta$ and $f^{-1}(R)_{a,b,c}^{\frac{1}{2},\frac{1}{2},\frac{1}{2}}$ is a fuzzy pre-open set, as $\text{int}(\text{cl}(f^{-1}(R))) = A$ and so $f^{-1}(R) \subset \text{int}(\text{cl}(f^{-1}(R)))$.

5. Comparisons

Theorem 5.1. *Let (X, τ) and (Y, δ) be fuzzy topological spaces. If $f : (X, \tau^{\alpha}) \to (Y, \delta^{\alpha})$ is a fuzzy quotient map then $f : (X, \tau) \to (Y, \delta)$ is a fuzzy α-quotient map.*

Proof. Let $V \in \delta$. So $V \in \delta^{\alpha}$. Since f is a fuzzy quotient map, $f^{-1}(V) \in \tau^{\alpha}$. Hence it is proved that when V is a fuzzy open set in Y, then $f^{-1}(V)$ is a fuzzy α- open set in X. So f is a fuzzy α-continuous map. Suppose $f^{-1}(V)$ is fuzzy open in (X, τ), then $f^{-1}(V) \in \tau^{\alpha}$. Since f is a fuzzy quotient map, $V \in \delta^{\alpha}$ and so V is a fuzzy α-open set in Y. Hence $f : (X, \tau) \to (Y, \delta)$ is a fuzzy α-quotient map. □

Definition 5.1. *Let (X, τ) and (Y, δ) be fuzzy topological spaces. A function $f : (X, \tau) \to (Y, \delta)$ is called fuzzy quasi α-open if the image of every fuzzy α-open set in X is fuzzy open in Y.*

Theorem 5.2. *Let (X, τ) and (Y, δ) be fuzzy topological spaces. If $f : (X, \tau^{\alpha}) \to (Y, \delta^{\alpha})$ is fuzzy quasi α-open then $f : (X, \tau) \to (Y, \delta)$ is fuzzy strongly α-open.*

Proof. Let V be a fuzzy α-open set in τ. So $V \in \tau^{\alpha}$. Since $f : (X, \tau^{\alpha}) \to (Y, \delta^{\alpha})$ is fuzzy quasi α-open, $f(V)$ is fuzzy open in (Y, δ^{α}). So $f(V)$ is a fuzzy α-open set in (Y, δ). Hence it follows that $f : (X, \tau) \to (Y, \delta)$ is a fuzzy strongly α-open map. □

Proposition 5.1. *A fuzzy α^*-quotient map is a fuzzy strongly α-quotient map.*

Proof. Let (X, τ) and (Y, δ) be fuzzy topological spaces. Let $f : (X, \tau) \to (Y, \delta)$ be a fuzzy α^*-quotient map. Suppose V is a fuzzy open set in Y. Then $f^{-1}(V)$ is a fuzzy α-open set in X as $V \in \delta^{\alpha}$ and f is fuzzy α-irresolute. Suppose $f^{-1}(V)$ is a fuzzy α-open set in X then V is a fuzzy open set in Y as f is a fuzzy α^*-quotient map. Hence f is a fuzzy strongly α-quotient map. □

Proposition 5.2. *Every fuzzy quotient map is a fuzzy α-quotient map.*

Proof. Let $f : (X, \tau) \to (Y, \delta)$ be a fuzzy quotient map where $(X, \tau), (Y, \delta)$ are fuzzy topological spaces. Let V be a fuzzy open set in Y. Since f is a fuzzy quotient map, $f^{-1}(V)$ is fuzzy open in X and so, $f^{-1}(V)$ is a fuzzy α-open set in X. So f is a fuzzy α-continuous map. Let $f^{-1}(V)$ be fuzzy open in X. Since f is a fuzzy quotient map, V is a fuzzy open set in Y and so V is a fuzzy α-open set in Y. Hence f is a fuzzy α-quotient map. □

Remark 5.1. Converse of the above Proposition 5.2 is not true as can be seen from the following example.

Example 5.1. The map f in Example 2.1 is a fuzzy α-quotient map. Now we claim that f is not a fuzzy quotient map. For, consider a fuzzy subset $R_{p,q}^{1,\frac{3}{4}}$ of Y. $f^{-1}(R) = B$. So $f^{-1}(R) \in \tau$ but $R \notin \delta$. Hence f is not a fuzzy quotient map.

Remark 5.2. A fuzzy quotient map need not be a fuzzy strongly α-quotient map as can be seen from the following example.

Example 5.2. Let $X = \{a, b, c\}$ and $Y = \{p, q\}$. Let $\tau = \{0, 1, A_{a,b,c}^{1,\frac{1}{2},\frac{1}{2}}, B_{a,b,c}^{1,\frac{3}{4},\frac{3}{4}}\}$, $\delta = \{0, 1, P_{p,q}^{1,\frac{1}{2}}, Q_{p,q}^{1,\frac{3}{4}}\}$. Clearly (X, τ) and (Y, δ) are fuzzy topological spaces.

$\tau^\alpha = \{0, 1, C_{a,b,c}^{1,r,s}/r, s \in [\frac{1}{2}, 1]\}$.
$\delta^\alpha = \{0, 1, R_{p,q}^{1,t}/t \in [\frac{1}{2}, 1]\}$.

Define $f : X \to Y$ by $f(a) = p, f(b) = f(c) = q$. Clearly f is a fuzzy quotient map. Consider a fuzzy subset $U_{p,q}^{1,.8}$ of Y. $f^{-1}(U)_{a,b,c}^{1,.8,.8} \in \tau^\alpha$. But $U \notin \delta$. So f is not a fuzzy strongly α-quotient map.

Remark 5.3. A fuzzy quotient map need not be a fuzzy α^*-quotient map which follows from Remark 5.2 and Proposition 5.1.

Remark 5.4. A fuzzy α^*-quotient map need not be a fuzzy quotient map as can be seen from the following example.

Example 5.3. Let $X = \{a, b, c, d\}$ and $Y = \{p, q, r\}$, $\tau = \{0, 1, A_{a,b,c,d}^{1,0,0,0}, B_{a,b,c,d}^{1,0,0,1}\}$, $\delta = \{0, 1, P_{p,q,r}^{1,r,s}/r, s \in [0, 1]\} = \delta^\alpha$. Clearly (X, τ) and (Y, δ) are fuzzy topological spaces. $\tau^\alpha = \{0, 1, C_{a,b,c,d}^{1,u,v,w}/u, v, w \in [0, 1]\}$. Define a map $f : X \to Y$ by $f(a) = p, f(b) = f(c) = q, f(d) = r$. Clearly f is a fuzzy α^*-quotient map. For, $f^{-1}(P)_{a,b,c,d}^{1,r,r,s} \in \tau^\alpha$. Also when $f^{-1}(U)$ is fuzzy α-open in X, U is clearly fuzzy open in Y. Obviously f is not a fuzzy quotient map. For, $Q_{p.q,r}^{1,\frac{1}{2},\frac{1}{2}} \in \delta$. But $f^{-1}(Q)_{a,b,c,d}^{1,\frac{1}{2},\frac{1}{2},\frac{1}{2}} \notin \tau$.

Remark 5.5. A fuzzy strongly α-quotient map need not be a fuzzy quotient map. For, in Example 3.2 we note that map f is fuzzy strongly α-quotient but not fuzzy quotient, since $P_{p,q,r}^{\frac{3}{4},\frac{3}{4},\frac{1}{2}} \in \delta$, but $(f^{-1}(P))_{a,b,c,d}^{\frac{3}{4},\frac{3}{4},\frac{3}{4},\frac{1}{2}} \notin \tau$.

Remark 5.6. A fuzzy α-quotient map need not be a fuzzy α^*-quotient map which follows from Remark 3.1 and Proposition 5.1.

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