BULLETIN *of the*
MALAYSIAN MATHEMATICAL
SCIENCES SOCIETY
http://math.usm.my/bulletin

Bull. Malays. Math. Sci. Soc. (2) **28***(2) (2005), 183–189*

Some Integral Properties of a General Class of Polynomials Associated with Feynman Integrals

V.B.L. CHAURASIA AND ASHOK SINGH SHEKHAWAT
Department of Mathematics, University of Rajasthan, Jaipur-302004, Rajasthan, India
csmaths2004@yahoo.com

Abstract. The object of the present paper is to discuss certain integral properties of a general class of polynomials and the $\overline{H}$-function, proposed by Inayat-Hussain which contain a certain class of Feynman integrals, the exact partition of a Gaussian model in statistical mechanics and several other functions as its particular cases. During the course of finding, we establish certain new double integral relations pertaining to a product involving a general class of polynomials and the $\overline{H}$-function. These double integral relations are unified in nature and act as a key formulae from which we can obtain as their special cases, double integral relations concerning a large number of simpler special functions and polynomials. For the sake of illustration, we record here some special cases of our main results which are also new and of interest by themselves. The results established here are basic in nature and are likely to find useful applications in several fields notably electrical networks, probability theory and statistical mechanics.

2000 Mathematics Subject Classification: 33C60, 26A33

Key words and phrases: Feynman integrals, $\bar{H}$-function, general class of polynomials, Hermite polynomials, Laguerre polynomials.

1. Introduction

The conventional formulation may fail pertaining to the domain of quantum cosmology but Feynman path integrals apply [5, 6]. Feynman path integrals reformulation of quantum mechanics are more fundamental than the conventional formulation in terms of operators. Feynman integrals are useful in the study and development of simple and multiple variable hypergeometric series which in turn are useful in statistical mechanics.

The $\bar{H}$-function [5] is a new generalization of the well known Fox's H-function [3]. The $\bar{H}$-function pertains the exact partition function of the Gaussian model in statistical mechanics, functions useful in testing hypothesis and several others as its special cases.

Received: April 12, 2004; *Revised:* March 23, 2005.

The $\bar{H}$-function will be defined and represented as follows [1]:

$$\bar{H}^{M,N}_{P,Q}[z] = \bar{H}^{M,N}_{P,Q}\left[z\left|{(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \atop (b_j,\beta_j)_{1,M},(b_j;\beta_j)_{M+1,Q}}\right.\right] = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\bar{\phi}(\xi)z^{\xi}d\xi \ , \tag{1.1}$$

where

$$\bar{\phi}(\xi) = \frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_j\xi)^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

which contains fractional powers of some of the gamma functions. Here, and through out the paper a_j $(j=1,\ldots,P)$ and b_j $(j=1,\ldots,Q)$ are complex parameters, $a_j \geq 0$ $(j=1,\ldots,P)$, $B_j \geq 0$ $(j=1,\ldots,Q)$ (not all zero simultaneously and the exponents $A_j(j=1,\ldots,N)$ and $B_j(j=M+1,\ldots,Q)$ can take on non-integer values.

The contour in (1.1) is imaginary axis $\Re(\xi)=0$. It is suitably indented in order to avoid the singularities of the gamma functions and to keep those singularities on appropriate sides. Again, for A_j $(j=1,\ldots,N)$ not an integer, the poles of the gamma functions of the numerator in (1.2) are converted to branch points. However, a long as there is no coincidence of poles from any $\Gamma(b_j-\beta_j\ \xi)$ $(j=1,\ldots,M)$ and $\Gamma(1-a_j+\alpha_j\ \xi)$ $(j=1,\ldots,N)$ pair, the branch cuts can be chosen so that the path of integration can be distorted in the useful manner. For the sake of brevity,

$$T = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}A_j\alpha_j - \sum_{j=M+1}^{Q}B_j\beta_j - \sum_{j=N+1}^{P}\alpha_j > 0\ .$$

The general class of polynomial introduced by Srivastava [7]:

$$S_n^m[x] = \sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}x^k, \quad n=0,1,2,... \tag{1.3}$$

2. Main results

We shall establish the following results:

$$\text{(A)} \int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\alpha}\left(\frac{1-y}{1-xy}\right)^{\beta}\frac{1-xy}{(1-x)(1-y)}S_n^m\left[\frac{1-x}{1-xy}vy\right]\ .\ \bar{H}^{\ M,N}_{\ P,Q}\left[\frac{1-y}{1-xy}v\right]dxdy$$
$$= \sum_{k=0}^{[n/m]}\frac{(-n)_{mk}A_{n,k}}{k!}v^k\Gamma(k+\alpha)$$
$$\times\ \bar{H}^{M,N+1}_{P+1,Q+1}\left[{(1-\beta:1),(a_j,\alpha_j;A_j)_{1,N}, \quad (a_j,\alpha_j)_{N+1,P} \atop (b_j,\beta_j)_{1,\mathrm{M}},(b_j,\beta_j:B_j)_{M+1,Q}, \quad (1-k-\alpha-\beta:1)}\Big|v\right], \tag{2.1}$$

provided that

$$\Re[\alpha+\beta+b_j/\beta_j]>0, |\arg v| < \frac{1}{2}T\pi,$$

m is an arbitrary positive integer and the coefficients $A_{n,k}$ $(n, k \geq 0)$ are arbitrary constants, real or complex.

Proof. We have

$$S_n^m \left[\frac{1-x}{1-xy} vy\right] \bar{H}_{P,Q}^{M,N} \left[\frac{1-y}{1-xy} v\right] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} \left(\frac{1-x}{1-xy} vy\right)^k \frac{1}{2\pi i}$$

$$\times \int_{-i\infty}^{i\infty} \frac{\prod_{j=1}^{M} \Gamma(b_j - \beta_j \xi)}{\prod_{j=M+1}^{Q} \{\Gamma(1 - b_j + \beta_j \xi)\}^{\beta_j}} \frac{\prod_{j=1}^{N} \{\Gamma(1 - a_j + \alpha_j \xi)\}^{A_j}}{\prod_{j=N+1}^{P} \Gamma(a_j - \alpha_j \xi)} \left(\frac{1-y}{1-xy} v\right)^{\xi} d\xi \tag{2.2}$$

Multiplying both sides of (2.2) by

$$\left[\frac{1-x}{1-xy} y\right]^{\alpha} \left[\frac{1-y}{1-xy}\right]^{\beta} \left[\frac{1-xy}{(1-x)(1-y)}\right]$$

and integrating with respect to x and y between 0 and 1 for both the variables and making a use of a known result [2, p.145], we get the result (2.1) after a little simplification.

(B) $\int_0^\infty \int_0^\infty \phi(u+v) v^{\beta-1} u^{\alpha-1} S_n^m[u] \bar{H}_{P,Q}^{M,N}[v] du dv$

$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k}\, \Gamma(k+\alpha) \int_0^\infty \phi(z) z^{\alpha+\beta+k-1}$$

$$\times\ \bar{H}\,{}_{P+1,Q+1}^{M,N+1} \left[{}^{(1-\beta:1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q};(1-k-\alpha-\beta:1)} \,|z\right] dz, \tag{2.3}$$

provided that $\Re(\alpha+\beta+b_j/\beta_j) > 0$, m is an arbitrary positive integer and coefficients $A_{n,k}(n, k \geq 0)$ are arbitrary constants, real or complex. $\square$

Proof. Using equations (1.1) and (1.3), we have

$$S_n^m[u] \bar{H}_{P,Q}^{M,N}[v] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} u^k \frac{1}{2\pi i}$$

$$\times \int_{-i\infty}^{i\infty} \frac{\prod_{j=1}^{M} \Gamma(b_j-\beta_j\xi)}{\prod_{j=M+1}^{Q} \{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}} \frac{\prod_{j=1}^{N} \{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=N+1}^{P} \Gamma(a_j-\alpha_j\xi)} v^{\xi} d\xi \ . \tag{2.4}$$

Multiplying both sides (2.4) by $\phi(u+v) v^{\beta-1} u^{\alpha-1}$ and integrating with respect to u and v between 0 and ∞ for both the variables and appealing to a known result [2, p.177], we easily arrive at the desired result.

Letting $\phi(z) = e^{-Pz}$ in (2.3) we get the particular case after simplification.

(C) $\int_0^1 \int_0^1 f(uv)(1-u)^{\alpha-1}(1-v)^{\beta-1} v^{\alpha} S_n^m[v(1-u)]\bar{H}_{P,Q}^{M,N}[1-v]dudv$

$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} \Gamma(k+2) \int_0^1 f(z)(1-z)^{\alpha+k-\beta-1}$$

$$\times \ \bar{H}\,{}^{M,N+1}_{P+1,Q+1} \left[{}^{(1-\beta:1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q};(1-k-\alpha-\beta:1)} \,|(1-z) \right] dz, \tag{2.5}$$

provided that $\Re(\alpha) > 0$, $\Re(\beta) > 0$, m is an arbitrary positive integer and coefficients $A_{n,k}(n, k \geq 0)$ are arbitrary constants, real or complex. □

Proof. Using equations (1.1) and (1.2), we have

$$S_n^m[v(1-u)]\bar{H}_{P,Q}^{M,N}[1-v] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} v^k (1-u)^k \frac{1}{2\pi i}$$

$$\times \int_{-i\infty}^{i\infty} \frac{\prod_{j=1}^{M} \Gamma(b_j-\beta_j\xi)}{\prod_{j=M+1}^{Q} \{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}} \frac{\prod_{j=1}^{N} \{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=N+1}^{P} \Gamma(a_j-\alpha_j\xi)} (1-v)^{\xi} d\xi \ . \tag{2.6}$$

Multiplying both sides of (2.6) by $f(uv)(1-u)^{\alpha-1}(1-v)^{\beta-1}v^{\alpha}$ and integrating with respect to u and v between 0 and 1 for both the variables and in view of the result [2, p.243] and by further simplification, this establishes the result (2.5).

Letting $f(z) = z^{\beta-1}$ in (2.5), we get the particular result after simplification.

(D) $\int_0^1 \int_0^1 \left[\frac{y(1-x)}{(1-xy)}\right]^{\alpha+\sigma} \left[\frac{1-y}{1-xy}\right]^{\beta} \frac{1}{(1-x)} \ S_n^m \left[\frac{y(1-x)}{1-xy}\right] \ .\bar{H}_{P,Q}^{M,N} \left[\frac{vy(1-x)}{1-xy}\right] dxdy$

$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} \ \Gamma(\beta+1)$$

$$\times \ \bar{H}\,{}^{M,N+1}_{P+1,Q+1} \left[{}^{(1-k-\alpha-\sigma:1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q};(-k-\alpha-\beta-\sigma\ :1)} \,|v \right], \tag{2.7}$$

provided that $\Re(\alpha+\beta+\sigma+b_j/\beta_j) > 0, |\arg v| < \frac{1}{2}T\pi$, m is an arbitrary integer and the coefficients $A_{n,k}$ $(n, k \geq 0)$ are arbitrary constants, real or complex. □

Proof. We have

$$S_n^m \left[\frac{y(1-x)}{1-xy}\right] \bar{H}_{P,Q}^{M,N} \left[\frac{vy(1-x)}{1-xy}\right] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} \left[\frac{y(1-x)}{1-xy}\right]^k \frac{1}{2\pi i}$$

$$\times \int_{-i\infty}^{i\infty} \frac{\prod_{j=1}^{M} \Gamma(b_j-\beta_j\xi) \prod_{j=1}^{N} \{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q} \{\Gamma(1-b_j+\beta_j\xi)\}^{B_j} \prod_{j=N+1}^{P} \Gamma(a_j-\alpha_j\xi)} \left[\frac{vy(1-x)}{1-xy}\right]^{\xi} d\xi \ . \tag{2.8}$$

Multiplying both sides of (2.8) by $\left[\frac{y(1-x)}{1-xy}\right]^{\alpha+\sigma} \left[\frac{1-y}{1-xy}\right]^{\beta} \frac{1}{(1-x)}$ and integrating with respect to x and y between 0 and 1 for both the variables, we easily arrive at the desired result (2.7). □

3. Special cases

(1) By applying our results given in (2.1), (2.3), (2.5) and (2.7) to the case of Hermite polynomial [8] and [9] and by setting

$$S_n^2(x) \to x^{n/2} H_n\left[\frac{1}{2\sqrt{x}}\right],$$

in which case $m = 2, A_{n,k} = (-1)^k$, we have the following interesting consequences of the main results.

$$\text{(A.1)} \int_0^1 \int_0^1 \left(\tfrac{1-x}{1-xy} y\right)\left(\tfrac{1-y}{1-xy}\right)^{\beta} \tfrac{1-xy}{(1-x)(1-y)} \left[\tfrac{1-x}{1-xy} vy\right]^{\frac{n}{2}}$$

$$\times\ H_n\left[\frac{1}{2\sqrt{\frac{1-x}{1-xy} vy}}\right] \bar{H}_{P,Q}^{M,N}\left[\frac{1-y}{1-xy} v\right] dxdy$$

$$= \sum_{k=0}^{[n/2]} \frac{(-n)_{2k}}{k!} (-1)^k v^k \ \Gamma(k+\alpha)$$

$$\times\ \bar{H}_{P+1,Q+1}^{M,N+1}\left[{}^{(1-\beta:1),(a_j;\alpha_j;A_j)_{1,N}, \quad (a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M},(b_j,\beta_j,B_j)_{M+1,Q}, \quad (1-k-\alpha-\beta:1)} |v\right],$$

valid under the same conditions as obtainable from (2.1).

$$\text{(A.2)} \int_0^\infty \int_0^\infty \phi(u+v) v^{\beta-1} u^{\alpha+\frac{n}{2}-1} H_n\left[\tfrac{1}{2\sqrt{u}}\right] \bar{H}_{P,Q}^{M,N}[v] dudv$$

$$= \sum_{k=0}^{[n/2]} \frac{(-n)_{2k}}{k!} (-1)^k \int_0^\infty \phi(z) z^{\alpha+b+k-1} \sqrt{k+\alpha}$$

$$\times\ \bar{H}_{P+1,Q+1}^{M,N+1}\left[{}^{(1-\beta:1),(a_j;\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q};(1-k-\alpha-\beta:1)} \mid z\right] dz,$$

valid under the same conditions as required for (2.3).

$$\text{(A.3)} \int_0^1 \int_0^1 f(uv)(1-u)^{\alpha-1}(1-v)^{\beta-1} v^{\alpha+\frac{n}{2}} (1-u)^{n/2}$$

$$H_n\left[\frac{1}{2\sqrt{v(1-u)}}\right] \bar{H}_{P,Q}^{M,N}[1-v] dudv$$

$$= \sum_{k=0}^{[n/2]} \frac{(-n)_{2k}}{k!} (-1)^k \Gamma(k+\alpha) \int_0^1 f(z)(1-z)^{\alpha+k-\beta-1}$$

$$\times\ \bar{H}_{P+1,Q+1}^{M,N+1}\left[{}^{(1-\beta:1),(a_j;\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1},(1-k-\alpha-\beta:1)} |(1-z)\right] dz,$$

valid under the same conditions as obtainable from (2.5).

$$\text{(A.4)} \int_0^1 \int_0^1 \left[\frac{y(1-x)}{(1-xy)}\right]^{\alpha+\sigma} \left[\frac{1-y}{1-xy}\right]^{\beta} \frac{1}{(1-x)^{1-\frac{n}{2}}} \frac{y^{n/2}}{(1-xy)^{n/2}}$$

$$\times\ H_n \left[\frac{1}{2\sqrt{\frac{y(1-x)}{1-xy}}}\right] \ \bar{H}^{M,N}_{P,Q}\left[\frac{vy(1-x)}{1-xy}\right] dxdy$$

$$= \sum_{k=0}^{[n/2]} \frac{(-n)_{2k}}{k!}(-1)^k\ \Gamma(\beta+1)$$

$$\times\ H\,^{M,N+1}_{P+1,Q+1}\left[{}^{(1-k-\alpha-\sigma:1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q};(-k-\alpha-\beta-\sigma\ :1)}\ |v\right],$$

valid under the same conditions as obtainable from (2.7).

(2) For the Laguerre polynomials ([8] and [9]) setting $S'_n(x) \to L_n^{(\alpha')}(x)$ in which case $m = 1$, $A_{n,k} = \left(\begin{array}{c} n+\alpha' \\ n \end{array}\right) \frac{1}{(\alpha'+1)_{k'}}$ the results (2.1), (2.3), (2.5) and (2.7) reduce to the following formulae:

$$\text{(B.1)} \int_0^1 \int_0^1 \left(\frac{1-x}{1-xy}y\right)^{\alpha} \left(\frac{1-y}{1-xy}\right)^{\beta} \left(\frac{1-xy}{(1-x)(1-y)}\right) L_n^{(\alpha')}\left(\frac{1-x}{1-xy}vy\right)$$

$$\times\ \bar{H}^{M,N}_{P,Q}\left[\frac{1-y}{1-xy}v\right] dxdy$$

$$= \sum_{k=0}^{n} \frac{(-n)_k}{k!} \left(\begin{array}{c} n+\alpha' \\ n \end{array}\right) \frac{1}{(\alpha'+1)_{k'}} v^k \Gamma(k+\alpha)$$

$$\times\ \bar{H}^{M,N+1}_{P+1,Q+1}\left[{}^{(1-\beta:1),(a_j;\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M},(b_j,\beta_j,B_j)_{M+1},(1-k-\alpha-\beta:1)}\ |v\right],$$

valid under the same conditions as required for (2.1).

$$\text{(B.2)} \int_0^\infty \int_0^\infty \phi(u+v) v^{\beta-1} u^{\alpha-1} L_n^{(\alpha')}(u) \bar{H}^{M,N}_{P,Q}[v] dudv$$

$$= \sum_{k=0}^{n} \frac{(-n)_k}{k!} \left(\begin{array}{c} n+\alpha' \\ n \end{array}\right) \frac{1}{(\alpha'+1)_{k'}} \int_0^\infty \phi(z) z^{\alpha+b+k-1}\ \Gamma(k+\alpha)$$

$$\times\ \bar{H}^{M,N+1}_{P+1,Q+1}\left[{}^{(1-\beta:1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q};(1-k-\alpha-\beta:1)}\ |\ z\right] dz\ ,$$

valid under the same conditions as required for (2.3).

$$\text{(B.3)} \int_0^1 \int_0^1 f(uv)(1-u)^{\alpha-1}(1-v)^{\beta-1} v^{\alpha} L_n^{(\alpha')}[v(1-u)]\ \bar{H}^{M,N}_{P,Q}[1-v] dudv$$

$$= \sum_{k=0}^{n} \frac{(-n)_k}{k!} \left(\begin{array}{c} n+\alpha' \\ n \end{array}\right) \frac{1}{(\alpha'+1)_{k'}} \Gamma(k+\alpha) \int_0^1 f(z)(1-z)^{\alpha+k-\beta-1}$$

$$\times\ \bar{H}^{M,N+1}_{P+1,Q+1}\left[{}^{(1-\beta:1),(a_j;\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M},(b_j,\beta_j;\ B_j)_{M+1,Q},(1-k-\alpha-\beta:1)}\ |(1-z)\right] dz\ ,$$

valid under the same conditions as required for (2.5).

$$\text{(B.4)} \int_0^1 \int_0^1 \left[\frac{y(1-x)}{(1-xy)}\right]^{\alpha+\sigma} \left[\frac{1-y}{1-xy}\right]^{\beta} \frac{1}{(1-x)} L_n^{\alpha'} \left[\frac{y(1-x)}{1-xy}\right] . \bar{H}_{P,Q}^{M,N} \left[\frac{vy(1-x)}{1-xy}\right] dxdy$$

$$= \sum_{k=0}^{n} \frac{(-n)_k}{k!} \begin{pmatrix} n+\alpha' \\ n \end{pmatrix} \frac{1}{(\alpha'+1)_{k'}} \; \Gamma(\beta+1)$$

$$\times \; \bar{H}_{P+1,Q+1}^{M,N+1} \left[{}^{(1-k-\alpha-\sigma:1),(a_j;\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q};(-k-\alpha-\beta-\sigma:1)} \, |(v) \right],$$

valid under the same conditions as obtainable from (2.7).

Acknowledgment. The authors are grateful to Professor H.M. Srivastava, University of Victoria, Canada for his kind help and valuable suggestions in the preparation of this paper.

References

[1] R. G. Buschman and H. M. Srivastava, The $\bar{H}$-function associated with a certain class of Feynman integrals, *J. Phys. A: Math. Gen.* **23** (1990), 4707–4710.
[2] J. Edwards, *A Treatise on Integral Calculus*, Chelsea Pub. Co., **2** (1922).
[3] C. Fox, The G and H-functions as symmetrical Fourier kernels, *Trans. Amer. Math. Soc.* **98** (1961), 395–429.
[4] C. Grosche and F. Steiner, *Hand Book of Feynman Path Integrals, Springer Tracts in Modern Physics Vol.145*, Springer-Verlag Berlin Heidelberg, New York, 1998.
[5] A. A. Inayat-Hussain, New properties of hypergeometric series derivable from Feynman integrals: I. Transformation and reduction formulae, *J. Phys. A: Math. Gen.* **20** (1987), 4109–4117.
[6] A. A. Inayat-Hussain, New properties of hypergeometric series derivable from Feynman integrals: II. A generalization of the H-function, *J. Phys. A: Math. Gen.* **20** (1987), 4119–4128.
[7] H. M. Srivastava, A contour integral involving Fox's H-function, *Indian J. Math.* **14** (1972), 1–6.
[8] H. M. Srivastava and N. P. Singh, The integration of certain products of the multivariable H-function with a general class of polynomials, *Rend. Circ. Mat. Palermo* **2**(32) (1983), 157–187.
[9] C. Szego, Orthogonal polynomials, *Amer. Math. Soc. Colloq. Publ.* **23** *Fourth edition, Amer. Math. Soc. Providence, Rhode Island* (1975).