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A Bitopological (1,2)* Semi-generalised Continuous Maps

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Abstract. We introduce a new type of generalized sets called $(1,2)^*$ semi-generalized closed sets and a new class of generalized functions called $(1,2)^*$ semi-generalized continuous maps. We obtain several characterizations of this class and study its bitopological properties and investigate the relationships with other new functions like $(1,2)^*$ g-continuous maps and $(1,2)^*$ gc-irresolute maps.

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1. Introduction

Levine [5] introduced the concept of Generalized closed sets in topological spaces. Also the notion of semi-open sets in topological spaces was initiated by the same Levine [4]. Bhattacharyya and Lahiri [1] introduced a class of sets called semi-generalized closed sets by means of semi-open sets of Levine [4] and obtained various topological properties corresponding to [5]. Sundaram *et al.* [12] introduced and studied the concept of a class of maps namely g-continuous maps which included the continuous maps and a class of gc-irresolute maps. In this paper, we generalize the concept of semi-generalised closed sets to $(1,2)^*$ semi-generalised closed sets and obtain various bitopological properties. The generalizations, in most of the cases, are substantiated by suitable examples.

2. Preliminaries

Throughout the present paper, (X,τ_1,τ_2), (Y,σ_1,σ_2) and (Z,U_1,U_2) (or simply X, Y, Z) denote bitopological spaces.

Definition 2.1. [10] *A subset S of X is called $\tau_1\tau_2$ open if $S \in \tau_1 \cup \tau_2$ and the complement of $\tau_1\tau_2$ open set is $\tau_1\tau_2$ closed.*

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Example 2.1. Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, X, \{a\}\}$ and $\tau_2 = \{\varphi, X, \{b\}\}$. The sets in $\{\varphi, X, \{a\}, \{b\}\}$ are called $\tau_1\tau_2$ open and the sets in $\{\varphi, X, \{b, c\}, \{a, c\}\}$ are called $\tau_1\tau_2$ closed.

Definition 2.2. [10] *Let S be a subset of X.*

i) The $\tau_1\tau_2$ closure of S, denoted by $\tau_1\tau_2 \operatorname{cl} S$, is defined by $\cap\{F/S \subset F$ and F is $\tau_1\tau_2$ closed $\}$.

ii) The $\tau_1\tau_2$ interior of S, denoted by $\tau_1\tau_2 \operatorname{int} S$, is defined by $\cup\{F/F \subset S$ and F is $\tau_1\tau_2$ open $\}$.

Definition 2.3. [10] *A subset S of X is said to be*

i) $(1, 2)^*$ α-open set if $S \subseteq \tau_1\tau_2 \operatorname{int}(\tau_1\tau_2 \operatorname{cl}(\tau_1\tau_2 \operatorname{int} S))$ and

ii) $(1, 2)^*$ semi-open set if $S \subseteq \tau_1\tau_2 \operatorname{cl}(\tau_1\tau_2 \operatorname{int} S)$. The complement of $(1, 2)^*$ semi-open $[(1, 2)^*\alpha$-open$]$ set is $(1, 2)^*$ semi-closed $[(1, 2)^*$ α-closed$]$.

Definition 2.4. [10] *A subset S of X is called pairwise α-open set in X if S is both $(1, 2)^*\alpha$-open set and $(2, 1)*$ α-open set. The family of all $(1, 2)^*$ semi-open $[(1, 2)^*$ semi-closed$]$ sets of X is denoted by $(1, 2)^*$ SO (X) $[(1, 2)^*SC$ $(X)]$. The intersection of all $(1, 2)^*$ semi-closed sets of X containing a subset S of X is called $(1, 2)^*$ semi-closure of S and is denoted by $(1, 2)^* \operatorname{scl}(S)$. Analogously, the $(1, 2)^*$ semi-interior of S, denoted by $(1, 2)^* \operatorname{sint}(S)$, is the union of all $(1, 2)^*$ semi-open sets contained in S.*

Remark 2.1. A subset S of X is $(1, 2)^*$ semi-closed if and only if $(1, 2)^* \operatorname{scl} S = S$.

Theorem 2.1. [11] *Let A be a subset of X. Then*

i) $(1, 2)^* \operatorname{scl}(A) = A \cup \tau_1\tau_2 \operatorname{int}(\tau_1\tau_2 \operatorname{cl} A)$ and

ii) $(1, 2)^* \operatorname{sint}(A) = A \cap \tau_1\tau_2 \operatorname{cl}(\tau_1\tau_2 \operatorname{int} A)$.

Definition 2.5. [9] *Let S be a subset of X. Then S is called $(1, 2)^*$ generalized closed (briefly $(1, 2)^*$ g-closed) set if and only if $\tau_1\tau_2 \operatorname{cl} S \subset F$ whenever $S \subset F$ and F is $\tau_1\tau_2$ open. The complement of $(1, 2)^*$ g-closed set is $(1, 2)^*$ g-open. Ravi and Thivagar [9] have proved that the intersection of two $(1, 2)^*$ g-closed sets is generally not a $(1, 2)^*$ g-closed set and a $\tau_1\tau_2$ closed set is always $(1, 2)^*$ g-closed set. Also, some properties of $(1, 2)^*$ g-closed sets were discussed.*

Remark 2.2. The union of two $(1, 2)^*$ g-open sets is generally not a $(1, 2)^*$ g-open set as seen from the following example.

Example 2.2. Let $X = \{a, b, c\}, \tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X\}$. So the sets in $\{\emptyset, X, \{a\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b, c\}\}$ are $\tau_1\tau_2$ closed. Clearly $\{c\}$ and $\{b\}$ are $(1, 2)^*$ g-open sets but $\{b, c\}$ is not $(1, 2)^*$ g-open.

Here, we introduce the new concept of $(1, 2)^*$ semi-generalized closed set.

Definition 2.6. *A subset S of X is said to be $(1, 2)^*$ semi-generalized closed (briefly $(1, 2)^*$ sg-closed) if and only if $(1, 2)^* \operatorname{scl}(S) \subset F$ whenever $S \subset F$ and F is $(1, 2)^*$ semi-open set. The complement of $(1, 2)^*$ semi-generalized closed set is $(1, 2)^*$ semi-generalized open.*

Example 2.3. A $(1,2)^*$ sg-closed set need not be $(1,2)^*$ g-closed set. Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$ and $\tau_2 = \{\emptyset, X\}$. So the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b,c\}, \{a,c\}, \{c\}\}$ are $\tau_1\tau_2$ closed. Clearly $\{a\}$ is $(1,2)^*$ sg-closed set but it is not $(1,2)^*$ g-closed since $\tau_1\tau_2\,\mathrm{cl}\{a\} = \{a,c\} \not\subset \{a\}$ whenever $\{a\} \subset \{a\} = F$ and F is $\tau_1\tau_2$ open.

Example 2.4. $(1,2)^*$ g-closed set need not be $(1,2)^*$ sg-closed. Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X\}$. So the sets in $\emptyset, X, \{a\}\}$ are $\tau_1\tau_2$ open and the sets in $\emptyset, X, \{b,c\}\}$ are $\tau_1\tau_2$ closed. Clearly $\{a,b\}$ is $(1,2)^*$ g-closed set but not $(1,2)^*$ sg-closed since $(1,2)^*\,\mathrm{scl}\{a,b\} = X \not\subset \{a,b\}$ whenever $\{a,b\} \subset \{a,b\}$ and $\{a,b\} \in (1,2)^*$ SO (X).

Remark 2.3. Examples 2.3 and 2.4 show that $(1,2)^*$ g-closed and $(1,2)^*$ sg-closed sets are, in general, independent.

Here we introduce a new class of maps as follows:

Definition 2.7. *A map $f : X \to Y$ is called*
i) $(1,2)^*$ sg-continuous if the inverse image of each $\sigma_1\sigma_2$ closed set in Y is $(1,2)^*$ sg-closed set in X.
ii) $(1,2)^*$ g-continuous if the inverse image of each $\sigma_1\sigma_2$ closed set in Y is $(1,2)^*$ g-closed set in X.
iii) $(1,2)^*$ gc-irresolute if the inverse image of each $(1,2)^*$ g-closed set in Y is $(1,2)^*$ g-closed in X.

Definition 2.8. [10, 11] *A map $f : X \to Y$ is called $(1,2)^*$ semi-continuous if the inverse image of each $\sigma_1\sigma_2$open set in Y is $(1,2)^*$ semi-open set in X.*

Remark 2.4. A map $f : X \to Y$ is $(1,2)^*$ semi-continuous if and only if the inverse image of each $\sigma_1\sigma_2$ closed set in Y is $(1,2)^*$ semi-closed set in X.

3. Characterizations

Lemma 3.1. *For any subset S of X, $(1,2)^*\,\mathrm{sint}[(1,2)^*\,\mathrm{scl}\,S - S] = \emptyset$.*

Proof. The proof is obvious. □

Proposition 3.1. *Every $\tau_1\tau_2$ open set is $(1,2)^*$ g-open set.*

Proof. Let S be an $\tau_1\tau_2$ open set in X. Then $X - S$ is $\tau_1\tau_2$ closed. Therefore $\tau_1\tau_2\,\mathrm{cl}(X - S) = (X - S) \subset X$ whenever $X - S \subset X$ and X is $\tau_1\tau_2$ open. It implies $X - S$ is $(1,2)^*$ g-closed. Thus, S is $(1,2)^*$ g-open set. □

Proposition 3.2. $(1,2)^*$ *g-open set need not be $\tau_1\tau_2$ open set. Refer Example: 2.10. Clearly $\{b\}$ is $(1,2)^*$ g-open set but it is not $\tau_1\tau_2$ open.*

Proposition 3.3. *For each $x \in X, \{x\} \in (1,2)^*SC(X)$ or $X - \{x\}$ is $(1,2)^*$ sg-closed in X.*

Proof. Suppose that $\{x\} \notin (1,2)^*SC(X)$. Since $X - \{x\}$ is not $(1,2)^*$ semi-open set, the space X itself is only $(1,2)^*$ semi-open set containing $X - \{x\}$. Therefore $(1,2)^*\,\mathrm{scl}[X - \{x\}] \subset X$ holds and so, $X - \{x\}$ is $(1,2)^*$ sg-closed. □

Theorem 3.1. *A sub set S of X is $(1,2)^*$ sg-closed if and only if $(1,2)^* \operatorname{scl}(S) - S$ contains no non-empty $(1,2)^*$ semi-closed set.*

Proof. Necessity: Let F be a $(1,2)^*$ semi-closed set such that $F \subset (1,2)^* \operatorname{scl}(S) - S$. Then

$$F \subset (1,2)^* \operatorname{scl}(S) \text{ and } F \not\subset S \Rightarrow F \subset X - S. \tag{3.1}$$

Since $X - F \in (1,2)^*$ SO (X) and $S \subset X - F$. By the definition of $(1,2)^*$ sg-closed set, it follows that

$$(1,2)^* \operatorname{scl}(S) \subset X - F \Rightarrow F \subset X - (1,2)^* \operatorname{scl}(S). \tag{3.2}$$

Thus, by (3.1) and (3.2), $F \subset [(1,2)^* \operatorname{scl}(S)] \cap [X - (1,2)^* \operatorname{scl}(S)] = \emptyset$.

Sufficiency: Let $S \subset G$ where $G \in (1,2)^*$ SO (X). If $(1,2)^* \operatorname{scl}(S) \not\subset G$, then $[(1,2)^* \operatorname{scl}(S)] \cap [X - G] \neq \emptyset$. As we have $[(1,2)^* \operatorname{scl}(S)] \cap [X - G] \subset (1,2)^* \operatorname{scl}(S) - S$ and $[(1,2)^* \operatorname{scl}(S)] \cap [X - G]$ is a non-empty $(1,2)^*$ semi-closed set, we obtain a contradiction. Hence the theorem. □

Corollary 3.1. *Let S be $(1,2)^*$ sg-closed set in X. Then S is $(1,2)^*$ semi-closed if and only if $(1,2)^* \operatorname{scl}(S) - S$ is $(1,2)^*$ semi-closed.*

Proof. Necessity: Let S be $(1,2)^*$ sg-closed set in X and $(1,2)^*$ semi-closed. Then $(1,2)^* \operatorname{scl}(S) - S = \emptyset$ which is $(1,2)^*$ semi-closed.

Sufficiency: Let $(1,2)^* \operatorname{scl}(S) - S$ be $(1,2)^*$ semi-closed and S be $(1,2)^*$ sg-closed set in X. Then $(1,2)^* \operatorname{scl}(S) - S$ does not contain any non empty $(1,2)^*$ semi-closed subset $\Rightarrow (1,2)^* \operatorname{scl}(S) - S = \emptyset$. Thus $(1,2)^* \operatorname{scl}(S) = S \Rightarrow S \in (1,2)^* SC(X)$. □

Theorem 3.2. *If A is $(1,2)^*$ sg-closed and $A \subset B \subset (1,2)^* \operatorname{scl} A$ then B is $(1,2)^*$ sg-closed set.*

Proof. Let $B \subset F$ where $F \in (1,2)^*$ SO (X). Since A is $(1,2)^*$ sg-closed and $A \subset F$, it follows that $(1,2)^* \operatorname{scl} A \subset F$. By hypothesis, $B \subset (1,2)^* \operatorname{scl} A$ and hence $(1,2)^* \operatorname{scl} B \subset (1,2)^* \operatorname{scl} A$. Consequently $(1,2)^* \operatorname{scl} B \subset F$ and B becomes $(1,2)^*$ sg-closed set. □

Theorem 3.3. *In $(X, \tau_1, \tau_2), (1,2)^*$ SO $(X) = (1,2)^*$ SC (X) if and only if every subset of X is $(1,2)^*$ sg-closed.*

Proof. Sufficiency: Let $A \subset F$ where $F \in (1,2)^*$ SO $(X) = (1,2)^*$ SC (X). Therefore $(1,2)^* \operatorname{scl}(A) \subset (1,2)^* \operatorname{scl}(F) = F$. Thus A is $(1,2)^*$ sg-closed set.

Necessity: Let $F \in (1,2)^*$ SO (X). Since every subset of X is $(1,2)^*$ sg-closed, F is $(1,2)^*$ sg-closed $\Rightarrow (1,2)^* \operatorname{scl}(F) \subset F \Rightarrow (1,2)^* \operatorname{scl}(F) = F$. Therefore $F \in (1,2)^*$ SC (X). Let $G \in (1,2)^*$ SC (X). Then $X - G \in (1,2)^*$ SO(X). Since $X - G$ is $(1,2)^*$ sg-closed, it may be seen as before that $X - G \in (1,2)^*$ SC $(X) \Rightarrow G \in (1,2)^*$ SO (X). This proves the theorem. □

Theorem 3.4. *A subset A of X is $(1,2)^*$ sg-open if and only if $F \subset (1,2)^* \operatorname{sint} A$ whenever $F \in (1,2)^*$ SC (X) and $F \subset A$.*

Proof. Necessity: Let A be $(1,2)^*$ sg-open set in X and suppose $F \subset A$ where $F \in (1,2)^*$ SC (X). Since $X - A$ is $(1,2)^*$ sg-closed set, $(1,2)^* \operatorname{scl}(X - A) \subset X - F$ whenever $X - A \subset X - F$ and $X - F \in (1,2)^*$ SO (X). Now $(1,2)^* \operatorname{scl}(X - A) =$

$X-(1,2)^*\operatorname{sint} A \subset X-F \Rightarrow F \subset (1,2)^*\operatorname{sint} A$.
Sufficiency: If $F \in (1,2)^*\mathrm{SC}\ (X)$ with $F \subset (1,2)^*\operatorname{sint} A$ whenever $F \subset A$, it follows that $X-A \subset X-F$ and $X-(1,2)^*\operatorname{sint} A \subset X-F \Rightarrow (1,2)^*\operatorname{scl}(X-A) \subset X-F \Rightarrow X-A$ is $(1,2)^*$ sg-closed $\Rightarrow A$ is $(1,2)^*$ sg-open. □

Theorem 3.5. *If $(1,2)^*\operatorname{sint} A \subset B \subset A$ and A is $(1,2)^*$ sg-open set then B is $(1,2)^*$ sg-open.*

Proof. By hypothesis, $X-A \subset X-B \subset X-(1,2)^*\operatorname{sint} A = (1,2)^*\operatorname{scl}(X-A)$ since $X-A$ is $(1,2)^*$ sg-closed set, By Theorem 3.2, $X-B$ is $(1,2)^*$ sg-closed $\Rightarrow B$ is $(1,2)^*$ sg-open. □

Theorem 3.6. *A subset A of X is $(1,2)^*$ sg-closed if and only if $(1,2)^*\operatorname{scl}(A)-A$ is $(1,2)^*$ sg-open set.*

Proof. Necessity: If A is $(1,2)^*$ sg-closed and F is a $(1,2)^*$ semi-closed set such that $F \subset (1,2)^*\operatorname{scl} A - A$ then by Theorem 3.1, $F = \{\emptyset\}$. Hence

$$F \subset (1,2)^*\operatorname{sint}[(1,2)^*\operatorname{scl}(A)-A]$$

by Lemma. 3.1 and by Theorem 3.4, $(1,2)^*\operatorname{scl} A - A$ is $(1,2)^*$ sg-open.

Sufficiency: Suppose $(1,2)^*\operatorname{scl}(A)-A$ is $(1,2)^*$ sg-open set. Let $A \subset F$ where $F \in (1,2)^*$ SO (X). Then $X-F \subset X-A$ that is $(1,2)^*\operatorname{scl}(A)\cap(X-F) \subset (1,2)^*\operatorname{scl}(A)\cap(X-A)$. Thus $(1,2)^*\operatorname{scl}(A)\cap(X-F)$ is a $(1,2)^*$ semi-closed subset of $(1,2)^*\operatorname{scl}(A)\cap(X-A) = (1,2)^*\operatorname{scl}(A)-A$. Therefore by Theorem 3.4 $(1,2)^*\operatorname{scl}(A)\cap(X-F) \subset (1,2)^*\operatorname{sint}[(1,2)^*\operatorname{scl}(A)-A] = \emptyset$ by Lemma 3.1. Hence $(1,2)^*\operatorname{scl}(A) \subset F \Rightarrow A$ is $(1,2)^*$ sg-closed set. □

Lemma 3.2. *Let A be $(1,2)^*$ semi-open set in X and suppose $A \subset B \subset \tau_1\tau_2 clA$. Then B is $(1,2)^*$ semi-open set in X.*

Proof. Since A is $(1,2)^*$ semi- open set in X, $A \subset \tau_1\tau_2\operatorname{cl}(\tau_1\tau_2\operatorname{int} A)$ and since $A \subset B, \tau_1\tau_2\operatorname{cl}(\tau_1\tau_2\operatorname{int} A) \subset \tau_1\tau_2\operatorname{cl}(\tau_1\tau_2\operatorname{int} B)$. Therefore

$$A \subset \tau_1\tau_2\operatorname{cl}(\tau_1\tau_2\operatorname{int} B) \Rightarrow \tau_1\tau_2\operatorname{cl} A \subset \tau_1\tau_2\operatorname{cl}(\tau_1\tau_2\operatorname{int} B)$$

and since

$$B \subset \tau_1\tau_2\operatorname{cl} A, B \subset \tau_1\tau_2\operatorname{cl}(\tau_1\tau_2\operatorname{int} B).$$

Thus B is $(1,2)^*$semi- open set in X. □

Theorem 3.7. *i) If a map $f: X \to Y$ is $(1,2)^*$ open and $(1,2)^*$ semi-continuous then $f^{-1}(V) \in (1,2)^*$ SO (X) for every $V \in (1,2)^*SO\ (Y)$.*
ii) If a map $f: X \to Y$ is $(1,2)^*$ open and $(1,2)^*$ semi-continuous then $f^{-1}(V) \in (1,2)^*$ SC (X) for every $V \in (1,2)^*$ SC (Y).

Proof. i) For an arbitrary $B \in (1,2)^*$ SO (Y), there exists an $\sigma_1\sigma_2$ open set V in Y such that $V \subset B \subset \sigma_1\sigma_2\operatorname{cl} V$. Since f is $(1,2)^*$ open map, we have $f^{-1}(V) \subset f^{-1}(B) \subset f^{-1}(\sigma_1\sigma_2\operatorname{cl} V) \subset \tau_1\tau_2\operatorname{cl} f^{-1}(V)$. Since f is $(1,2)^*$ semi-continuous and V is $\sigma_1\sigma_2$ open set in Y, $f^{-1}(V) \in (1,2)^*$ SO (X). By Lemma 3.2, we obtain $f^{-1}(B) \in (1,2)^*$ SO (X).
ii) For an arbitrary $B \in (1,2)^*$ SC $(Y), Y-B \in (1,2)^*$ SO (Y). By i) $f^{-1}(Y-B) \in (1,2)^*$ SO $(X) \Rightarrow X - f^{-1}(B) \in (1,2)^*$ SO $(X) \Rightarrow f^{-1}(B) \in (1,2)^*$SC (X). □

Theorem 3.8. *For any $(1,2)^*$ gc-irresolute map $f : X \to Y$ and any $(1,2)^*$ g-continuous map $g : Y \to Z$, the composition $g \circ f : X \to Z$ is $(1,2)^*$ g-continuous map.*

Proof. Let V be any U_1U_2 closed set in Z. Since $g : Y \to Z$ is $(1,2)^*$ g-continuous map, $g^{-1}(V)$ is $(1,2)^*$ g-closed in Y. Since $f : X \to Y$ is $(1,2)^*$ gc-irresolute map, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is $(1,2)^*$ g-closed in X. Thus $g \circ f : X \to Z$ is $(1,2)^*$ g-continuous map. □

Theorem 3.9. *If $f : X \to Y$ is bijective, $(1,2)^*$ open and $(1,2)^*$ g-continuous map then f is $(1,2)^*$ gc-irresolute.*

Proof. Let A be a $(1,2)^*$ g-closed set in Y. Let $f^{-1}(A) \subset F$ where F is an $\tau_1\tau_2$ open set in X. Therefore $A \subset f(F)$ holds. Since $f(F)$ is $\sigma_1\sigma_2$ open and A is $(1,2)^*$ g-closed in Y, $\sigma_1\sigma_2 \operatorname{cl} A \subset f(F)$ holds and hence $f^{-1}(\sigma_1\sigma_2 clA) \subset F$. Since f is $(1,2)^*$ g-continuous map and $\sigma_1\sigma_2 \operatorname{cl} A$ is $\sigma_1\sigma_2$ closed set in Y, $f^{-1}(\sigma_1\sigma_2 \operatorname{cl} A)$ is $(1,2)^*$ g-closed in X. Then $\tau_1\tau_2 \operatorname{cl}(f^{-1}(\sigma_1\sigma_2 \operatorname{cl} A)) \subset F$ and so, $\tau_1\tau_2 \operatorname{cl}(f^{-1}(A)) \subset F \Rightarrow f^{-1}(A)$ is $(1,2)^*$ g-closed in X. Thus, f is $(1,2)^*$ gc-irresolute map. □

Remark 3.1. The following three examples show that no assumption of the Theorem 3.9 can be removed.

Example 3.1. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. So the sets in $\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b, c\}, \{a, b\}, \{b\}\}$ are $\tau_1\tau_2$ closed. Let $\sigma_1 = \{\emptyset, Y, \{a\}, \{a, b\}\}$ and $\sigma_2 = \{\emptyset, Y, \{a\}\}$. So the sets in $\{\emptyset, Y, \{a\}, \{a, b\}\}$ are $\sigma_1\sigma_2$ open and the sets in $\{\emptyset, Y, \{b, c\}, \{c\}\}$ are $\sigma_1\sigma_2$ closed. Let $f : X \to Y$ be defined by $f(a) = f(c) = a; f(b) = b$. Clearly f is $(1,2)^*$ g-continuous and $(1,2)^*$ open map. But f is neither bijective nor $(1,2)^*$ gc-irresolute map.

Example 3.2. Let $X = Y = \{a, b, c\}, \tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. So the sets in $\{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b, c\}, \{a, b\}, \{b\}\}$ are $\tau_1\tau_2$ closed. Let $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y\}$. So the sets in $\{\emptyset, Y, \{a\}\}$ are $\sigma_1\sigma_2$ open and the sets in $\emptyset, Y, \{b, c\}\}$ are $\sigma_1\sigma_2$ closed. Let $f : X \to Y$ be the identity map. Clearly f is $(1,2)^*$ g-continuous and bijective. But f is neither $(1,2)^*$ open nor $(1,2)^*$ gc-irresolute map.

Example 3.3. Let $X = Y = \{a, b, c\}, \tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X\}$. So the sets in $\{\emptyset, X, \{a\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b, c\}\}$ are $\tau_1\tau_2$ closed. Let $\sigma_1 = \{\emptyset, Y, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and $\sigma_2 = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$. So the sets in $\{\emptyset, Y, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ are both $\sigma_1\sigma_2$ open and $\sigma_1\sigma_2$ closed. Let $f : X \to Y$ be the identity map. Clearly f is bijective and $(1,2)^*$ open map. But f is neither $(1,2)^*$ g-continuous nor $(1,2)^*$ gc-irresolute map.

Remark 3.2. If $f : X \to Y$ and $g : Y \to Z$ are both $(1,2)^*$ sg-continuous maps then the composition $g \circ f : X \to Z$ need not be $(1,2)^*$ sg-continuous as per the following.

Example 3.4. Let $X = Y = Z = \{a, b, c\}$. Let $\tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. So the sets in $\{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b, c\}, \{a, b\}, \{b\}\}$ are $\tau_1\tau_2$ closed. Let $\sigma_1 = \{\emptyset, Y, \{a, b\}\}$ and $\sigma_2 = \{\emptyset, Y\}$. So the sets in $\{\emptyset, Y, \{a,b\}\}$ are $\sigma_1\sigma_2$ open and the sets in $\{\emptyset, Y, \{c\}\}$ are $\sigma_1\sigma_2$ closed. Let $U_1 = \{\emptyset, Z, \{a\}\}$ and $U_2 = \{\emptyset, Z, \{b\}\}$. So the sets in $\{\emptyset, Z, \{a\}, \{b\}\}$ are U_1U_2 open and the sets in $\{\emptyset, Z, \{b,c\}, \{a,c\}\}$ are U_1U_2 closed. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity map and $g : Y \to Z$ be the identity map. Then f is $(1,2)^*$ sg-continuous map and g is $(1,2)^*$ sg-continuous map but $g \circ f$ is not $(1,2)^*$ sg-continuous map since $(g \circ f)^{-1}(\{a,c\}) = \{a,c\}$ is not $(1,2)^*$ sg-closed set in (X, τ_1, τ_2).

4. Comparisons

Remark 4.1. From the subsets defined above, we have the following diagram of implications:

$$\begin{array}{ccc} \tau_1\tau_2 \text{ closed} & \overset{\not\leftarrow}{\rightarrow} & (1,2)^* \ g\text{-closed} \\ \downarrow\not\uparrow & & \not\downarrow \ \not\uparrow \\ (1,2)^*\text{semi-closed} & \overset{\not\leftarrow}{\rightarrow} & (1,2)^* \ sg\text{-closed} \end{array}$$

where $A \nrightarrow B$ means A does not necessarily imply B.

Proposition 4.1. *Every $\tau_1\tau_2$ closed set is $(1,2)^*$ semi-closed.*

Proof. Let A be $\tau_1\tau_2$ closed set in X. Then $X - A$ is $\tau_1\tau_2$ open in X. Since every $\tau_1\tau_2$ open is $(1,2)^*$ semi-open, $X - A \in (1,2)^*$ SO (X). Thus $A \in (1,2)^*$ SC (X). □

Example 4.1. $(1,2)^*$ semi-closed set need not be $\tau_1\tau_2$ closed. Refer Example 2.2. Clearly $\{b\}$ is $(1,2)^*$ semi-closed set but not $\tau_1\tau_2$ closed.

Proposition 4.2. *Every $(1,2)^*$ semi-closed set is $(1,2)^*$ sg-closed.*

Proof. Since A is $(1,2)^*$ semi-closed, $(1,2)^* \operatorname{scl} A = A \subset X$ whenever $A \subset X$ and $X \in (1,2)^*$ SO (X). It implies that A is $(1,2)^*$ sg-closed set. □

Example 4.2. A $(1,2)^*$ sg-closed set need not be $(1,2)^*$ semi-closed. Let $X = \{a,b,c\}, \tau_1 = \{\emptyset, X, \{a,b\}\}$ and $\tau_2 = \{\emptyset, X\}$. So the sets in $\{\emptyset, X, \{a,b\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{c\}\}$ are $\tau_1\tau_2$ closed. Clearly $A = \{a,c\}$ is $(1,2)^*$ sg-closed set but it is not $(1,2)^*$ semi-closed.

Remark 4.2. The following example shows that the union of two $(1,2)^*$ sg-closed sets is not, in general, $(1,2)^*$ sg-closed.

Example 4.3. Refer Example 2.3. Clearly $\{a\}$ and $\{b\}$ are $(1,2)^*$ sg-closed sets. But $\{a,b\}$ is not $(1,2)^*$ sg-closed since $(1,2)^* \operatorname{scl}(\{a,b\}) = X \not\subset \{a,b\}$ whenever $\{a,b\} \subset \{a,b\}$ and $\{a,b\} \in (1,2)^*SO(X)$.

Remark 4.3. The intersection of two $(1,2)^*$ sg-closed sets is $(1,2)^*$ sg-closed.

Remark 4.4. From the maps we stated above, we have the following diagram of implications. Where $A \longrightarrow B$ does not necessarily imply B.

$$(5) \overset{\not\leftarrow}{\rightarrow} (4) \overset{\leftarrow}{\nrightarrow} (1) \overset{\not\leftarrow}{\rightarrow} (2) \overset{\not\leftarrow}{\rightarrow} (3)$$

where $(1) = (1,2)^*$ continuity, $(2) = (1,2)^*$ semi- continuity, $(3) = (1,2)^*$ sg-continuity, $(4) = (1,2)^*$ g-continuity and $(5) = (1,2)^*$ gc-irresolute.

Proposition 4.3. *Every* $(1,2)^*$ *semi-continuous map is* $(1,2)^*$ *sg-continuous.*

Proof. Let V be any $\sigma_1\sigma_2$ closed set in Y. Since $f : X \to Y$ is $(1,2)^*$ semi-continuous, $f^{-1}(V)$ is $(1,2)^*$ semi-closed set in X. By Proposition 4.2, $f^{-1}(V)$ is $(1,2)^*$ sg-closed set in X. Thus, f is $(1,2)^*$ sg-continuous map. □

Example 4.4. The converse of Proposition 4.3 is false. Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X\}$. So the sets in $\{\emptyset, X, \{a, b\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{c\}\}$ are $\tau_1\tau_2$ closed. Let $Y = \{a, b, c\}, \sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y, \{a, b\}\}$. So the sets in $\{\emptyset, Y, \{a\}, a, b\}$ are $\sigma_1\sigma_2$ open and the sets in $\{\emptyset, Y, \{c\}, b, c\}$ are $\sigma_1\sigma_2$ closed. Let $f : X \to Y$ be the identity map. Clearly f is $(1,2)^*$ sg-continuous map but not $(1,2)^*$ semi-continuous since $f^{-1}(\{a\}) = \{a\} \notin (1,2)^*SO(X)$.

Proposition 4.4. *Every* $(1,2)^*$ *continuous map is* $(1,2)^*$ *semi-continuous.*

Proof. It is obvious. □

Example 4.5. The converse of Proposition 4.4 is false. Let$X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X\}$. So the sets in $\{\emptyset, X, \{a\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b, c\}\}$ are $\tau_1\tau_2$ closed. Let $\sigma_1 = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\sigma_2 = \{\emptyset, X, \{a\}\}$. So the sets in $\{\emptyset, X, \{a\}, \{a, b\}\}$ are $\sigma_1\sigma_2$ open and the sets in $\{\emptyset, X, \{b, c\}, \{c\}\}$ are $\sigma_1\sigma_2$ closed. Let $F : X \to Y$ be the identity map. Clearly f is $(1,2)^*$ semi-continuous map but not $(1,2)^*$ continuous since $f^{-1}(\{a, b\}) = \{a, b\}$ is not $\tau_1\tau_2$ open.

Example 4.6. The composition map of two $(1,2)^*$ semi-continuous maps is not always $(1,2)^*$ semi-continuous. Let $X = Y = Z = \{a, b, c\}$. Let $\tau_1 = \{\emptyset, X, \{b\}, \{b, c\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. So the sets in $\{\emptyset, X, \{a\}, \{b\}, \{b, c\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b, c\}, \{a, c\}, \{a\}\}$ are $\tau_1\tau_2$ closed. Let $\sigma_1 = \{\emptyset, Y, \{a\}, \{a, b\}\}$ and $\sigma_2 = \{\emptyset, Y, \{a\}\}$. So the sets in $\{\emptyset, Y, \{a\}, \{a, b\}\}$ are $\sigma_1\sigma_2$ open and the sets in $\{\emptyset, Y, \{b, c\}, \{c\}\}$ are $\sigma_1\sigma_2$ closed. Let $U_1 = \{\emptyset, Z, \{a, b\}\}$ and $U_2 = \{\emptyset, Z, \{b, c\}\}$. So the sets in $\{\emptyset, Z, \{a, b\}, \{b, c\}\}$ are U_1U_2 open and the sets in $\{\emptyset, Z, \{c\}, \{a\}\}$ are U_1U_2 closed. Let $F : X \to Y$ be the identity map and define $g : Y \to Z$ as $g(a) = b, g(b) = a$ and $g(c) = c$. Clearly, f is $(1,2)^*$ semi-continuous map and g is $(1,2)^*$ semi-continuous map but $g \circ f$ is not $(1,2)^*$ semi-continuous map since $f^{-1}(g^{-1}\{b, c\}) = f^{-1}(\{a, c\}) = \{a, c\}$ is not $(1,2)^*$ semi-open set in X.

However, we obtain the following Remark as an immediate consequence of the example.

Remark 4.5. If $f : X \to Y$ is an $(1,2)^*$ open and $(1,2)^*$ semi-continuous map and $g : Y \to Z$ is a $(1,2)^*$ semi-continuous map, then $g \circ f : X \to Z$ is $(1,2)^*$ semi-continuous.

Proposition 4.5. *Every* $(1,2)^*$ *continuous map is* $(1,2)^*$ *g-continuous.*

Proof. It is proved from definitions. □

Example 4.7. The converse of Proposition 4.5 is false. Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X\}$. So the sets in $\{\emptyset, X, \{a\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b, c\}\}$ are $\tau_1\tau_2$ closed. Let $Y = \{p, q\}, \sigma_1 = \{\emptyset, Y, \{p\}\}$ and $\sigma_2 = \{\emptyset, Y\}$.

So the sets in $\{\emptyset, Y, \{p\}\}$ are $\sigma_1\sigma_2$ open and the sets in $\{\emptyset, Y, \{q\}\}$ are $\sigma_1\sigma_2$ closed. Define $f : X \to Y$ as follows $f(a) = f(c) = q, f(b) = p$. Clearly f is $(1,2)^*$ g-continuous map but not $(1,2)^*$ continuous since $f^{-1}(\{q\}) = \{a, c\}$ is not $\tau_1\tau_2$ closed.

Remark 4.6. The composition of two $(1,2)^*$ g-continuous maps is not, in general, $(1,2)^*$ g-continuous map as is illustrated in the following example.

Example 4.8. Let $X = Y = Z = \{a, b, c\}, \tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X\}$. So the sets in $\{\emptyset, X, \{a, b\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{c\}\}$ are $\tau_1\tau_2$ closed. Let $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y\}$. So the sets in $\{\emptyset, Y, \{a\}\}$ are $\sigma_1\sigma_2$ open and the sets in $\{\emptyset, Y, \{b, c\}\}$ are $\sigma_1\sigma_2$ closed. Let $U_1 = \{\emptyset, Z, \{a, c\}\}$ and $U_2 = \{\emptyset, Z\}$. So U_1U_2 open $= U_1$ and U_1U_2 closed $= \{\emptyset, Z, \{b\}\}$. Let $f : X \to Y$ be the identity map. Let $g : Y \to Z$ be the identity map. Clearly f is $(1,2)^*$ g-continuous map and g is $(1,2)^*$ g-continuous map but $g \circ f$ is not $(1,2)^*$ g-continuous mapping. Since $f^{-1}(g^{-1}\{b\}) = f^{-1}(\{b\}) = \{b\}$ is not $(1,2)^*$ g-closed $[\tau_1\tau_2 cl(\{b\}) = X \not\subset \{a, b\}$ whenever $\{b\} \subset \{a, b\}$ and $\{a, b\}$ is $\tau_1\tau_2$ open.]

Remark 4.7. A map $f : X \to Y$ is $(1,2)^*$ gc-irresolute if and only if the inverse image of every $(1,2)^*$ g-open in Y is $(1,2)^*$ g-open in X.

Proposition 4.6. *Every $(1,2)^*$ gc-irresolute map is $(1,2)^*$ g-continuous.*

Proof. Since every $\tau_1\tau_2$ closed set is $(1,2)^*$ g-closed, it is easily proved by straightforward. □

Example 4.9. The converse of Proposition 4.6 is false. Let $X = Y = \{a, b, c\}, \tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. So the sets in $\{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ are $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b, c\}, \{a, b\}, \{b\}\}$ are $\tau_1\tau_2$ closed. Let $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y\}$. So the sets in $\{\emptyset, Y, \{a\}\}$ are $\sigma_1\sigma_2$ open and the sets in $\{\emptyset, Y, \{b, c\}\}$ are $\sigma_1\sigma_2$ closed. Define $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ as follows $f(a) = f(c) = a$ and $f(b) = b$. Clearly f is $(1,2)^*$ g-continuous map but not $(1,2)^*$ gc-irresolute since the inverse image of $(1,2)^*$ g-closed set $\{a, c\}$ in Y is not $(1,2)^*$ g-closed in X.

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