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Numerical Solution of Mass Transfer Effects on Unsteady Flow Past an Accelerated Vertical Porous Plate with Suction

[1]S.S. DAS, [2]S.K. SAHOO AND [3]G.C. DASH
[1]Department of Physics, KBDAV College Nirakarpur, Khurda – 752 019, Orissa, India
[2]Department of Mathematics, Nayagarh College Nayagarh – 752 069, Orissa, India
[3]Department of Mathematics, University Engineering College Burla – 768 018, Orissa, India

Abstract. The unsteady free convection and mass transfer boundary layer flow past an accelerated infinite vertical porous flat plate with suction is considered when the plate accelerates in its own plane. The governing equations are solved both analytically and numerically using finite difference scheme. The flow phenomenon has been characterized with the help of flow parameters such as suction parameter (a), porosity parameter (α), Grashof number (G_r, G_c), Schmidt number (S_c) and Prandtl number (P_r). The effects of these parameters on the velocity field, temperature field and concentration distribution have been studied and the results are presented graphically and discussed quantitatively. This type of problem is significantly relevant to geophysical and astrophysical studies.

2000 Mathematics Subject Classification: 76S05

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1. Introduction

The study of convective flow with mass transfer along a vertical porous plate is receiving considerable attention of many researchers because of its varied applications in the field of cosmical and geophysical sciences. Permeable porous plates are used in the filtration processes and also for a heated body to keep its temperature constant and to make the heat insulation of the surface more effective. The study of stellar structure on the solar surface is connected with mass transfer phenomena. Its origin is attributed to difference in temperature caused by the non-homogeneous production of heat, which in many cases can rest not only in the formation of convective currents but also in violent explosions. Mass transfer certainly occurs within the mantle and cores of planets of the size of or larger than the earth. It is therefore interesting to investigate this phenomenon and to study in particular, the case of mass transfer on the free convection flow.

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Several workers have studied the problem of free convection flow with mass transfer. Acharya *et al.* [1] have investigated the effect of chemical and thermal diffusion with Hall current on unsteady hydromagnetic flow near an infinite vertical porous plate. Chandran *et al.* [2] have discussed the unsteady free convection flow with heat flux and accelerated motion. Dash and Das [3] have analyzed the effect of Hall current on MHD free convection flow along an accelerated porous flat plate with mass transfer and internal heat generation. Hasimoto [4] has discussed the boundary layer growth on a flat plate with suction or injection. Israel-Cookey *et al.* [5] have studied the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Jha [6] has reported the effects of applied magnetic field on transient convective flow in a vertical channel. Kim [7] has investigated the problem of unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Mishra and Dash [8] have studied the free convection of non-Newtonian fluids between parallel walls. Panda *et al.* [9] have analyzed the unsteady free convective flow and mass transfer of a rotating elastico-viscous liquid through porous media past a vertical porous plate.

Pop and Soundalgekar [10] have investigated the free convection flow past an accelerated infinite plate. Raptis *et al.* [11] have studied the unsteady free convective flow through a porous medium adjacent to a semi-infinite vertical plate using finite difference scheme. Sattar [12] has discussed the free convection and mass transfer flow through a porous medium past an infinite vertical porous plate with time dependent temperature and concentration. Singh [13] has analyzed the MHD free convective flow past an accelerated vertical porous plate by finite difference method. Singh and Dikshit [14] have studied the hydromagnetic flow past a continuously moving semi-infinite plate at large suction. Singh and Soundalgekar [15] have investigated the problem of transient free convection in cold water past an infinite vertical porous plate. Soundalegekar [16] has discussed the free convection effects on steady MHD flow past a vertical porous plate. Soundalgekar *et al.* [17] have analyzed the transient free convection flow of a viscous dissipative fluid past a semi-infinite vertical plate. Vajravelu [18] has studied the natural convection at a heated semi-infinite vertical plate with internal heat generation.

The proposed study considers the unsteady free convection and mass transfer boundary layer flow past an accelerated infinite vertical porous flat plate with suction when the plate accelerates in its own plane. The governing equations are solved both analytically and numerically using finite difference scheme. The effects of the flow parameters on the velocity, temperature and the concentration distribution of the flow field have been studied with the help of graphs and tables. This type of problem has some significant relevance to geophysical and astrophysical studies.

2. Formulation of the problem

Consider the unsteady flow of an incompressible viscous fluid past an accelerating vertical porous plate. Let the x-axis be directed upward along the plate and the y-axis normal to the plate. Let u and v be the velocity components along the x- and y- axes respectively. Let us assume that the plate is accelerating with a velocity $u = Ut$ in its own plane at time $t \geq 0$. Then the unsteady boundary layer equations in the Boussinesq's approximation, together with Brinkman's empirical modification of Darcy's law, are

$$\frac{\partial v}{\partial y} = 0\ , \tag{2.1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k^*}u + g\beta\left(T - T_\infty\right) + g\beta^*\left(C - C_\infty\right)\ , \tag{2.2}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2}\ , \tag{2.3}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}\ , \tag{2.4}$$

where k is the thermal diffusivity, v is the kinematic viscosity, k^* is the permeability coefficient, β is the volumetric expansion coefficient for heat transfer, β^* is the volumetric expansion coefficient for mass transfer, ρ is the density, g is the acceleration due to gravity, T is the temperature, T_∞ is the temperature of the fluid far away from the plate, C is the concentration, C_∞ is the concentration far away from the plate and D is the molecular diffusivity.

The necessary boundary conditions are

$$\begin{aligned} &u = Ut, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0,\\ &u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty \quad \text{for} \quad t \to 0. \end{aligned} \tag{2.5}$$

We introduce the similarity variables and dimensionless quantities

$$\begin{aligned} &\eta = \frac{y}{2\sqrt{\nu t}} \quad u = Utf\left(\eta\right) \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \quad C = \frac{C - C_\infty}{C_w - C_\infty}, \quad \alpha = 4\nu\frac{t}{k^*}\\ &P_r = \frac{\nu}{k} \quad S_c = \frac{\nu}{D} \quad G_r = 4g\beta\frac{(T_w - T_\infty)}{U} \text{ and } G_c = 4g\beta^*\frac{(C_w - C_\infty)}{U} \end{aligned} \tag{2.6}$$

where

P_r = Prandtl number,
G_c = Grashof number for mass transfer,
G_r = Grashof number for heat transfer,
S_c = Schmidt number,
α = Porosity parameter

and the primes denote the derivative with respect to η.

Following Hasimoto [4], Singh and Soundalgekar [15], we choose

$$v = -a\left(\frac{\nu}{t}\right)^{\frac{1}{2}} \tag{2.7}$$

where $a > 0$, the suction parameter. Using equations (2.6) and (2.7), equations (2.2), (2.3) and (2.4) become

$$f'' + 2(\eta + a)f' - \alpha f = -G_r\theta - G_cC \tag{2.8}$$

$$\theta'' + 2(\eta + a)P_r\theta' = 0 \tag{2.9}$$

$$C'' + 2(\eta + a)S_cC' = 0 \tag{2.10}$$

with the boundary conditions

$$f(0) = 1,\ \theta(0) = 1,\ C(0) = 1,\ f(\infty) = 0,\ \theta(\infty) = 0,\ C(\infty) = 0. \tag{2.11}$$

Skin friction. The skin friction is given by

$$\tau = \rho\nu \left(\frac{\partial u}{\partial y}\right) = -\frac{\rho U}{2}\sqrt{\nu t}\ f'(0)$$

In non-dimensional form, we get

$$\tau' = \frac{2\tau}{\rho U \sqrt{\nu t}} = -f'(0).$$

Heat transfer. The non-dimensional local heat flux in terms of Nusselt number (N_u) at the plate is given by

$$N_u = \frac{2q_w\sqrt{\nu t}}{k\left(T_w - T_\infty\right)} = -\theta'(0)$$

where q_w is the heat flux per unit area.

3. Method of solution

Solving equations (2.9) and (2.10) exactly by error function subject to boundary conditions (2.11),

$$\theta = \frac{\operatorname{erfc}\left((\eta + a)\sqrt{P_r}\right)}{\operatorname{erfc}\left(a\sqrt{P_r}\right)}$$

$$C = \frac{\operatorname{erfc}\left((\eta + a)\sqrt{S_c}\right)}{\operatorname{erfc}\left(a\sqrt{S_c}\right)}.$$

In order to solve equation (2.8), we set up the following difference approximations

$$f' = \frac{f_{i+1} - f_{i-1}}{2h}, \quad f'' = \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2}.$$

Introducing these difference approximations in equation (2.8), we obtain

$$A_0 f_{i+1} + A_1 f_i + A_2 f_{i-1} = B_1 + B_2$$

where

$$A_0 = \frac{1 + h\left(\eta_i + a\right)}{h^2}, \quad A_1 = \frac{2 + h^2\alpha}{h^2}, \quad A_2 = \frac{1 - h\left(\eta_i + a\right)}{h^2},$$

$$B_1 = -G_r\theta_i, \quad B_2 = -G_c C_i, \quad \eta_i = ih \quad h = \frac{L}{N+1} \quad 0 \le \eta_i \le L$$

we take $L = 2.0$ and $N = 200$, since it lies well outside the boundary layer.

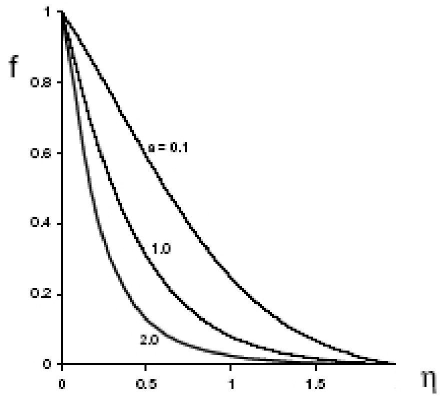

Figure 1. Velocity profiles against η for different values of a with $\alpha = 1$, $G_r = 1$, $G_c = 1$, $Sc = 0.22$, $P_r = 0.71$

4. Discussions and results

The problem of unsteady free convection and mass transfer boundary layer flow past an accelerated infinite vertical porous flat plate with suction has been formulated, analyzed and solved both analytically and numerically using finite difference scheme. The effects of the flow parameters on the velocity, temperature and concentration distribution of the flow field are presented with the help of velocity profiles, (Figures (1)–(4)), temperature profiles (Figures (5)–(6)) and concentration profiles (Figures (7)–(8)).

4.1. Velocity field. The velocity of the flow field is found to change more or less with the variation of the flow parameters. The effect of the flow parameters on the velocity field is analyzed with the help of Figures (1)–(4).

4.1.1. *Effect of suction parameter (a).* Figure (1) shows the velocity profiles against η for several values of the suction parameter (a). The suction parameter is found to retard the velocity of the flow field at all points. The reduction in velocity at any point of the flow field is faster as the suction parameter becomes larger. One interesting inference of this finding is greater suction leads to a faster reduction in the velocity of the flow field.

4.1.2. *Effect of porosity parameter (α).* Figure (2) depicts the effect of porosity parameter (α) on the velocity of the flow field. Here, the velocity profiles are drawn against η for three different values of α. The porosity parameter is found to decelerate the velocity of the flow field at all points. Higher the porosity parameter, the more sharper is the reduction in velocity.

4.1.3. *Effect of Grashof numbers for heat and mass transfer (G_r, G_c).* The effects of Grashof numbers for heat transfer (G_r) and mass transfer (G_c) on the velocity of the flow field are presented in Figure (3). In the figure, the velocity of the flow field is plotted against η for different values of the respective Grashof numbers keeping other parameters of the flow field constant. A study of the curves of the figure shows that the Grashof number for heat transfer (G_r) and mass transfer

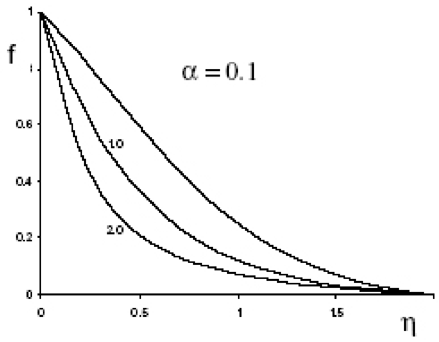

Figure 2. Velocity profiles against η for different values of α with $a = 0.1,\ G_r = 1,\ G_c = 1,\ Sc = 0.22,\ P_r = 0.71$

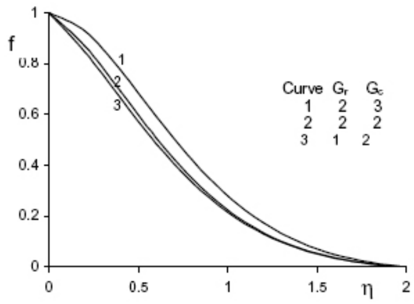

Figure 3. Velocity profiles against for η different values of G_r and G_c with $\alpha = 1,\ a = 0.1,\ S_c = 0.22,\ P_r = 0.71$

(G_c) accelerate the velocity of the flow field at all points. Comparing the curves of Figure (3), it is further observed that the increase in velocity of the flow field is more significant in presence of mass transfer. Thus, mass transfer has a dominant effect on the flow field.

4.1.4. *Effect of Schmidt number (S_c).* The nature of velocity profiles in presence of foreign species such as H_2 ($S_c = 0.22$), CO_2 ($S_c = 0.30$), NH_3 ($S_c = 0.78$) is shown in Figure (4). The flow field suffers a decrease in velocity at all points in presence of heavier diffusing species.

4.2. Temperature field. The temperature of the flow field is mainly affected by two flow parameters, namely, suction parameter (a) and the Prandtl number (P_r). The effects of these parameters on temperature of the flow field are shown in Figures (5) and (6) respectively.

4.2.1. *Effect of suction parameter (a).* Figure (5) depicts the temperature profiles against η for various values of suction parameter (a) keeping Prandtl number (P_r) as constant. Suction parameter is found to decrease the temperature of the

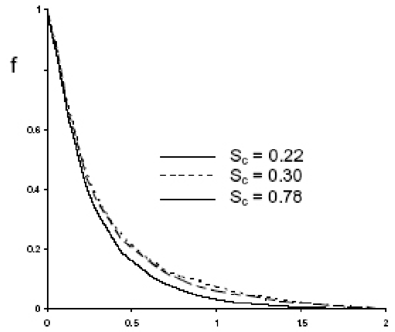

Figure 4. Velocity profiles against η for different values of S_c with $\alpha = 1$, $a = 0.1$, $G_r = 1$, $G_c = 1$, $P_r = 0.71$

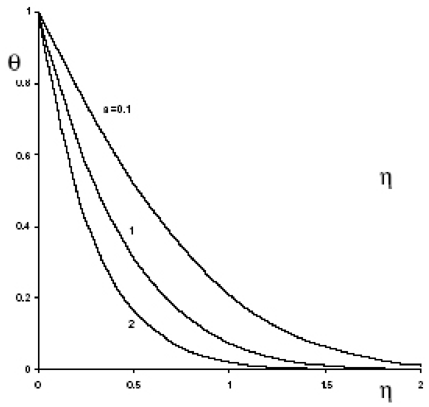

Figure 5. Temperature profiles against η for different values of a with $P_r = 0.71$

flow field at all points. In other words, cooling of the plate is faster as the suction parameter becomes larger. Thus it may be concluded that larger suction leads to faster cooling of the plate.

4.2.2. *Effect of Prandtl number (P_r).* Figure (6) shows the plot of temperature of the flow field against for different values of Prandtl number (P_r) taking suction parameter (a) as constant. Comparing both the curves of Figure (6), it is observed that the temperature of the flow field decreases in magnitude as P_r increases. Thus higher Prandtl number (P_r) leads to faster cooling of the plate.

4.3. Concentration distribution. The concentration distribution of the flow field in presence of foreign species, such as H_2 ($Sc = 0.22$), CO_2 ($Sc = 0.30$) and NH_3 ($Sc = 0.78$) is shown in Figures (7) and (8). It is affected by two flow parameters, namely Schmidt number (Sc) and suction parameter (a).

4.3.1. *Effect of Schmidt number (S_c).* The concentration distribution is vastly affected by the presence of foreign species (S_c) in the flow field. Figure (7) depicts

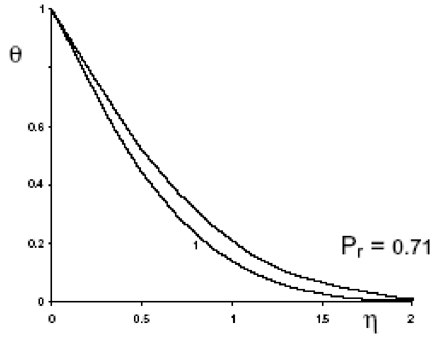

Figure 6. Temperature profiles against η for different values of P_r with $a = 0.1$

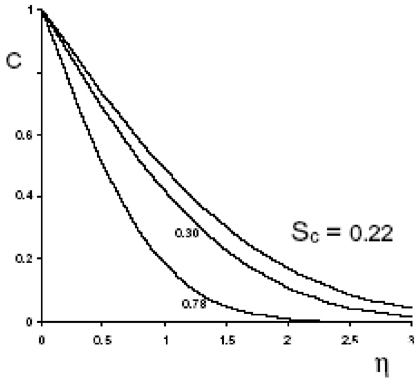

Figure 7. Concentration profiles against η for different values of S_c with $a = 0.1$

the effect of S_c on the concentration distribution of the flow field. The concentration distribution is found to decrease faster as the diffusing foreign species becomes heavier. Thus higher S_c leads to a faster decrease in concentration of the flow field.

4.3.2. *Effect of porosity parameter (a).* Figure (8) shows the plot of concentration distribution against η for different values of porosity parameter (a) and fixed S_c (= 0.22). A comparative study of the curves of the above figure shows that the concentration distribution of the flow field decreases faster as the porosity parameter (a) becomes larger. Thus greater suction leads to a faster decrease in concentration of the flow field.

4.4. Heat flux and skin friction. The heat flux in terms of Nusselt number (N_u) and the non-dimensional skin friction (τ') are entered in table (1) for different values of suction parameter (a). Both the heat flux and skin friction at the wall increase with an increase in suction.

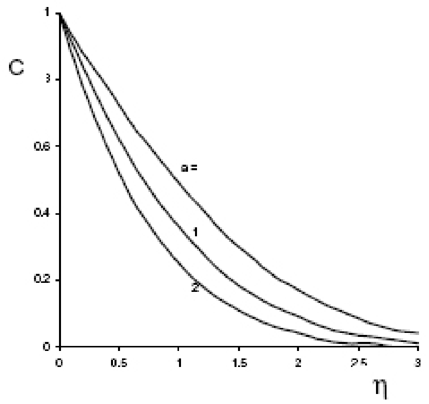

Figure 8. Concentration profiles against η for different values of a with $S_c = 0.22$

Table 1. Variation of heat flux (N_u) and skin friction (τ') with a for $\alpha = 0.1$ and $P_r = 0.71$

a	N_u	τ'
0.1	1.042761	0.884583
1.0	2.002307	2.156250
2.0	3.236602	3.758333

5. Conclusions

The above study brings out the following inferences of physical interest on the velocity, temperature and concentration distribution of the flow field.

(1) The reduction in velocity at any point of the flow field is faster as the suction parameter (a) becomes larger. Thus greater suction leads to a faster reduction in the velocity of the flow field.

(2) The porosity parameter (α) retards the velocity of the flow field at all points. Higher the porosity parameter, the more sharper is the reduction in velocity.

(3) The Grashof number for heat transfer (G_r) and mass transfer (G_c) accelerate the velocity of the flow field at all points. But the increase in velocity of the flow field is more significant in presence of mass transfer.

(4) In presence of heavier diffusing species, the flow field suffers a greater reduction in velocity at all points.

(5) At any point in the flow field, the cooling of the plate is faster as the suction parameter (a) and Prandtl number (P_r) become larger. Thus greater suction/Prandtl number leads to faster cooling of the plate.

(6) The flow field suffers a faster decrease in concentration at all points as the diffusing foreign species present in the flow field becomes heavier and a greater suction leads to a faster reduction of concentration of the flow field.

(7) Both the skin friction (τ') and the heat flux (N_u) at the wall increase with an increase in suction.

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