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Random Nonlinear Variational Inclusions Involving $H(\cdot,\cdot)$-accretive Operator for Random Fuzzy Mappings in Banach Spaces

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Abstract. In this paper we introduce and study a new class of random nonlinear variational inclusions involving $H(\cdot,\cdot)$-accretive operator for fuzzy mapping in Banach space. By using the new resolvent operator technique for $H(\cdot,\cdot)$-accretive operators duo to Zou and Huang, we construct a new iterative algorithm for solving such random nonlinear variational inclusion problem. Under some suitable conditions, we prove the existence of random solution and the convergence of random iterative sequences generated by the algorithm. The results presented in this paper improve and generalize some known corresponding results in the literature.

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1. Introduction

Variational inclusions is an important and useful generalization of variational inequalities, which have wide application in mechanics, physics, optimization and control, nonlinear programming, economics and transportation equilibrium, and engineering sciences, etc, and which have been widely studied extensively by many authors (see, for example, [1, 2, 4, 10, 17–19, 28, 31–35, 37–39] and the references therein). Many efficient ways have been studied to find solutions for variational inclusions, among them, the resolvent operator technique was of great concern.

In 2001, Huang and Fang [24] first introduced the concept of a generalized m-accretive mapping, which is a generalization of an m-accretive mapping, and gave the definition of the resolvent operator for the generalized m-accretive mapping in Banach spaces. After that, Fang and Huang [12–14,16], Fang, Cho and Kim [15], Huang

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and Fang [25], Huang, Fang and Deng [27], Huang [28], Lan [31], Lan and Verma [32], Lan, Cho and Verma [33], Lan, Liu and Li [34], Verma [36] introduced and studied many generalized operators such as H-monotone, H-accretive, η-monotone, (H,η)-accretive, (A,η)-monotone, etc. Recently, Zou and Huang [38] and [39] introduced and studied a new class of $H(\cdot,\cdot)$-accretive operator in Banach spaces, which provided unifying work for the H-accretive, (H,η)-accretive and (A,η)-accretive mappings in Banach spaces.

Random variational inequality theories is an important part of random function analysis. These topics have attracted many scholars and experts due to the extensive applications of the random problems. In 1997, Huang [20] first introduced the concept of random fuzzy mapping and studied the random nonlinear quasi-complementarity problem for random fuzzy mappings. Further, Huang [21] studied the random generalized nonlinear variational inclusions for random fuzzy mappings. Ahmad and Bazán [2] studied a class of random generalized nonlinear mixed variational inclusions for random fuzzy mappings and constructed an iterative algorithm for solving such random problems. Some works concerned with random variational inequalities and random variational inclusion problems. Very recently Lan, Cho and Xie [30] study general nonlinear random equations with random multi-valued operator in Banach spaces. We refer to Ahmad and Farajzadeh [3], Chang and Huang [5, 7], Cho and Huang [8], Cho and Lan [9, 11], Huang [21, 22, 23], Huang, Long and Cho [26], Khan, Salahuddin and Verma [29] and the references therein.

Motivated and inspired by recent research works in this field, in this paper we introduce and study a new kind of random nonlinear variational inclusions involving $H(\cdot,\cdot)$-accretive operator for random fuzzy mappings in Banach spaces. We construct an iterative algorithm for solving such random variational inclusion problems. Under some suitable conditions, we prove the existence of random solution for the random variational inclusion problem in Banach space and the convergence of iterative sequences generated by the algorithm.

2. Preliminaries and formulation

Throughout this paper, we suppose that $(\Omega,\mathcal{A})$ is a measurable space, where Ω is set and $\mathcal{A}$ is a σ-algebra over Ω. E is a separable real Banach space, E^* is dual space of E, $\mathcal{B}(E)$ is a class of Borel σ-algebra in E, $CB(E)$ is the family of all nonempty closed and bounded subsets of E, $\langle\cdot,\cdot\rangle$ is the dual pair between E and E^*, $\|\cdot\|$ is the norm of E. $J\colon E\to 2^{E^*}$ is the normalized duality mapping defined by

$$J(x)=\{f\in E^*\colon \langle x,f\rangle=\|x\|^2, \|f\|=\|x\|\},\quad x\in E.$$

In this paper, we will use the following definitions and lemmas (see, for example, [4–6, 36]).

Definition 2.1.

(i) *A mapping $x\colon\Omega\to E$ is said to be measurable if,*

$$\{t\in\Omega\colon x(t)\in B\}\in\mathcal{A}$$

for any $B\in\mathcal{B}(E)$;

(ii) *A mapping* $T\colon \Omega \times E \to E$ *is called a random mapping if for any given* $x \in E$, $T(t,x) = y(t)$ *is measurable. A random mapping* T *is said to be continuous if for any* $t \in \Omega$, *the mapping* $T(t,\cdot)\colon E \to E$ *is continuous;*

(iii) *A set-valued mapping* $T\colon \Omega \to 2^E$ *is said to be measurable if for any* $B \in \mathcal{B}(E)$,

$$T^{-1}(B) = \{t \in \Omega : T(t) \cap B \neq \emptyset\} \in \mathcal{A};$$

It is well-known that the measurable mapping is necessary to a random mapping.

(iv) *A set-valued mapping* $T\colon \Omega \times E \to 2^E$ *is called random set-valued if for any given* $x \in E$, $T(\cdot,x)\colon \Omega \to 2^E$ *is a measurable set-valued mapping;*

(v) *A mapping* $u\colon \Omega \to E$ *is called a measurable selection of a set-valued measurable mapping* $V\colon \Omega \to 2^E$, *if* u *is measurable and* $u(t) \in V(t)$ *for any* $t \in \Omega$.

Definition 2.2.

(i) *A random set-valued mapping* $T\colon \Omega \times E \to CB(E)$ *is said to be* ξ-H-*Lipschitz continuous, if there exists a measurable function* $\xi\colon \Omega \to (0,\infty)$, *such that*

$$D(T(t,u_1(t)), T(t,u_2(t))) \leq \xi(t)||u_1(t) - u_2(t)||,$$

$\forall\, u_i(t) \in E, \forall\, t \in \Omega, i = 1,2;$ *where* $D(\cdot,\cdot)$ *is the Hausdorff metric on* $CB(E)$ *defined by*

$$D(A,B) = \max\big\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(A,y)\big\}, \qquad \forall\, A,B \in CB(E).$$

(ii) *Suppose that* $s\colon \Omega\times E \to E$ *is a random single-valued mapping,* $N\colon E\times E\times E \to E$ *is a single-valued mapping. The mapping* N *is said to be* β-*Lipschitz continuous with respect to the mapping* s *in the first argument, if there exists measurable function* $\beta\colon \Omega \to (0,\infty)$, *such that*

$$\|N(s(t,x(t)),u(t),v(t)) - N(s(t,y(t)),u(t),v(t))\| \leq \beta(t)\|x(t) - y(t)\|,$$

for all $t \in \Omega, x(t), y(t), u(t), v(t) \in E$;

Similarly, we can define the β-*Lipschitz continuity respect to the second argument and the third argument of* $N(\cdot,\cdot,\cdot)$;

(iii) *Let* $f\colon \Omega \times E \to E$ *be a random single-valued mapping,* f *is said to be* γ-*Lipschitz continuous, if there exists a measurable function* $\gamma : \Omega \to (0,\infty)$, *such that*

$$\|f(t,x(t)) - f(t,y(t))\| \leq \gamma(t)\|x(t) - y(t)\|$$

for all $t \in \Omega, x(t), y(t) \in E$.

Definition 2.3. *Let* $f,g\colon \Omega\times E \to E$ *be two random single-valued mappings,* $H\colon E\times E \to E$ *be a single-valued mapping and* $j(x(t)-y(t)) \in J(x(t)-y(t))$.

(i) f *is said to be accretive if,*

$$\langle f(t,x(t)) - f(t,y(t)), j(x(t) - y(t))\rangle \geq 0;$$

for all $t \in \Omega, x(t), y(t) \in E$,

(ii) f *is said to be strictly accretive if* f *is accretive and* $\langle f(t,x(t)) - f(t,y(t)), j(x(t) - y(t))\rangle = 0$ *if and only if* $x(t) = y(t)$, *for all* $t \in \Omega$;

(iii) *$H(f,\cdot)$ is said to be α-strongly accretive with respect to f in the first argument, if there exists a measurable function $\alpha:\Omega\to(0,\infty)$, such that*

$$\begin{aligned}&\langle H(f(t,x(t)),u(t))-H(f(t,y(t)),u(t))),j(x(t)-y(t))\rangle\\&\geq\alpha(t)\|x(t)-y(t)\|^2,\end{aligned}$$

for all $t\in\Omega, x(t),y(t),u(t)\in E$,

(iv) *$H(\cdot,g)$ is said to be β-relaxed accretive with respect to g in the second argument, if there exists a measurable function $\beta\colon\Omega\to(0,\infty)$, such that*

$$\begin{aligned}&\langle H(u(t),g(t,x(t)))-H(u(t),g(t,y(t))),j(x(t)-y(t))\rangle\\&\geq-\beta(t)\|x(t)-y(t)\|^2\end{aligned}$$

for all $t\in\Omega, x(t),y(t),u(t)\in E$;

Definition 2.4. *Let $\eta:E\times E\to E$ be a single-valued mapping, $M\colon E\to 2^E$ be a set-valued mapping and $j(x(t)-y(t))\in J(x(t)-y(t))$.*

(i) *M is said to be accretive if*

$$\langle u(t)-v(t),j(x(t)-y(t))\rangle\geq 0,$$

for all $t\in\Omega, x(t),y(t)\in E, u(t)\in M(x(t)), v(t)\in M(y(t))$;

(ii) *M is said to be η-accretive if,*

$$\langle u(t)-v(t),j(\eta(x(t),y(t)))\rangle\geq 0,$$

for $t\in\Omega, x(t),y(t)\in E, u(t)\in M(x(t)), v(t)\in M(y(t))$;

(iii) *M is said to be strongly η-accretive, if M is η-accretive and the equality*

$$\langle u(t)-v(t),j(\eta(x(t),y(t)))\rangle=0$$

holds if and only if $x(t)=y(t)$, for all $t\in\Omega$;

(iv) *M is said to be γ-strongly η-accretive if there exists a measurable functions $\gamma(t)>0$, such that*

$$\langle u(t)-v(t),j(\eta(x(t),y(t)))\rangle\geq\gamma(t)\|x(t)-y(t)\|^2,$$

for $t\in\Omega, x(t),y(t)\in E, u(t)\in M(x(t)), v(t)\in M(y(t))$;

(v) *M is said to be α-relaxed η-accretive if there exists a measurable functions $\alpha(t)>0$, such that*

$$\langle u(t)-v(t),j(\eta(x(t),y(t)))\rangle\geq-\alpha(t)\|x(t)-y(t)\|^2,$$

for $t\in\Omega, x(t),y(t)\in E, u(t)\in M(x(t)), v(t)\in M(y(t))$.

Definition 2.5. *Let $\eta\colon E\times E\to E$ be a single-valued mapping, $H,A\colon E\to E$ be two single-valued mappings, $M\colon E\to 2^E$ be a set-valued mapping.*

(i) *M is said to be m-accretive if M is accretive and $(I+\lambda M)(E)=E$ for all $\lambda>0$, where I is identity operator on E;*

(ii) *M is said to be generalized m-accretive if M is η-accretive and $(I+\lambda M)(E)=E$ for all $\lambda>0$;*

(iii) *M is said to be H-accretive if M is accretive and $(H+\lambda M)(E)=E$ for all $\lambda>0$;*

(iv) *) M is said to be (H,η)-accretive if M is η-accretive and $(H+\lambda M)(E)=E$ for all $\lambda>0$;*

(v) *M is said to be (A,η)-accretive if M is m-relaxed η-accretive and $(A+\lambda M)(E)=E$ for all $\lambda>0$.*

Definition 2.6. *Let $f,g\colon \Omega\times E\to E$ be two random single-valued mappings, $H\colon E\times E\to E$ be a single-valued mapping, $M\colon E\to 2^E$ be a set-valued mapping. M is said to be $H(\cdot,\cdot)$-accretive with respect to operators f and g (or simply $H(\cdot,\cdot)$-accretive in the sequel), if M is accretive and $(H(f,g)+\lambda M)(E)=E$, for every $\lambda>0$.*

Definition 2.7. *Suppose E is a Banach space, E is said to be uniformly smooth if for any $\varepsilon>0$, there exists $\delta>0$, such that*

$$\|x+y\|+\|x-y\|<2+\varepsilon\|y\|$$

for all $x,y\in E, x\in S_x, \|y\|<\delta$, where $S_x=\{x\in E, \|x\|=1\}$.

It is well-known that if E is uniformly smooth then the normalized duality mapping is single-valued mapping.

Let $\mathcal{F}(E)$ be the family of all fuzzy sets over E, A mapping $F\colon E\to\mathcal{F}(E)$ is called a fuzzy mapping over E.

If F is a fuzzy mapping over E, then $F(x)$ (denoted by F_x) is a fuzzy set on E, and $F_x(y)$ is the membership degree of the point y in F_x. Let $A\in\mathcal{F}(E)$, $\alpha\in[0,1]$. Then the set

$$(A)_\alpha=\{x\in E\colon A(x)\geq\alpha\}$$

is called a α-cut set of fuzzy set A.

Definition 2.8.

(i) *A fuzzy mapping $F\colon\Omega\to\mathcal{F}(E)$ is called measurable if, for any given $\alpha\in(0,1]$, $(F(\cdot))_\alpha\colon\Omega\to 2^E$ is a measurable set-valued mapping.*

(ii) *A fuzzy mapping $F\colon\Omega\times E\to\mathcal{F}(E)$ is called a random fuzzy mapping if, for any $x\in E$, $F(\cdot,x)\colon\Omega\to\mathcal{F}(E)$ is a measurable fuzzy mapping.*

Let $F,G\colon\Omega\times E\to\mathcal{F}(E)$ be two random fuzzy mappings satisfying the following condition $(\star\star)$: There exist two mappings $a,b\colon E\to(0,1]$, such that

$$(\star\star)\qquad (F_{t,x})_{a(x)}\in CB(E),\ (G_{t,x})_{b(x)}\in CB(E),\quad \forall\,(t,x)\in\Omega\times E.$$

By using the random fuzzy mappings F and G, we can define the two set-valued mappings $\widetilde{F}$ and $\widetilde{G}$ as follows, respectively.

$$\widetilde{F}\colon\Omega\times E\to CB(E),\ \ (t,x)\to(F_{t,x})_{a(x)},\forall\,(t,x)\in\Omega\times E,$$

$$\widetilde{G}\colon\Omega\times E\to CB(E),\ \ (t,x)\to(G_{t,x})_{b(x)},\forall(t,x)\in\Omega\times E.$$

It means that

$$\widetilde{F}(t,x)=(F_{t,x})_{a(x)}=\{z\in E,(F_{t,x})(z)\geq a(x)\}\in CB(E),$$

$$\widetilde{G}(t,x)=(G_{t,x})_{b(x)}=\{z\in E,(G_{t,x})(z)\geq b(x)\}\in CB(E).$$

It is easy to see that $\widetilde{F}$ and $\widetilde{G}$ are the random set-valued mappings. We call $\widetilde{F}$ and $\widetilde{G}$ are random set-valued mappings induced by fuzzy mappings F and G, respectively.

Problem 2.1. *Let $f, g, s, p\colon \Omega \times E \to E$ be four random single-valued mappings, $H\colon E \times E \to E$ and $N\colon E \times E \times E \to E$ be two single-valued mappings. Suppose that $M\colon E \to 2^E$ is an $H(\cdot,\cdot)$-accretive operator with respect to f and g, $F, G\colon \Omega \times E \to \mathcal{F}(E)$ are two random fuzzy mappings satisfying the condition $(\star\star)$. Given mappings $a, b\colon E \to (0,1]$, we consider the following problem: Find measurable mappings x, u, v, $: \Omega \to E$, such that*

$$\begin{cases} F_{t,x(t)}(u(t)) \geq a(x(t)), \\ G_{t,x(t)}(v(t)) \geq b(x(t)), \\ 0 \in N(s(t,x(t)), u(t), v(t)) + M(p(t,x(t))) \end{cases} \tag{2.1}$$

for all $t \in \Omega$.

The Problem (2.1) is called random nonlinear variational inclusions involving $H(\cdot,\cdot)$-accretive operator for random fuzzy mappings in Banach spaces. The set of measurable mappings (x,u,v) is called a solution of the Problem (2.1).

Remark 2.1. In the paper of Zou and Huang [36], the author have gave a description that if $H(f,g) = g$, the $\eta(x,y)$ is Lipschitz continuous, and the set-valued mapping M is η-accretive, then the $H(\cdot,\cdot)$-accretive operator become the (A,η)-accretive operator.

Remark 2.2. If the fuzzy mapping F and G are Classic single-valued mappings, $\eta(x,y)$ is Lipschitz continuous, and the set-valued mapping M is η-accretive, the $H(f,g) = g$, $N(s(t,x(t)), u(t), v(t)) = \frac{1}{\gamma(t)}[s(t,x(t)) + (u(t) - c(t)) + 0 \cdot v(t)]$, then from Remark 2.1, we know that the Problem (2.1) is equivalent to the problem of finding measurable mappings $x, u\colon \Omega \to E$, such that

$$c(t) \in s(t,x(t)) + u(t) + \gamma(t)M(p(t,x(t))), \tag{2.2}$$

for all $t \in \Omega$ and $u(t) \in E$, where $c(t)\colon \Omega \to E$ is measurable function. The determinate form of the Problem (2.2) was considered and studied by Cho and Lan [9].

Remark 2.3. If the mappings F, G, M, η, H, N are the same as in the Remark 2.2 and $c(t) = 0$ for all $t \in \Omega$, then the Problem 2.2 is equivalent to the problem of finding $x\colon \Omega \to E$, such that

$$0 \in s(t,x(t)) + u(t) + \gamma(t)M(p(t,x(t)), \tag{2.3}$$

for all $t \in \Omega$ and $u(t) \in E$. The determinate form of the problem (2.3) was considered and studied by Lan [31].

Remark 2.4. If $E = E^* = H$ is Hilbert space, the mappings F, G, η, H are the same as in the Remark 2.2 and $N(s(t,x(t)), u(t), v(t)) = s(t,x(t)) - (u(t) - v(t)))$, for all $t \in \Omega$, and $M(\cdot) = \partial\phi(\cdot)$, where $\partial\phi(\cdot)$ denotes the subdifferential of a lower semicontinuous and η-subdifferentiable function $\phi : H \to R \cup \{+\infty\}$, then the Problem (2.1) is equivalent to the problem of finding measurable function $x, u, v\colon \Omega \to E$, such that

$$\langle s(t,x(t)) - (u(t) - v(t))), z(t) - p(t,x(t))\rangle \geq \phi(p(t,x(t))) - \phi(z(t)) \tag{2.4}$$

for all $t \in \Omega$ and $z(t) \in H$. This problem is called the random generalized nonlinear mixed variational inclusions for random fuzzy mappings, which was introduced and studied by Ahmad and Bazán [2].

Remark 2.5. If $E = E^* = H$ is Hilbert space, the mappings η, H are the same as in the Remark 2.2 and $N(s(t, x(t)), u(t), v(t)) = u(t) - v(t)$, for all $t \in \Omega$, and $M(\cdot) = \partial\phi(\cdot)$, where $\partial\phi(\cdot)$ denotes the subdifferential of a lower semi-continuous and η-subdifferentiable function $\phi : H \to R \cup \{+\infty\}$, then the Problem (2.1) is equivalent to the problem of finding measurable function $x, u, v \colon \Omega \to E$, such that

$$(2.5) \qquad \begin{cases} F_{t,x(t)}(u(t)) \geq a(x(t)), \\ G_{t,x(t)}(v(t)) \geq b(x(t)), \\ \langle u(t) - v(t), z(t) - p(t, x(t))\rangle \geq \phi(p(t, x(t))) - \phi(z(t)) \end{cases}$$

for all $t \in \Omega$ and $z(t) \in H$. This problem is called the random generalized nonlinear variational inclusion for random fuzzy mappings, which was introduced and studied by Huang [18].

3. Random iterative algorithms

Based on the formulation in Section 2, we now construct a new algorithm for solving Problem (2.1).

Lemma 3.1. [6] *Suppose $T : \Omega \times E \to E$ is a continuous random mapping, then for any measurable mapping $x : \Omega \to E$, $T(t, x(t))$ is a measurable mapping.*

Lemma 3.2. [6] *Let $T : \Omega \times E \to CB(E)$ be an H-continuous random set-valued mapping, then for any measurable mapping $u : \Omega \to E$, $T(\cdot, u(\cdot)) : \Omega \to CB(E)$ is measurable.*

Lemma 3.3. [6] *Let $U, V : \Omega \to CB(E)$ be two measurable set-valued mappings and $u : \Omega \to E$ be a selection of U, then there exists a measurable selection of V such that, for all $t \in \Omega$ and $\varepsilon > 0$,*

$$(3.1) \qquad \|u(t) - v(t)\| \leq (1 + \varepsilon)D\Big(U(t), V(t)\Big).$$

Lemma 3.4. *Let E be a real Banach space and $J : E \to 2^{E^*}$ be the normalized duality mapping. Then*

$$(3.2) \qquad \|x + y\|^2 \leq \|x\|^2 + 2\langle y, j(x + y)\rangle,$$

for any $x, y \in E$ and for all $j(x + y) \in J(x + y)$.

Lemma 3.5. [36] *Let $H(f, g)$ be α-strongly accretive with respect to f, β-relaxed accretive with respect to g, and $\alpha > \beta$. Suppose M is an $H(\cdot,\cdot)$-accretive operator with respect to f and g. Then the mapping $(H(f, g) + \lambda M)^{-1}(\cdot)$ is a single-valued mapping.*

Lemma 3.6. [36] *Suppose that $H, f, g, M, \alpha, \beta$ are the same as in the Lemma 3.5, then the resolvent operator $R^{H(\cdot,\cdot)}_{M,\lambda}$ is $(\frac{1}{\alpha-\beta})$-Lipschitz continuous. That is*

$$\|R^{H(\cdot,\cdot)}_{M,\lambda}(u) - R^{H(\cdot,\cdot)}_{M,\lambda}(v)\| \leq \frac{1}{\alpha - \beta}\|u - v\|,$$

for all $u, v \in E$, where $R^{H(\cdot,\cdot)}_{M,\lambda}(u) = (H(f, g) + \lambda M)^{-1}(u)$.

Lemma 3.7. *Measurable operator $x, u, v\colon \Omega \to E$ are solution of the Problem* (2.1) *if and only if,*

$$x(t) = R_{M,\lambda}^{H(\cdot,\cdot)}\Big(H(f(t,x(t)),g(t,x(t))) - \lambda(t)N(s(x(t)),u(t),v(t))\Big), \tag{3.3}$$

where $\lambda : \Omega \to (0,\infty)$.

Proof. The proof directly follows from the definition of $R_{M,\lambda}^{H(\cdot,\cdot)}$ and so it is omitted. ∎

Remark 3.1. Let $z(t) = H(f(t,x(t)),g(t,x(t))) - \lambda(t)N(s(t,x(t)),u(t),v(t))$, then $(x(t),u(t),v(t))$ is the solution of the Problem (2.1) if and only if,

$$x(t) = R_{M,\lambda}^{H(\cdot,\cdot)}z(t) \quad \forall\, t \in \Omega. \tag{3.4}$$

Now we use the Lemma 3.7 to suggest the following algorithms for solving the Problem (2.1).

For any given measurable mapping $x_0 : \Omega \to E$, then the set-valued mappings $\widetilde{F}(\cdot,x_0(\cdot))$ and $\widetilde{G}(\cdot,x_0(\cdot)) : \Omega \to CB(E)$ are measurable by the condition $(\star\star)$, hence there exist measurable selections $u_0 : \Omega \to E$ of $\widetilde{F}(\cdot,x_0(\cdot))$ and $v_0 : \Omega \to E$ of $\widetilde{G}(\cdot,x_0(\cdot))$. Let

$$z_1(t) = H(f(t,x_0(t)),g(t,x_0(t))) - \lambda(t)N(s(t,x_0(t)),u_0(t),v_0(t)), \tag{3.5}$$

from (3.4), we take

$$x_1(t) = R_{M,\lambda}^{H(\cdot,\cdot)}z_1(t).$$

By the Lemma 3.3, there exist $u_1(t) \in \widetilde{F}(t,x_1(t))$ and $v_1(t) \in \widetilde{G}(t,x_1(t))$, such that

$$\begin{cases} \|u_0(t) - u_1(t)\| \leq (1+1)D(\widetilde{F}(t,x_0(t)),\widetilde{F}(t,x_1(t))), \\ \|v_0(t) - v_1(t)\| \leq (1+1)D(\widetilde{G}(t,x_0(t)),\widetilde{G}(t,x_1(t))), \end{cases} \tag{3.6}$$

for all $t \in \Omega$, where $D(\cdot,\cdot)$ is the Hausdorff metric on $CB(E)$. Let

$$z_2(t) = H(f(t,x_1(t)),g(t,x_1(t))) - \lambda(t)N(s(t,x_1(t)),u_1(t)),v_1(t)), \tag{3.7}$$

put

$$x_2(t) = R_{M,\lambda}^{H(\cdot,\cdot)}z_2(t),$$

by the Lemma 3.3, there exist $u_2(t) \in \widetilde{F}(t,x_2(t))$ and $v_2(t) \in \widetilde{G}(t,x_2(t))$, such that

$$\begin{cases} \|u_1(t) - u_2(t)\| \leq (1+1/2)D(\widetilde{F}(t,x_1(t)),\widetilde{F}(t,x_2(t))), \\ \|v_1(t) - v_2(t)\| \leq (1+1/2)D(\widetilde{G}(t,x_1(t)),\widetilde{G}(t,x_2(t))), \end{cases} \tag{3.8}$$

for any $t \in \Omega$.

Algorithm 3.1. By induction, we can define sequences $x_n(t)$, $u_n(t)$, $v_n(t)$ and $z_n(t)$ inductively satisfying

$$(3.9)\quad \begin{cases} x_n(t) = R_{M,\lambda}^{H(\cdot,\cdot)} z_n(t), \\ z_n(t) = H(f(t, x_{n-1}(t)), g(t, x_{n-1}(t))) \\ \qquad\qquad -\lambda(t) N(s(t, x_{n-1}(t)), u_{n-1}(t), v_{n-1}(t)), \\ u_n(t) \in \widetilde{F}(t, x_n(t)), \\ \quad \|u_{n-1}(t) - u_n(t)\| \le (1 + 1/n) D(\widetilde{F}(t, x_{n-1}(t)), \widetilde{F}(t, x_n(t))), \\ v_n(t) \in \widetilde{G}(t, x_n(t)), \\ \quad \|v_{n-1}(t) - v_n(t)\| \le (1 + 1/n) D(\widetilde{G}(t, x_{n-1}(t)), \widetilde{G}(t, x_n(t))), \end{cases}$$

for any $t \in \Omega$.

4. Existence and convergence

In this section we will prove the existence of random solution of the Problem (2.1) and the convergence of random iterative sequences generated by the Algorithm 3.1.

Theorem 4.1. *Let E be a uniformly smooth and separable real Banach space. Suppose that $f, g, s, p\colon \Omega \times E \to E$ and $H\colon E \times E \to E$ are five single-valued mappings. Assume that*

(i) *$M\colon E \to 2^E$ is an $H(\cdot,\cdot)$-accretive with respect to operators f and g;*
(ii) $\mathrm{Range}(p) \cap \mathrm{dom} M \neq \emptyset$;
(iii) *$N\colon E \times E \times E \to E$ is the β_1-Lipschitz continuous with respect to mapping s in the first argument, β_2-Lipschitz continuous with respect to the second argument and β_3-Lipschitz continuous with respect to the third argument.*
(iv) *Let $F, G\colon \Omega \times E \to \mathcal{F}(E)$ be two random fuzzy mappings satisfying the condition $(\star\star)$, $\widetilde{F}, \widetilde{G}$ be two random set-valued mappings induced by the mappings F and G, respectively, $\widetilde{F}$ and $\widetilde{G}$ are ξ_1-H-Lipschitz and ξ_2-H-Lipschitz continuous, respectively.*
(v) *$H(f, g)$ is ρ_1-Lipschitz continuous with respect to f and ρ_2-Lipschitz continuous with respect to g;*
(vi) *$H(f, g)$ is δ_1-strongly accretive with respect to f and δ_2-relaxed accretive with respect to g, where $\delta_1 > \delta_2$.*

If the following conditions are satisfied,

$$(4.1)\quad \begin{cases} 0 \le \rho_1(t) + \rho_2(t) < \delta_1(t) - \delta_2(t), \\ 0 < \xi_1(t),\ \xi_2(t) < \frac{1}{2}, \\ 0 < \lambda(t) < \min\Big\{ \frac{1}{\beta_1(t) + \beta_2(t)\xi_1(t) + \beta_3(t)\xi_2(t)}, \\ \qquad \frac{(\delta_1(t) - \delta_2(t))^2 - (\rho_1(t) + \rho_2(t))^2}{((\delta_1(t) - \delta_2(t))^2 + 1)(\beta_1(t) + \beta_2(t)\xi_1(t) + \beta_3(t)\xi_2(t))} \Big\}, \end{cases}$$

for any $t \in \Omega$, then there exist measurable mappings $x^*, u^*, v^* : \Omega \to E$ is the solution of the Problem (2.1) and

$$x_n(t) \to x^*(t),\ u_n(t) \to u^*(t),\ v_n(t) \to v^*(t),$$

as $n \to \infty$, where $\{x_n(t)\}$, $\{u_n(t)\}$ and $\{v_n(t)\}$ are iterative sequences generated by the Algorithm 3.1.

Proof. Since $\widetilde{F}$ and $\widetilde{G}$ are ξ_1-H-Lipschitz and ξ_2-H-Lipschitz continuous, respectively, hence there exist two measurable functions $\xi_1(t)$ and $\xi_2(t)$, such that

$$D(\widetilde{F}(t,x_n(t)),\widetilde{F}(t,x_{n-1}(t))) \le \xi_1(t)\|x_n(t)-x_{n-1}(t)\|, \qquad \forall\, t\in\Omega \tag{4.2}$$

$$D(\widetilde{G}(t,x_n(t)),\widetilde{G}(t,x_{n-1}(t))) \le \xi_2(t)\|x_n(t)-x_{n-1}(t)\|, \qquad \forall\, t\in\Omega. \tag{4.3}$$

Hence

$$\begin{aligned}\|u_n(t)-u_{n-1}(t)\| &\le (1+\frac{1}{n})D(\widetilde{F}(t,x_n(t)),\widetilde{F}(t,x_{n-1}(t)))\\ &\le (1+\frac{1}{n})\xi_1(t)\|x_n(t)-x_{n-1}(t)\|\end{aligned} \tag{4.4}$$

$$\begin{aligned}\|v_n(t)-v_{n-1}(t)\| &\le (1+\frac{1}{n})D(\widetilde{G}(t,x_n(t)),\widetilde{G}(t,x_{n-1}(t)))\\ &\le (1+\frac{1}{n})\xi_2(t)\|x_n(t)-x_{n-1}(t)\|\end{aligned} \tag{4.5}$$

Since N is the β_1-Lipschitz continuous with respect to mapping s in the first argument, β_2-Lipschitz continuous with respect to the second argument and β_3-Lipschitz continuous with respect to the third argument, hence there exist three measurable function $\beta_1(t),\beta_2(t),\beta_3(t)$, such that

$$\begin{aligned}&\|N(s(t,x_n(t)),u_n(t),v_n(t))-N(s(t,x_{n-1}(t)),u_n(t),v_n(t))\|\\ &\quad\le \beta_1(t)\|x_n(t)-x_{n-1}(t)\|, \qquad \forall\, t\in\Omega\end{aligned} \tag{4.6}$$

$$\begin{aligned}&\|N(s(t,x_{n-1}(t)),u_n(t)),v_n(t)))-N(s(t,x_{n-1}(t)),u_{n-1}(t),v_n(t))\|\\ &\quad\le \beta_2(t)\|u_n(t)-u_{n-1}(t)\|, \qquad \forall\, t\in\Omega\end{aligned} \tag{4.7}$$

$$\begin{aligned}&\|N(s(t,x_{n-1}(t)),u_{n-1}(t),v_n(t))-N(s(t,x_{n-1}(t)),u_{n-1}(t),v_{n-1}(t))\|\\ &\quad\le \beta_3(t)\|v_n(t)-v_{n-1}(t)\|, \qquad \forall\, t\in\Omega\end{aligned} \tag{4.8}$$

From (4.2)–(4.8), we have

$$\begin{aligned}&\|N(s(t,x_n(t)),u_n(t),v_n(t))-N(s(t,x_{n-1}(t)),u_{n-1}(t),v_{n-1}(t))\|\\ &\quad\le (\beta_1(t)+\beta_2(t)\xi_1(t)(1+\tfrac{1}{n})+\beta_3(t)\xi_2(t)(1+\tfrac{1}{n}))\|x_n(t)-x_{n-1}(t)\|\\ &\forall\, t\in\Omega,\end{aligned} \tag{4.9}$$

$$\begin{aligned}&\|H(f(t,x_n(t)),g(t,x_n(t))-H(f(t,x_{n-1}(t)),g(t,x_{n-1}(t))\|\\ &\quad\le (\rho_1(t)+\rho_2(t))\|x_n(t)-x_{n-1}(t)\|, \qquad \forall\, t\in\Omega.\end{aligned} \tag{4.10}$$

For the sake of brevity, let

$$\begin{aligned}\varphi(n) &= H(f(t,x_n(t)),g(t,x_n(t))),\\ \varphi(n-1) &= H(f(t,x_{n-1}(t)),g(t,x_{n-1}(t)))\\ q(n) &= N(s(t,x_n(t)),u_n(t),v_n(t))\\ q(n-1) &= N(s(t,x_{n-1}(t)),u_{n-1}(t),v_{n-1}(t)).\end{aligned} \tag{4.11}$$

Hence, from the Lemma 3.4 and (4.9)–(4.11), we have

$$\|z_{n+1}(t)-z_n(t)\|^2 = \|\varphi(n)-\varphi(n-1)-\lambda(t)(q(n)-q(n-1))\|$$

$$\le \|\varphi(n)-\varphi(n-1)\|^2 - 2\lambda(t)\langle q(n)-q(n-1), j(z_{n+1}(t)-z_n(t))\rangle$$

$$\le \|\varphi(n)-\varphi(n-1)\|^2 + 2\lambda(t)\|q(n)-q(n-1)\|\|z_{n+1}(t)-z_n(t)\| \tag{4.12}$$

From (4.4)–(4.12), we have

$$\begin{aligned}\|z_{n+1}(t)-z_n(t)\|^2 &\le (\rho_1(t)+\rho_2(t))^2\|x_n(t)-x_{n-1}(t)\|^2\\ &\quad + 2\lambda(t)\alpha_n(t)\|x_n(t)-x_{n-1}(t)\|\|z_{n+1}(t)-z_n(t)\|)\\ &\le (\rho_1(t)+\rho_2(t))^2\|x_n(t)-x_{n-1}(t)\|^2\\ &\quad + \lambda(t)\alpha_n(t)(\|x_n(t)-x_{n-1}(t)\|^2+\|z_{n+1}(t)-z_n(t)\|^2)\end{aligned}$$

which implies that

$$\|z_{n+1}(t)-z_n(t)\|^2 \le \frac{(\rho_1(t)+\rho_2(t))^2+\lambda(t)\alpha_n(t)}{1-\lambda(t)\alpha_n(t)}\|x_n(t)-x_{n-1}(t)\|^2 \tag{4.13}$$

where

$$\alpha_n(t) = \beta_1(t)+\beta_2(t)\xi_1(t)\left(1+\frac{1}{n}\right)+\beta_3(t)\xi_2(t)\left(1+\frac{1}{n}\right).$$

Put

$$\theta_n(t) = \frac{(\rho_1(t)+\rho_2(t))^2+\lambda(t)\alpha_n(t)}{1-\lambda(t)\alpha_n(t)},$$

from (4.13), we have

$$\|z_{n+1}(t)-z_n(t)\| \le \sqrt{\theta_n(t)}\|x_n(t)-x_{n-1}(t)\|. \tag{4.14}$$

On the other hand, from the Lemma 3.6, we have

$$\begin{aligned}\|x_n(t)-x_{n-1}(t)\| &= \|R^{H(\cdot,\cdot)}_{M,\lambda}(z_n(t))-R^{H(\cdot,\cdot)}_{M,\lambda}(z_{n-1}(t))\|\\ &\le \frac{1}{\delta_1(t)-\delta_2(t)}\|z_n(t)-z_{n-1}(t)\|.\end{aligned} \tag{4.15}$$

From (4.14) and (4.15), we have

$$\|z_{n+1}(t)-z_n(t)\| \le \frac{\sqrt{\theta_n(t)}}{\delta_1(t)-\delta_2(t)}\|z_n(t)-z_{n-1}(t)\|. \tag{4.16}$$

Put

$$\zeta_n(t) = \frac{\sqrt{\theta_n(t)}}{\delta_1(t)-\delta_2(t)},$$

from (4.16), we have

$$\|z_{n+1}(t)-z_n(t)\| \le \zeta_n(t)\|z_n(t)-z_{n-1}(t)\| \tag{4.17}$$

and

$$\lim_{n\to\infty}\zeta_n(t) = \frac{\sqrt{\theta(t)}}{\delta_1(t)-\delta_2(t)}$$

where

$$\theta(t) = \frac{(\rho_1(t)+\rho_2(t))^2+\lambda(t)(\beta_1(t)+\beta_2(t)\xi_1(t)+\beta_3(t)\xi_2(t))}{1-\lambda(t)(\beta_1(t)+\beta_2(t)\xi_1(t)+\beta_3(t)\xi_2(t))},$$

we now prove that $0 < \frac{\sqrt{\theta(t)}}{\delta_1(t)-\delta_2(t)} < 1$, for all $t \in \Omega$. In fact, from the condition (4.1), it follows that

$$0 < \lambda(t) < \frac{1}{\beta_1(t) + \beta_2(t)\xi_1(t) + \beta_3(t)\xi_2(t)}, \tag{4.18}$$

$$(\delta_1(t) - \delta_2(t))^2 - (\rho_1(t) + \rho_2(t))^2 > 0 \tag{4.19}$$

and

$$((\delta_1(t)-\delta_2(t))^2+1)\lambda(t)(\beta_1(t)+\beta_2(t)\xi_1(t)+\beta_3(t)\xi_2(t))<(\delta_1(t)-\delta_2(t))^2-(\rho_1(t)+\rho_2(t))^2 \tag{4.20}$$

which implies that

$$\begin{aligned}(\rho_1(t) + \rho_2(t))^2 &+ \lambda(t)(\beta_1(t) + \beta_2(t)\xi_1(t) + \beta_3(t)\xi_2(t)) \\ &< (\delta_1(t) - \delta_2(t))^2(1 - \lambda(t)(\beta_1(t) + \beta_2(t)\xi_1(t) + \beta_3(t)\xi_2(t)))\end{aligned} \tag{4.21}$$

for any $t \in \Omega$.

This implies that

$$0 < \frac{(\rho_1(t) + \rho_2(t))^2 + \lambda(t)(\beta_1(t) + \beta_2(t)\xi_1(t) + \beta_3(t)\xi_2(t))}{(\delta_1(t) - \delta_2(t))^2(1 - \lambda(t)(\beta_1(t) + \beta_2(t)\xi_1(t) + \beta_3(t)\xi_2(t)))} < 1$$

for any $t \in \Omega$. That is

$$0 < \frac{\sqrt{\theta(t)}}{\delta_1(t) - \delta_2(t)} < 1.$$

Since $0 < \frac{\sqrt{\theta(t)}}{\delta_1(t)-\delta_2(t)} < 1$, there exists a measurable function $l(t)$: $0 < l(t) < 1$ and $N > 0$, such that $0 < \zeta_n(t) < l(t)$, as $n > N$, for all $t \in \Omega$. Hence we have

$$\|z_{n+1}(t) - z_n(t)\| \leq l(t)\|z_n(t) - z_{n-1}(t)\|, \ \forall\, t \in \Omega, \ \textit{as } n > N. \tag{4.22}$$

Therefore, $\{z_n(t)\}$ is a Cauchy sequence in E. Since E is a Banach space, there exists $z^*(t) \in E$, such that $z_n(t) \to z^*(t)$, as $n \to \infty$.

From (4.15), we know that the sequence $x_n(t)$ is also a Cauchy sequence in E. From (4.4) and (4.5), we know that the sequences $u_n(t)$ and $v_n(t)$ both are Cauchy sequence in E. Hence there exist $u^*(t)$ and $v^*(t) \in E$, such that $u_n(t) \to u^*(t)$ and $v_n(t) \to v^*(t)$ as $n \to \infty$, respectively.

From the continuity of $R^{H(\cdot,\cdot)}_{M,\lambda}$, $H(\cdot,\cdot)$ and $N(\cdot,\cdot,\cdot)$, we have

$$x^*(t) = R^{H(\cdot,\cdot)}_{M,\lambda} z^*(t) \quad \forall\, t \in \Omega. \tag{4.23}$$

$$z^*(t) = H(f(t, x^*(t)), g(t, x^*(t))) - \lambda(t)N(s(t, x^*(t)), u^*(t), v^*(t)), \tag{4.24}$$

Finally, we prove that $u^*(t) \in \widetilde{F}(t, x^*(t))$, and $v^*(t) \in \widetilde{G}(t, x^*(t))$. In fact, since $u_n(t) \in \widetilde{F}(t, x_n(t))$, we have

$$\begin{aligned} d(u^*(t), \widetilde{F}(t, x^*(t))) &\leq \|u^*(t) - u_n(t)\| + d(u_n(t), \widetilde{F}(t, x^*(t))) \\ &\leq \|u^*(t) - u_n(t)\| + D(\widetilde{F}(t, x_n(t)), \widetilde{F}(t, x^*(t))) \\ &\leq \|u^*(t) - u_n(t)\| + \xi_1(t)\|x_n(t) - x^*(t)\| \to 0, \quad (n \to \infty), \end{aligned} \tag{4.25}$$

which implies that $d(u^*(t), \widetilde{F}(t, x^*(t))) = 0$. Since $\widetilde{F}(t, x^*(t)) \in CB(E)$, it follows that $u^*(t) \in \widetilde{F}(t, x^*(t))$. Similarly, we can prove that $v^*(t) \in \widetilde{G}(t, x^*(t))$. From

Lemma 3.7 and (4.23), (4.24), we know that $(x^*(t), u^*(t), v^*(t))$ is solution of the Problem (2.1). This completes the proof. ∎

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