# THE 31ST INTERNATIONAL CONFERENCE ON FINITE OR INFINITE DIMENSIONAL COMPLEX ANALYSIS AND APPLICATIONS

# ABSTRACT BOOK

# ICFIDCAA

20-24 AUGUST 2025

# THE 31ST INTERNATIONAL CONFERENCE ON FINITE OR INFINITE DIMENSIONAL COMPLEX ANALYSIS AND APPLICATIONS

# ABSTRACT BOOK

# Organizing Committee

**Advisors**
Professor Dato' Indera Dr. Rosihan M. Ali
Professor V. Ravichandran (NIT, Tiruchirappalli)
**Chairman**
Associate Professor See Keong Lee
**Secretary**
Dr. Maisarah Haji Mohd
**Vice Secretary**
Madam Azizah Ibrahim
**Treasurer**
Dr. Zhen Chuan Ng
**Vice Treasurer**
Madam Hasliza Razali
**Secretariat**
Dr. Shamani Supramaniam
Professor Maslina Darus (Universiti Kebangsaan Malaysia)
**Technical**
Dr. Wen Eng Ong
Ms. Hartini Ahmad
**Accommodation & Logistics**
Dr. Yao Liang Chung (INTI International College Penang)
**Publicity**
Dr. Chandrashekar Ramasamy (Universiti Tun Hussein Onn Malaysia)

# Contents

# ICFIDCAA 2025

The School of Mathematical Sciences, Universiti Sains Malaysia (USM) is delighted to host the 31st International Conference on Finite or Infinite Dimensional Complex Analysis and Applications (ICFIDCAA 2025) from August 20-24, 2025, in our beautiful Penang campus.

ICFIDCAA, which began as a colloquium at Pusan National University in 1993, has become a leading forum for researchers to share cutting-edge work in finite and infinite dimensional complex analysis. This annual gathering provides invaluable opportunities for academics, researchers, educators, and students to connect and exchange ideas and experiences.

We believe that ICFIDCAA 2025 will continue to inspire and connect researchers from around the globe.

# Schedule

# CONFERENCE SCHEDULE DAY 1
# 20TH AUGUST 2025

| | |
|---|---|
| 8:45-9:45 | **REGISTRATION & REFRESHMENT**<br>**Venue**: ELL Room |
| 9:45-10:00 | **WELCOMING REMARK**<br>**Venue**: Auditorium |
| 10:00-10:50 | **PLENARY TALK 1**<br>Nevanlinna Currents and Entire Curves in Certain Projective Hypersurfaces<br>**Speaker:** Patrick, Tuen Wai Ng (The University of Hong Kong)<br>**Chair:** See Keong Lee<br>**Venue:** Auditorium |
| 11:00-11:50 | **PLENARY TALK 2**<br>Coefficient Estimates of Various Subclasses of Analytic Univalent Functions and Operators<br>**Speaker:** Maslina Darus (Universiti Kebangsaan Malaysia)<br>**Chair:** Toshiyuki Sugawa<br>**Venue:** Auditorium |
| 11:50-14:00 | **GROUP PHOTO & LUNCH BREAK**<br>**Venue:** ELL Room |
| 14:00-14:30 | **INVITED TALK 1**<br>Radius Problems in Geometric Function Theory<br>**Speaker:** V. Ravichandran (National Institute of Technology Tiruchirappalli)<br>**Chair:** Maslina Darus<br>**Venue:** Auditorium |
| 14:35-15:05 | **INVITED TALK 2**<br>Equidistribution of Fekete Points and its Speed Convergence on Projective Manifolds<br>**Speaker:** Ngoc Cuong Nguyen (Korea Advanced Institute of Science and Technology)<br>**Chair:** Maslina Darus<br>**Venue:** Auditorium |
| 15:10-16:00 | **PLENARY TALK 3**<br>Quantization of the Theory of Topological Insulators<br>**Speaker:** Armen Sergeev (Russian Academy of Sciences)<br>**Chair:** Patrick, Tuen Wai Ng<br>**Venue:** Auditorium |
| 16:00-16:20 | **TEA BREAK**<br>**Venue:** ELL Room |

# CONFERENCE SCHEDULE DAY 2
# 21ST AUGUST 2025

| | | |
|---|---|---|
| 9:00-9:50 | **PLENARY TALK 4**<br>Near-parabolic Renormalization in Holomorphic Dynamics<br>**Speaker:** Fei Yang (Nanjing University)<br>**Chair:** V. Ravichandran<br>**Venue:** Auditorium |
| 10:00-10:30 | **INVITED TALK 3**<br>Grunsky Coefficients and Some Applications in the Theory of Univalent Functions<br>**Speaker:** Milutin Obradovic (University of Belgrade)<br>**Chair:** Swaminathan Anbhu<br>**Venue:** Auditorium |
| 10:30-10:45 | **COFFEE BREAK**<br>**Venue:** ELL Room |
| 10:45-11:35 | **PLENARY TALK 5**<br>Totally Monotone Sequences, Pick Functions and Hypergeometric Functions<br>**Speaker:** Toshiyuki Sugawa (Tohoku University)<br>**Chair:** Fei Yang<br>**Venue:** Auditorium |
| 11:40-12.10 | **INVITED TALK 4**<br>Ratio of Hypergeometric Type Functions in Function Spaces<br>**Speaker:** Swaminathan Anbhu (Indian Institute of Technology Roorkee)<br>**Chair:** Milutin Obradovic<br>**Venue:** Auditorium |
| 12:10-14:00 | **LUNCH BREAK**<br>**Venue:** ELL Room |
| 14:00-16:50 | **CONTRIBUTED PRESENTATION 1**<br>**Venue:** Auditorium |
| 16:50-17:20 | **TEA BREAK**<br>**Venue:** ELL Room |

## CONTRIBUTED PRESENTATION 1

**Session Chair: Weiwei Cui**

| | |
|---|---|
| 14:00-14:20 | Fejer-Riesz Composition Operators<br>**Speaker:** Boo Rim Choe (Korea University) |
| 14:25-14:45 | On the Size of the Fourier Coefficients<br>**Speaker:** Mohammad Sababheh (Abdullah Al Salem University) |
| 14:50-15:10 | Exploring the Bounds and Geometric Aspect of $q$-Hypergeometric Functions via the $q$-Hohlov Operator<br>**Speaker:** Bilal Khan (Henan Academy of Sciences) |
| 15:15-15:35 | Singularity Analysis of Non-linear Partial Differential Equations and their Application Towards the Solution of the Coupled Ramani Equation<br>**Speaker:** Rajeswari Seshadri (Pondicherry University) |
| 15:40-16:00 | Devaney's Chaos in Non-Autonomous Discrete Dynamical Systems<br>**Speaker:** Mohammad Salman (Shyama Prasad Mukherjee College) |
| 16:05-16:25 | Radii Of Convexity Associated with Various Subclasses of Analytic Functions for Functions Related to Kaplan Classes<br>**Speaker:** Nisha Bohra (Sri Venkateswara College) |
| 16:30-16:50 | Exponential Radii of Starlikeness and Convexity of Some Special Functions<br>**Speaker:** Adiba Naz (University of Delhi) |

# CONFERENCE SCHEDULE DAY 3
# 22ND AUGUST 2025

| | |
|---|---|
| 9:00-9:50 | **PLENARY TALK 6**<br>New Results in $H^P$ theory for Quasiregular Mappings<br>**Speaker:** Vesna Todorcevic (Mathematical Institute SANU, Serbia)<br>**Chair:** Aihua Fan<br>**Venue:** Auditorium |
| 10:00-10:30 | **INVITED TALK 5**<br>A Naive Generalization of the Hyperbolic and the Quasihyperbolic Metrics<br>**Speaker:** Swadesh Kumar Sahoo (Indian Institute of Technology Indore)<br>**Chair:** Diganta Borah<br>**Venue:** Auditorium |
| 10:30-10:45 | **COFFEE BREAK**<br>**Venue:** ELL Room |
| 10:45-11:35 | **PLENARY TALK 7**<br>Some Interactions of Ergodic Theory, Number Theory and Harmonic Analysis<br>**Speaker:** Aihua Fan (Picardie University/Wuhan University)<br>**Chair:** Vesna Todorcevic<br>**Venue:** Auditorium |
| 11:40-12.10 | **INVITED TALK 6**<br>Limits of an Increasing Sequence of Riemann Surfaces<br>**Speaker:** Diganta Borah (Indian Institute of Science Education and Research Pune)<br>**Chair:** Swadesh Kumar Sahoo<br>**Venue:** Auditorium |
| 12:10-14:00 | **LUNCH BREAK**<br>**Venue:** ELL Room |
| | **FREE AND EASY** |

# CONFERENCE SCHEDULE DAY 4
# 23RD AUGUST 2025

| | |
|---|---|
| 9:00-9:30 | **INVITED TALK 7**<br>Collet-Eckmann Maps in the Unicritical Family<br>**Speaker:** Weiwei Cui (Shandong University)<br>**Chair:** Paweł Zaprawa<br>**Venue:** Auditorium |
| 9:35-10:05 | **INVITED TALK 8**<br>On Certain $(p, q)$-Extended Hypergeometric Type Higher Transcendental Functions<br>**Speaker:** Rakesh Kumar Parmar (Pondicherry University)<br>**Chair:** Boo Rim Choe<br>**Venue:** Auditorium |
| 10:05-10:20 | **COFFEE BREAK**<br>**Venue:** ELL Room |
| 10:20-12:20 | **CONTRIBUTED PRESENTATION 2**<br>**Venue:** Auditorium |
| 12:10-14:00 | **LUNCH BREAK**<br>**Venue:** ELL Room |
| 14:00-16:25 | **CONTRIBUTED PRESENTATION 3**<br>**Venue:** Auditorium |
| 16:25-16:40 | **TEA BREAK**<br>**Venue:** ELL Room |
| 18:00-22:00 | **CONFERENCE DINNER**<br>**Venue:** The Tamarra by Irama Dining |

## CONTRIBUTED PRESENTATION 2

**Session Chair: Sung-Ho Kim**

10:20-10:40 Bank-Laine Functions with Preassigned Number of Zeros
**Speaker:** Yueyang Zhang (University of Science and Technology, Beijing)

10:45-11:05 Minimal Surfaces in $\mathbb{R}^3$ and Planar Harmonic Mappings
**Speaker:** Zhen Chuan Ng (Universiti Sains Malaysia)

11:10-11:30 Coefficient Invariances for Non-Convex Functions
**Speaker:** Paweł Zaprawa (Lublin University of Technology)

11:35-11:55 Normality from One Family of Meromorphic Functions to Another
**Speaker:** Manish Kumar (University of Jammu)

12:00-12:20 Planar Harmonic Mapping Associated with Four Parameter Wright Function
**Speaker:** Anish Kumar (Dr. Shyama Prasad Mukherjee University)

## CONTRIBUTED PRESENTATION 3

**Session Chair: Rakesh Kumar Parmar**

14:00-14:20 Initial Successive Coefficients for Certain Classes of Univalent Functions
**Speaker:** Vibhuti Arora (National Institute of Technology Calicut)

14:25-14:45 Asymptotic Value of the Multidimensional Bohr Radius
**Speaker:** Shankey Kumar (Indian Institute of Technology Madras)

14:50-15:10 Dynamics of Chebyshev's Method for Exponential Maps
**Speaker:** Pooja Phogat (Indian Institute of Technology Bhubaneswar)

15:15-15:35 On The Dynamics of Newton's Method Applied to Rational Functions
**Speaker:** Soumen Pal (Indian Institute of Technology Madras)

15:40-16:00 On The Zeros of Some Complex Harmonic Polynomials
**Speaker:** Sarika Verma (University of Jammu)

16:05-16:25 Conjugate Function Theorems for Harmonic Quasiregular Mappings
**Speaker:** Suman Das (Guangdong Technion)

# CONFERENCE SCHEDULE DAY 5
# 24TH AUGUST 2025

| | |
|---|---|
| 9:00-11:25 | **CONTRIBUTED PRESENTATION 4**<br>**Venue:** Auditorium |
| 11:25-11:30 | **CLOSING REMARKS**<br>**Venue:** Auditorium |
| 11:30-13:00 | **BRUNCH BREAK**<br>**Venue:** ELL Room |

## CONTRIBUTED PRESENTATION 4

**Session Chair: Chandrashekar Ramasamy**

| | |
|---|---|
| 9:00-9:20 | Bounds and Asymptotic Expansions for the Radii of Convexity and Uniform Convexity of Normalized Bessel Functions<br>**Speaker:** Pranav Kumar (Indian Institute of Technology Madras) |
| 9:25-9:45 | Monotone Coefficients of Univalent Harmonic Functions with Applications to Special Functions<br>**Speaker:** Sheetal Wankhede (Indian Institute of Technology Indore) |
| 9:50-10:10 | Length Distortion of Curves Under Meromorphic Univalent Mappings<br>**Speaker:** Deblina Maity (Indian Institute of Technology Kharagpur) |
| 10:15-10:35 | On Univalence Criteria and Quasiconformal Extensions<br>**Speaker:** Xiaoyuan Wang (Liaocheng University) |
| 10:40-11:00 | Application of Markov Properties for Graph Merging<br>**Speaker:** Sung-Ho Kim (Korea Advanced Institute of Science and Technology) |

# Plenary Talks

## Nevanlinna Currents and Entire Curves in Certain Projective Hypersurfaces

**Patrick, Tuen Wai NG,** Department of Mathematics, The University of Hong Kong
E-mail: ntw@maths.hku.hk

In this talk, we will first introduce the basics of Nevanlinna currents. We will then outline a new approach based on Nevanlinna currents to study entire curves on some specific hypersurfaces in the complex projective spaces.

This is a joint work with Sai-Kee Yeung.

## Coefficient Estimates of Various Subclasses of Analytic Univalent Functions And Operators

**Maslina Darus,** Department of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia
E-mail: maslina@ukm.edu.my

Complex function theory has been a remarkable field of study for decades. It deals with the transformation of a unique function mapping from one domain to another. The history of the theory is full of enriching results of a one-to-one function (univalent) denoted by $S$ and defined in the open unit disc. It started in 1904 when Koebe introduced the largest univalent function given by $f(z) = z/(1-z)^2$ maps the open unit disc conformally onto the complex plane except at $-1/4$ to $-\infty$. Without loss of generality, we can normalise $f$ and write $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. Later, Bieberbach in 1916 conjectured that the coefficient estimate of the Koebe function was $|a_n| \leq n$ for all $n \geq 2$. In 1984, De Branges proved the conjecture and later known as de Branges's Theorem. As time goes by, numerous problems are innovated, generalised and solved. In this talk, some work on analytic and univalent functions will be highlighted. These include operators and some orthogonal polynomials.

## Quantization of the Theory of Topological Insulators

**Armen Sergeev,** Steklov Mathematical Institute, Moscow
E-mail: sergeev@mi-ras.ru

After the discovery of the quantum Hall effect by von Klitzing in 1980 and its topological explanation proposed in the papers by Laghlin and Thouless et al. the mathematical methods, based on the topology and K-theory of $C^*$-algebras, entered firmly into the arsenal of solid state physics. In our work we pay main attention to the class of solid bodies called the topological insulators. They are characterized by having a broad energy gap stable under small deformations which motivates the usage of topological methods for their study. A key role in the solid state theory is played by the investigation of their symmetry groups. Kitaev has pointed out that the symmetry algebras of topological insulators belong to the class of Clifford algebras. According to this observation the quantization of topological insulators should reduce to the theory of irreducible representations of Clifford algebras. The next important step was done by Kennedy and Zirnbauer who introduced the notion of pseudosymmetries. The observables, given by Hamiltonians of topological insulators, correspond to the quantum observables, given by the complex structures on the Nambu space of observables. The original observables, satisfying the commutation relations with the symmetries of topological insulators, will correspond to the quantum observables, satisfying the anticommutation relations with pseudosymmetries, determined by the representation of the Clifford algebra.

The study was carried out with the financial support of the Ministry of Science and Higher Education of the Russian Federation in the framework of a scientific project under agreement No. 075-15-2024-631.

## Near-parabolic Renormalization in Holomorphic Dynamics

**Fei Yang,** School of Mathematics, Nanjing University, China
E-mail: yangfei@nju.edu.cn

In 2008, Inou and Shishikura introduced a class of holomorphic maps which is invariant under the action of parabolic and near-parabolic renormalization operators. As one of the essential ingredients in their proof, Buff and Chéritat used it to prove the existence of quadratic Julia sets with positive area (Ann. Math., 2012). Since then, some other important progresses on the dynamics of quadratic polynomials have been harvested.For examples, the Marmi-Moussa-Yoccoz conjecture for high type rotation numbers by Cheraghi- Chéritat (Invent. Math., 2015); the existence of Feigenbaum Julia sets with positive area by Avila-Lyubich (Ann. Math., 2022); the local connectivity of the Mandelbrot set at some infinitely satellite renormalization points by Cheraghi-Shishikura (arXiv, 2015) etc. In this talk I will review some recent developments in this area.

## Totally Monotone Sequences, Pick Functions and Hypergeometric Functions

**Toshiyuki Sugawa,** GSIS, Tohoku University, Japan
E-mail: sugawa@tohoku.ac.jp

A sequence of real numbers $c_0, c_1, c_2, \dots$ is called totally monotone (or completely monotone) if $\Delta^m c_n \geq 0$ for all $m, n \geq 0$, where $\Delta^m c_n$ are defined inductively in $m$ by $\Delta^0 c_n = c_n$ and $\Delta^{m+1} c_n = \Delta^m c_n - \Delta^m c_{n+1}$ for $n \geq 0$. Hausdorff showed in 1921 that a sequence $c_0, c_1, c_2, \dots$ with $c_0 = 1$ is totally monotone if and only if there exists a Borel probability measure $\mu$ on the interval $[0, 1]$ such that

$$c_n = \int_0^1 t^n d\mu(t), \quad n = 0, 1, 2, \dots.$$

Therefore, totally monotone sequences are also called Hausdorff moment sequences. The generating function of such a sequence is expressed by

$$F(z) = \sum_{n=0}^{\infty} c_n z^n = \sum_{n=0}^{\infty} \int_0^1 t^n z^n d\mu(t) = \int_0^1 \frac{d\mu(t)}{1 - tz}.$$

By the above integral representation, we observe that the function $F$ is analytically continued to the slit domain $\Lambda = \mathbb{C} \setminus [1, +\infty)$. In particular, $F$ is a Pick function. Conversely, a Pick function on $\Lambda$ is known to be represented as the generating function of a totally monotone sequence under some conditions. In this way, we have a criterion for a given sequence to be totally monotone.

As an application of such an approach, we give a necessary and sufficient condition for the coefficients $(a)_n(b)_n/(c)_n n!$ of the hypergeometric function to be totally monotone. As is immediately recognized, the necessity part is hard to prove by merely using the definition of total monotonicity.

This talk is based on joint work with Li-Mei Wang (UIBE, China).

## New Results in $H^p$-theory for Quasiregular Mappings

**Vesna Todorčević,** Mathematical Institute, Serbian Academy of Sciences and Arts
E-mail: `gligorovska0@gmail.com`

We present some new developments in the $H^p$-theory for quasiregular mappings in space.

In particular, we will consider conditions on multiplicity growth and omitted sets in teh codomain and their consequences.

## Some Interactions of Ergodic Theory, Number Theory and Harmonic Analysis

**Aihua Fan,** Department of Mathematics, University of Picardie, France
E-mail: `ai-hua.fan@@u-picardie.fr`

We shall present some interesting interactions of Ergodic theory, Number theory and Harmonic analysis. The following topics will be concerned: convergence of ergodic Hilbert transform and even of general ergodic series, Kronecker and Weyl theorems in Diophantine approximation and representation of quasi-periodic functions by periodic functions. As we shall see, irrational numbers tie these three topics.

# Invited Talks

## Radius Problems in Geometric Function Theory

**V. Ravichandran,** Department of Mathematics, National Institute of Technology, India
E-mail: ravic@nitt.edu, vravi68@gmail.com

Complex function theory has been a remarkable field of study for decades. It deals with the transformation of a unique function mapping from one domain to another. The history of the theory is full of enriching results of a one-to-one function (univalent) denoted by $S$ and defined in the open unit disc. It started in 1904 when Koebe introduced the largest univalent function given by $f(z) = z/(1-z)^2$ maps the open unit disc conformally onto the complex plane except at $-1/4$ to $-\infty$. Without loss of generality, we can normalise $f$ and write $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. Later, Bieberbach in 1916 conjectured that the coefficient estimate of the Koebe function was $|a_n| \leq n$ for all $n \geq 2$. In 1984, De Branges proved the conjecture and later known as de Branges's Theorem. As time goes by, numerous problems are innovated, generalised and solved. In this talk, some work on analytic and univalent functions will be highlighted. These include operators and some orthogonal polynomials.

## Equidistribution of Fekete Points and its Speed Convergence on Projective Manifolds

**Ngoc Cuong Nguyen,** Korea Advanced Institute of Science & Technology
E-mail: cuongnn@kaist.ac.kr

We survey recent developments on the speed of convergence of Fekete points on projective manifolds where the equidistribution was proved by Berman and Boucksom (2011). In particular, the convergence speed can be obtained for a large class of polynomially cuspidal compact sets introduced by Pawłucki and Pleśniak (1988). These sets are $(\mathscr{C}^{\alpha}, \mathscr{C}^{\alpha'})$-regular in the sense of Dinh, Ma and Nguyen (2017). This based partially on a joint work with Hyunsoo Ahn.

## Grunsky Coefficients and Some Applications in the Theory of Univalent Functions

**Milutin Obradović,** Faculty of Civil Engineering, University of Belgrade, Serbia
E-mail: obrad@grf.bg.ac.rs

Let $\mathcal{A}$ be the class of functions $f$ which are analytic in the open unit disc $D$ of the form

$$f(z) = z + a_2z^2 + a_3z^3 + \cdots,$$

and let $\mathcal{S}$ be the subclass of $\mathcal{A}$ consisting of functions that are univalent in $D$ .

Let $f \in \mathcal{S}$ and let

$$\log \frac{f(t) - f(z)}{t - z} = \sum_{p,q=0}^{\infty} \omega_{p,q} t^p z^q,$$

where $\omega_{p,q}$ are called Grunsky's coefficients with property $\omega_{p,q} = \omega_{q,p}$ (see Duren's or Lebedev's book).

Using certain properties of these coefficients in the theory of univalent functions, we can obtain new results, as well as the improved version of the previous results.

This is a joint work with N.Tuneski.

## Ratio of Hypergeometric Type Functions in Function Spaces

**A. Swaminathan,** Department of Mathematics, I.I.T. Roorkee, India
E-mail: a.swaminathan@ma.iitr.ac.in

Extremal functions of many subclasses of univalent function theory have the hypergeometric type representation. Hence the role of hypergeometric functions in determining the theory of classes of univalent functions in well known. In this talk, the role of ratio of hypergeometric type functions and their q-analogues in determining certain properties of functions in various function spaces will be underlined. Recent attention is finding the behaviour of such ratios in determining certain properties of orthogonal polynomials in the Hilbert space. Open problems and directions for future research will be outlined.

## A Naive Generalization of the Hyperbolic and the Quasihyperbolic Metrics

**Swadesh Kumar Sahoo,** Department of Mathematics, Indian Institute of Technology Indore, India
E-mail: swadesh.sahoo@iiti.ac.in

Although the hyperbolic metric possesses many interesting properties, it is not defined on arbitrary subdomains of $\mathbb{R}^n$, for $n \geq 2$. This talk introduces a hyperbolic-type metric that offers a new approach to addressing this limitation. The proposed metric coincides with the classical hyperbolic metric in the case of a ball and, interestingly, also agrees with the quasihyperbolic metric not only in half-spaces but in all unbounded domains.

Furthermore, we show that the proposed metric is bi-Lipschitz equivalent to the hyperbolic metric in all simply connected planar domains, and to the quasihyperbolic metric in any bounded domain. As a consequence, we establish characterizations of uniform domains and John disks in terms of the proposed metric. We also explore several geometric properties of this metric, including the existence of geodesics and the minimal length of non-trivial closed curves in multiply connected domains.

This is a joint work with Bibekananda Maji and Pritam Naskar.

## Limits of an Increasing Sequence of Riemann Surfaces

**Diganta Borah,** Department of Mathematics, Indian Institute of Science Education and Research (IISER) Pune, India
E-mail: dborah@iiserpune.ac.in

Complex manifolds that can be expressed as increasing unions of biholomorphic images of model domains (such as balls or polydiscs) arise naturally in complex analysis. Characterizing such manifolds in terms of these model domains is a classical question, known as the *union problem*. In this talk, we focus on the case of complex dimension one. Specifically, let $M$ be a Riemann surface that admits an exhaustion by open subsets $M_j$ each biholomorphic to a fixed domain $\Omega \subset \mathbb{C}$. We describe $M$ in terms of $\Omega$ under various assumptions on the boundary components of $\Omega$.

This is joint work with Prachi Mahajan and Jiju Mammen.

## Collet-Eckmann Maps in the Unicritical Family

**Weiwei Cui,** Research Center for Mathematics and Interdisciplinary Sciences, Shandong University, China
E-mail: weiwei.cui@sdu.edu.cn

We study perturbations of Collet-Eckmann maps in the complex unicritical family and show that Collet-Eckmann parameters are Lebesgue density points of the complement of the Multibrot set.

This is a joint work with Magnus Aspenberg and Mats Bylund.

## On Certain $(p, q)$–Extended Hypergeometric Type Higher Transcendental Functions

**Rakesh K. Parmar,** Department of Mathematics, Ramanujan School of Mathematical Sciences, Pondicherry University, India
E-mail: rakeshparmar27@gmail.com

In this present talk, we present certain $(p, q)$–extended hypergeometric type higher transcendental functions such as Beta function, Gauss's hypergeometric function, Kummer's confluent hypergeometric function, Bessel and modified bessel functions, Struve and modified Struve function, Bessel Struve function, elliptic integrals and so on.

Collaboration work with Prof. Tibor K. Pogány and Prof. Junesang Choi

# Contributed Presentations

## Fejér-Riesz Composition Operators

**Boo Rim Choe,** Department of Mathematics, Korea University
E-mail: cbr@korea.ac.kr

We introduce the *Fejér-Riesz composition operator* $F_\varphi$ induced by a holomorphic self-map $\varphi$ of the unit disk $\mathbf{D}$, and defined on the space of holomorphic functions on $\mathbf{D}$ by $F_\varphi f = \chi_{(-1,1)} f \circ \varphi$. Denote by $A^p_\alpha$ the standard weighted Bergman space of holomorphic functions on $\mathbf{D}$ and denote the Hardy space $H^p$ by $A^p_{-1}$. We study the operators $F_\varphi : A^p_\alpha \to L^p(m_\alpha)$, where $m_\alpha$ is the weighted Lebesgue measure $dm_\alpha(x) = (1-x^2)^{\alpha+1}dx$, $x \in (-1,1)$. For $0 < p < \infty$ and $\alpha \geq -1$, every such $F_\varphi$ is bounded, which in the case $\alpha = -1$ is related to the classical Fejér-Riesz Inequality. We provide characterizations for when $F_\varphi$ is compact and for when $F_\varphi - F_\psi$ is compact. Our characterizations reveal a new phenomenon which is not the case for ordinary composition operators.

This presentation is based on a recent joint work with Hyungwoon Koo and Wayne Smith.

## On the Size of the Fourier Coefficients

**Mohammad Sababheh,** Department of Mathematics, Abdullah Al Salem University, Kuwait. Princess Sumaya University for Technology, Jordan
E-mail: mohammad.sababheh@aasu.edu.kw, sababheh@psut.edu.jo

Fourier series has been an important tool for investigating many phenomenon in real and complex analysis, with a wide range of applications in many fields.

A key concern in this area is the investigation of the size of the Fourier coefficients, which affect convergence of the Fourier series.

In this paper, we explore this concern by providing some partial results related to an open problem in this regard.

## Exploring the Bounds and Geometric Aspect of $q$-Hypergeometric Functions via the $q$-Hohlov Operator

**Bilal Khan,** Institute of Mathematics, Henan Academy of Sciences, Henan, China
E-mail: `bilalmaths789@gmail.com`

This work investigates the hypergeometric form of an analytic function under the subordination of the $q$-Hohlov operators in the open unit disc and examines the related bounds. We derive bounds for two distinct classes and of $q$-Hohlov operators, presenting the results both mathematically and graphically. The rate of convergence of the $q$-analytic function and classical function are analyzed graphically, highlighting key differences in their behavior. In addition, several corollaries are discussed, providing further insight and elaborating derived bounds. Graphical representations of the bounds offer a clear visualization of their boundaries, enriching the understanding of these results within the context of the open unit disk.

This is a joint work with Muhammad Uzair Shah and Serkan Araci.

## Lie Symmetry and Singularity Analysis of Non linear Partial Differential Equation governing Ramani Equation

**Rajeswari Seshadri,** Department of Mathematics, Pondicherry University, INDIA
E-mail: `seshadrirajeswari@pondiuni.ac.in`

Nonlinear evolution equations which describes the several important physical phenomena has been a great source of study to understand their behaviour. The propagation of waves in the plasma medium is modelled as a sixth order nonlinear partial differential equation known as Ramani Equation. In this talk, we present a detailed study of Lie point symmetries, similarity reductions, and possible closed-form solutions to the Ramani equation. For the symmetry-reduced ODEs with no point symmetries, singularity analysis is carried out to obtain its corresponding Laurent series solution.

This research work is supported by the grant CRG/2023/005418 to the author from ANRF, Govt. of India. This research is partially supported by the DST-FIST grant SR/FST/MS-I/2024/173 to the Department of Mathematics, Pondicherry University, Puducherry, India. This is a joint work with Sherin Agnus, Research Scholar, PU.

## Devaney's Chaos in Non-autonomous Discrete Dynamical Systems

**Mohammad Salman,** Shyama Prasad Mukherji College, Department of Mathematics, University of Delhi, India
E-mail: salman25july@gmail.com

A non-autonomous system is said to be Devaney chaotic if it is topologically transitive, it has dense set of periodic points and it has sensitive dependence on initial conditions. Banks et al. proved one of the most celebrated results in topological dynamics that topological transitivity and dense periodicity imply sensitivity. We explore this problem in non-autonomous dynamical systems by giving different sufficient conditions under which the result is true.

This is a joint work with Prof. Ruchi Das.

## Radii of Convexity Associated with Various Subclasses of Analytic Functions for Functions Related to Kaplan Classes

**Nisha Bohra,** Department of Mathematics, Sri Venkateswara College, University of Delhi, India
E-mail: nishabohra@svc.ac.in

A normalized analytic function $f$ defined on the open unit disc $\mathbb{D}$ is called Ma-Minda convex if $1 + zf''(z)/f'(z)$ is subordinate to the function $\varphi$. For $0 \leqslant \alpha \leqslant \beta$, the Kaplan class $\mathcal{K}(\alpha, \beta)$ of type $\alpha$ and $\beta$ consists of normalized analytic functions of the form $p^{\alpha} g$ defined on $\mathbb{D}$ where $p$ with $p(0) = 1$ is an analytic function taking values in the right half-plane and $g$ is an analytic function with $g(0) = 1$ satisfying $\mathrm{Re}(zg'(z)/g(z)) > (\alpha - \beta)/2$. For functions $f$ with $f' \in \mathcal{K}(\alpha, \beta)$, we obtain the radius of Ma-Minda convexity for various choices of $\varphi$. The radius of lemniscate convexity, lune convexity, nephroid convexity, exponential convexity and several other radius estimates are examined. The results obtained are sharp.

This is a joint work with V. Ravichandran and B.B. Janani.

## Exponential Radii of Starlikeness and Convexity of Some Special Functions

**Adiba Naz,** Department of Mathematics, University of Delhi, India
E-mail: adibanaz81@gmail.com

Using the Hadamard factorization, the exponential radii of starlikeness and convexity for various special functions like Wright function, Lommel function, Struve function, Ramanujan type entire function, cross product and product of Bessel function have been investigated. For certain ranges of the parameters appearing in these special functions, the precise values of the exponential radii of starlikeness and convexity are calculated as the solutions of transcendental equations. The interlacing property of the zeros of special functions and their derivatives is the fundamental technique utilized to demonstrate these results.

This is a joint work with Sumit Nagpal and V. Ravichandran.

## Bank–Laine Functions with Preassigned Number of Zeros

**Yueyang Zhang,** School of Mathematics and Physics, University of Science and Technology Beijing, China
E-mail: zhangyueyang@ustb.edu.cn

A Bank-Laine function $E$ is written as a product of two linearly independent solutions of the second order differential equation $f'' + A(z)f = 0$, where $A(z)$ is an entire function. Let $n \in \mathbb{N}$ be a positive integer. In this report, I will talk about how to modify the method of quasiconformal surgery by Bergweiler and Eremenko to construct entire functions $A$ for which the associated Bank–Laine functions $E = f_1 f_2$ have preassigned exponent of convergence of number of zeros $\lambda(E)$ of three types:

(1) for every two numbers $\lambda_1, \lambda_2 \in [0, n]$ such that $\lambda_1 \leq \lambda_2$, there exists an entire function $A$ of order $\rho(A) = n$ such that $E = f_1 f_2$ satisfies $\lambda(f_1) = \lambda_1$, $\lambda(f_2) = \lambda_2$ and $\lambda(E) = \lambda_2 \leq \rho(E) = n$;

(2) for every number $\rho \in (n/2, n)$ and $\lambda \in [0, \infty)$, there exists an entire function $A$ of order $\rho(A) = \rho$ such that $E = f_1 f_2$ satisfies $\lambda(f_1) = \lambda$, $\lambda(f_2) = \infty$ and, moreover, $E_c = f_1(cf_1 + f_2)$ satisfies $\lambda(E_c) = \infty$ for any constant $c$;

(3) for every number $\lambda \in [0, n]$, there exists an entire function $A$ of order $\rho(A) = n$ such that $E = f_1 f_2$ satisfies $\lambda(f_1) = \lambda$, $\lambda(f_2) = \infty$ and, moreover, $E_c = f_1(cf_1 + f_2)$ satisfies $\lambda(E_c) = \infty$ for any constant $c$.

I will also talk about how to further change the way of gluing the quasiregular functions along the positive real axis and the negative real axis in the above three constructions to provide a full-fledged theory on the existence of finite order Bank–Laine functions.

## Minimal Surfaces in $\mathbb{R}^3$ and Planar Harmonic Mappings

**Zhen Chuan Ng,** School of Mathematical Sciences, Universiti Sains Malaysia
E-mail: `zhenchuanng@usm.my`

In this talk, we apply the shear constructions to develop families of univalent harmonic functions that can be lifted to some minimal surfaces. This generalizes certain results in Dorff and Muir (2014).

## Coefficient Invariances for Non-convex Functions

**Paweł Zaprawa,** Department of Mathematics, Lublin University of Technology, Poland
E-mail: `p.zaprawa@pollub.pl`

In recent years a number of problems involving coefficient functionals have been considered. It is observed that within the class $\mathcal{K}$ of analytic convex functions, some functionals defined on the coefficients of functions $f \in \mathcal{K}$ and on the coefficients of inverse functions $f^{-1}$ have the same sharp bounds. This phenomenon also occurs for some proper subclasses of $\mathcal{K}$, especially for $\mathcal{K}_\beta$ the class of strongly convex functions of order $\beta$.

In this presentation we discuss similar problems, but they are considered for non-convex functions.

## Normality from One Family of Meromorphic Functions to Another

**Manish Kumar,** Department of Mathematics, University of Jammu, India
E-mail: manishbarmaan@gmail.com

The study of normal families of meromorphic functions under various sharing hypotheses dates back to Montel's classical results and has since been developed extensively, with applications in both complex dynamics and Nevanlinna value distribution theory. In this talk, we consider two families, $\mathcal{G}$ and $\mathcal{F}$, of meromorphic functions and show that the normality of $\mathcal{G}$ on a domain $D$ implies the normality of $\mathcal{F}$ on the same domain, provided that the members of these families satisfy certain sharing conditions.

This is a joint work with Kuldeep Singh Charak and Rahul Kumar.

## Planar Harmonic Mappings Associated with Four-Parameter Wright Functions

**Anish Kumar,** Department of Mathematics, DSPMU, Ranchi, India
E-mail: ak8107690@gmail.com

Harmonic functions play a vital role in several problems in applied mathematics and are also important because of their use in the minimal surface. A continuous complex value mapping $f = u + iv$ is said to be harmonic in a domain $D$ connected simply in the complex plane if it holds $f_{z\bar{z}} = 0$ in $D$, that is, $u$ and $v$ are real harmonic functions in $D$.

The primary aim of this work is to construct harmonic mapping associated with four parameter Wright functions as follows: Let

$$W_{(\alpha_1,\beta_1),(\gamma_1,\delta_1)}(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(\alpha_1)\Gamma(\gamma_1)A_n z^n}{\Gamma(\alpha_1 + (n-1)\beta_1)\Gamma(\gamma_1 + (n-1)\delta_1)}$$

and

$$W_{(\alpha_2,\beta_2),(\gamma_2,\delta_2)}(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(\alpha_2)\Gamma(\gamma_2)B_n z^n}{\Gamma(\alpha_2 + (n-1)\beta_2)\Gamma(\gamma_2 + (n-1)\delta_2)}.$$

Now define a linear convolution operator corresponding to $f = h + \bar{g}$
$L : \hat{H} \to \hat{H}$ as:

$$\begin{aligned} L(f) &= f * (W_{(\alpha_1,\beta_1),(\gamma_1,\delta_1)}(z) + \sigma \overline{W_{(\alpha_2,\beta_2),(\gamma_2,\delta_2)}(z)}) \\ &= H(z) + \sigma\bar{G}(z) = h * W_{(\alpha_1,\beta_1),(\gamma_1,\delta_1)}(z) + \sigma \overline{g * W_{(\alpha_2,\beta_2),(\gamma_2,\delta_2)}(z)}, \quad |\sigma| < 1, \end{aligned}$$

where

$$\begin{aligned} H(z) &= z + \sum_{n=2}^{\infty} \frac{\Gamma(\alpha_1)\Gamma(\gamma_1)A_n z^n}{\Gamma(\alpha_1 + (n-1)\beta_1)\Gamma(\gamma_1 + (n-1)\delta_1)}, \\ G(z) &= \sum_{n=1}^{\infty} \frac{\Gamma(\alpha_2)\Gamma(\gamma_2)B_n z^n}{\Gamma(\alpha_2 + (n-1)\beta_2)\Gamma(\gamma_2 + (n-1)\delta_2)}. \end{aligned}$$

Further sufficient conditions have been established so that this harmonic mapping satisfies geometric properties such as harmonic starlikeness and convexity. Moreover, we examine relation between several subclasses of family of harmonic close-to-convexity mappings. Several consequences has been obatined conjuction with derived results.

## Initial Successive Coefficients for Certain Classes of Univalent Functions

**Vibhuti Arora,** Department of Mathematics, National Institute of Technology Calicut, India
E-mail: vibhuti@nitc.ac.in

We consider functions of the type $f(z) = z + a_2 z^2 + a_3 z^3 + \cdots$ from a family of all analytic and univalent functions in the unit disk. The aim is to investigate the bounds of the difference of moduli of initial successive coefficients, *i.e.* $\big||a_{n+1}| - |a_n|\big|$ for $n = 1, 2$ and for some subclasses of analytic univalent functions. In addition, we found that all the estimations are sharp in nature by constructing some extremal functions.

## Bounds on Mixed Bohr Radii of Vector-valued Holomorphic Functions on Banach Spaces

**Shankey Kumar,** Department of Mathematics, Indian Institute of Technology Madras, India
E-mail: shankeygarg93@gmail.com

This article is motivated by the concepts of mixed Bohr radius for scalar-valued functions defined in Banach sequence spaces. More precisely, it aims to determine bounds of mixed Bohr radii for holomorphic functions defined on Banach sequence spaces with values in Banach spaces. We determine upper bound of the mixed Bohr radius by establishing a connection between the mixed Bohr radius and the arithmetic Bohr radius. However, the lower bound is obtained through the implementation of techniques developed recently by Defant, Galicer, Maestre, Mansilla, Muro, and Schwarting.

This is a joint work with Prof. S. Ponnusamy.

## Dynamics of Chebyshev's Method for Exponential Maps

**Pooja Phogat,** Department of Mathematics, IIT Bhubaneswar, India
E-mail: pj18@iitbbs.ac.in

Finding roots of a function is a classical problem in the mathematical sciences. Numerous root-finding methods, often expressed as rational maps, are developed to address this problem. The process mainly depends on the iterations of the respective rational map. For a non-constant entire function $f$, the Chebyshev's method $C_f$ is defined as $C_f(z) = z - \left(1 + \frac{1}{2}\frac{f(z)f''(z)}{(f'(z))^2}\right)\frac{f(z)}{f'(z)}$. Note that, for every affine map $T$, we have $C_f = T \circ C_{\lambda f \circ T} \circ T^{-1}$, where $\lambda$ is a nonzero complex number. This property is called Scaling property. We characterize the Chebyshev's method applied to entire functions by proving that for a non-constant entire function $f$, $C_f$ is a rational map if and only if $f(z) = p(z)e^{q(z)}$, where $p$ and $q$ are polynomials. We consider $g(z) = p(z)e^{\lambda[p(z)]^n + c}$, where $p$ is a linear polynomial and $\lambda, c$ are complex numbers where $\lambda \neq 0$. Then we explore the dynamics of $C_g$. Using the Scaling property it is proved that $C_g$ is affine conjugate to $C_n$, where $C_n$ is the Chebyshev's method applied to $ze^{z^n}$. It is shown that all fixed points of $C_n$, other than 0 and $\infty$, are repelling.

We prove that the Julia set $\mathcal{J}(C_1)$ is connected, and for $n \geq 2$, the connectedness of $\mathcal{J}(C_n)$ is proved under certain assumptions. For $n$ even, the non-existence of a rotation domain (a Herman ring or a Siegel disk) of $C_n$ is proved. The same is proved for odd $n$ under some additional hypothesis. We also explores the Euclidean isometries that preserve $\mathcal{J}(C_n)$. More precisely, we proved that for $n \geq 2$, the rotations about origin of order $n$ preserves $\mathcal{J}(C_n)$.

## On the Dynamics of Newton's Method Applied to Rational Functions

**Soumen Pal,** Department of Mathematics, IIT Madras, India
E-mail: ma24r007@smail.iitm.ac.in

For a polynomial $p$ having at least two distinct roots, the Newton's method applied to $p$ is defined by $N_p(z) = z - \frac{p(z)}{p'(z)}$. It is a classical root-finding method whose dynamics is extensively studied. A notable property of $N_p$ is that the point at $\infty$ is the only extraneous fixed point, i.e., a fixed point of $N_p$ which is not a root of $p$. This property ensures that the Julia set of $N_p$ is always connected. In contrast, when Newton's method is applied to a rational function, the dynamics are considerably more intricate, and the connectivity of the associated Julia set is no longer guaranteed.

Suppose $R = \frac{P}{Q}$ is a rational function, where $P$ and $Q$ are polynomials without having any common root. The Newton's method applied to $R$ is defined by $N_R(z) = z - \frac{R(z)}{R'(z)}$. We call a such map as Newton's map. The degree of $N_R$ depends on the number of distinct roots and poles of $R$, as well as on $\deg(P)$ and $\deg(Q)$. We completely describe the dynamics of all quadratic Newton maps of rational functions. The Julia set of such a map is found to be either a Jordan curve or totally disconnected. Furthermore, in the cubic case, we prove that if $N_R$ is conjugate to a polynomial then its Julia set is connected. We also construct two distinct families of polynomials whose associated Newton maps have totally disconnected Julia sets.

## On the Zeros of Some Complex Harmonic Polynomials

**Sarika Verma,** Department of Mathematics, University of Jammu, India
E-mail: `sarika.16984@gmail.com`

It is well known that the *Fundamental Theorem of Algebra* doesn't hold true in the case of complex harmonic polynomials. Although researchers have made progress in counting and locating the zeros of complex harmonic polynomials, they have considered only sparse polynomials. We introduce a special class of non-sparse polynomials $F(z)$ of degree $(n+m)$. In particular, we prove that the sum of the orders of the zeros of $F(z)$ is $-(n+m)$. We also show that $F(z)$ has a total $(n+m+2)$ or $(n+m+2k+2)$ number of zeros under different conditions on the coefficients. In addition to the count of zeros, we locate the zeros of $F(z)$ and obtain that all non-trivial zeros of $F(z)$ lie in the given annular region.

This is a joint work with Adithya Mayya, Raj Kumar and Kuncham Syam Prasad.

## Conjugate Function Theorems for Harmonic Quasiregular Mappings

**Suman Das,** Guangdong Technion, Shantou, China
E-mail: `suman.das@gtiit.edu.cn`

A celebrated theorem of M. Riesz says that if $f = u + iv$ is analytic in the unit disk $\mathbb{D}$, and $u$ is in the Hardy space $h^p$ for some $p > 1$, then $v \in h^p$. The theorem fails for $p = 1$ and $p = \infty$. However, another theorem of Kolmogorov ensures that the conjugate of an $h^1$-function does belong to $h^p$ for all $p < 1$. Finally, Zygmund's theorem gives the "minimal" growth restriction on $u$ which implies $v \in h^1$. Clearly, these results do not hold if $f = u + iv$ is a complex-valued harmonic function. Remarkably, Liu and Zhu (Adv. Math., 2023) established that Riesz's theorem holds for harmonic functions if one assumes, in addition, that $f$ is quasiregular in $\mathbb{D}$. In this talk, we present recent developments along this line, for harmonic quasiregular mappings. In particular, we show that the integral means $M_p(r, u)$ and $M_p(r, v)$ have the same "order of infinity" for all $p \in (0, \infty]$, as well as that $u$ and $v$ have the same degree of smoothness on the boundary.

Joint work with Antti Rasila.

## Bounds and Asymptotic Expansions for the Radii of Convexity and Uniform Convexity of Normalized Bessel Functions

**Pranav Kumar,** Department of Mathematics, Indian Institute of Technology Madras, India
E-mail: pranavarajchauhan@gmail.com

Let $r^c(g_\nu)$ and $r^{uc}(g_\nu)$ denote the radii of convexity and uniform convexity, respectively, for normalized Bessel functions

$$z \mapsto g_\nu(z) = 2^\nu \Gamma(\nu+1) z^{1-\nu} J_\nu(z).$$

In this talk we explore, the asymptotic behavior of $r^c(g_\nu)$ and $r^{uc}(g_\nu)$ with respect to large order $\nu$. We provide detailed asymptotic expansions for these radii and establish recurrence relations for the associated coefficients. In this investigation, the asymptotic inversion method serves as the primary analytical tool, combined with Rayleigh sum techniques to establish the requisite estimates. Similar results are also discussed for another normalized Bessel functions: $z \mapsto h_\nu(z) = 2^\nu \Gamma(\nu+1) z^{1-\frac{\nu}{2}} J_\nu(\sqrt{z})$. Generalized bounds for these radii, derived using the Euler-Rayleigh inequality and potential polynomials, provide sharp estimates and asymptotic form of these radii for large $\nu$.

This is a joint work with Á. Baricz, and Sanjeev Singh.

## Monotone Coefficients of Univalent Harmonic Functions with Applications to Special Functions

**Sheetal Wankhede,** Department of Mathematics, IIT Indore, India
E-mail: shitalwankhede1995@gmail.com

Let $\mathcal{A}$ denote the class of all analytic functions $h$ in the unit disk $\mathbb{D}$ normalized by $h(z) = z + \sum_{n=2}^{\infty} a_n z^n$, and $\mathcal{S} \subset \mathcal{A}$ consists of all univalent (one-to-one) functions. The celebrated Bieberbach conjecture, resolved by de Branges in 1985, states that for any $h \in \mathcal{S}$, the coefficients satisfy $|a_n| \leq n$. Motivated by this result, subsequent studies have investigated conditions on the coefficients that guarantee univalence of h. In this direction, Alexander and later MacGregor established monotonicity criteria for univalent analytic functions with real, non-negative coefficients.

In this talk, we extend these ideas to the harmonic setting by considering complex-valued functions of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n + \overline{\sum_{n=2}^{\infty} b_n z^n}$, and investigate coefficient conditions under which such functions are harmonic close-to-convex and hence univalent in $\mathbb{D}$. We focus on monotonicity properties of the sequences $\{n\, a_n\}$ and $\{n\, b_n\}$, and apply these to study the univalence of specific harmonic mappings associated with Gaussian hypergeometric functions.

This is a joint work with Prof. Swadesh Kumar Sahoo.

## Length Distortion of Curves Under Meromorphic Univalent Mappings

**Deblina Maity,** Department of Mathematics, Indian Institute of Technology Kharagpur, India
E-mail: deblinamaity1997@gmail.com

Let $f$ be a conformal (analytic and univalent) map defined on the open unit disk $\mathbb{D}$ of the complex plane $\mathbb{C}$ that remains continuous on the semi-circle $\partial\mathbb{D}^+ = \{z \in \mathbb{C} : |z| = 1, \operatorname{Im} z > 0\}$. The existence of a uniform upper bound for the ratio of the length of the image of the horizontal diameter $(-1, 1)$ to the length of the image of $\partial\mathbb{D}^+$ under $f$ was proved by Gehring and Hayman. We extend this result by introducing a simple pole for $f$ in $\mathbb{D}$ and considering the ratio of the length of the image of the vertical diameter $I = \{z : \operatorname{Re} z = 0; |\operatorname{Im} z| < 1\}$ to the length of the image of the semi-circle $C' = \{z : |z| = 1; \operatorname{Re} z < 0\}$ under such $f$. Finally, we further extend this result by replacing the vertical diameter $I$ with a hyperbolic geodesic symmetric with respect to the real line, and by replacing $C'$ with the corresponding arc of the unit circle passing through the point $-1$.

This is a joint work with Dr. Bappaditya Bhowmik.

## On Univalence Criteria and Quasiconformal Extensions

**Xiaoyuan Wang,** School of Mathematical Sciences, Liaocheng University, China
E-mail: mewangxiaoyuan@163.com

In this talk, I will introduce some univalence and quasiconformal extension criteria. The topic includes Loewner chains, Nehari class, John constant, pre-Schwarzian and Schwarzian derivatives. I will present some new results by using these fundamental tools. Furthermore, this work considers potential extensions of these concepts to broader classes of mappings, such as harmonic mappings, log-harmornic mappings, bi-harmonic mappings, $\alpha$-harmonic mappings. Selected generalizations will be introduced and discussed in the context of these new developments.

## Application of Markov Properties for Graph Merging

**Sung-Ho Kim,** Department of Mathematical Sciences, Korea Advanced Institute of Science and Technology, Korea
E-mail: sungkim@kaist.ac.kr

Let $G_1$ and $G_2$ be undirected graphs. The nodes in $G_i$ represent the random variables involved in probability model $M_i$, and $G_i$ represents Markov properties lying in $M_i$.

Let $V_i$ be the node set of $G_i$ and $V = V_1 \cup V_2$. Denote by $\mathbf{X}_V$ the random vector corresponding to the nodes in $V$. Suppose we want to construct a graph $G$ which represents Markov properties lying in the probability model $M$ of $\mathbf{X}_V$. It is not possible to build $M$ unless a data set of $\mathbf{X}_V$ is available. We can however find a set $\mathcal{H}$ of graphs for the node set $V$ based on $G_1$ and $G_2$ without data, where graph $H \in \mathcal{H}$ satisfies the following:

$$H_{\_V_i} = G_i, \quad \text{for } i = 1, 2.$$

In this equation, $H_{\_V_i}$ is a type of subgraph of $H$ upon $V_i$ such that the Markov properties among the nodes in $V_i$ are the same between $H$ and $H_{\_V_i}$. In this context, we call this subgraph a Markovian subgraph.

In the presentation, we will demonstrate how $\mathcal{H}$ is constructed and show that this method is useful for statistical structure learning.

This work is supported by National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. RS-2024-00336424).

# Sponsors

School of Mathematical Sciences
Universiti Sains Malaysia
11800 USM Penang Malaysia
Tel : +604 653 3284

www.math.usm.my
dean_mat@usm.my
matematikUSM