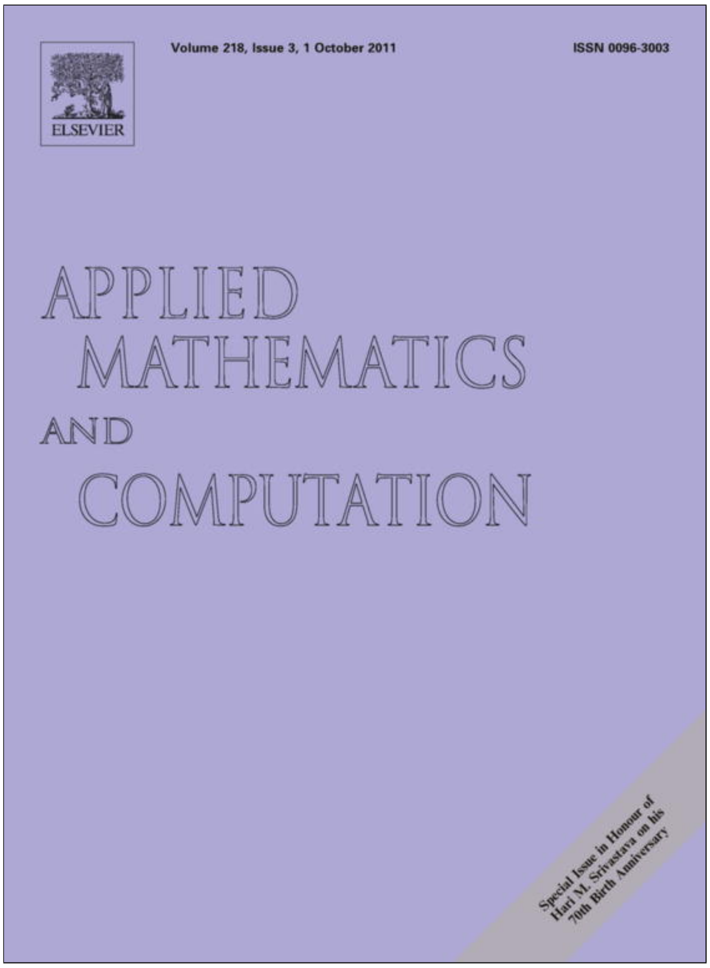
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Convolutions of meromorphic multivalent functions with respect to n-ply points and symmetric conjugate points ☆

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ABSTRACT

It is well-known that the classes of starlike, convex and close-to-convex univalent functions are closed under convolution with convex functions. In this paper, closure properties under convolution of general classes of meromorphic p-valent functions that are either starlike, convex or close-to-convex with respect to n-ply symmetric, conjugate and symmetric conjugate points are investigated.

1. Introduction

Let $\mathcal{H}(\mathbb{D})$ be the set of all analytic functions on the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$, and let $\mathcal{A} \subset \mathcal{H}(\mathbb{D})$ be the subclass of normalized functions of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. An analytic function f is *subordinate* to an analytic function g, written $f(z) \prec g(z)$, if there exists a Schwarz function w analytic in $\mathbb{D}$ with $w(0) = 0$ and $|w(z)| < 1$, satisfying $f(z) = g(w(z))$. In particular, if the function g is univalent in $\mathbb{D}$, then $f(z) \prec g(z)$ is equivalent to $f(0) = g(0)$ and $f(\mathbb{D}) \subset g(\mathbb{D})$. The *convolution* or the *Hadamard product* of two series $f(z) = \sum a_n z^n$ and $g(z) = \sum b_n z^n$ is defined by $(f * g)(z) = \sum a_n b_n z^n$. For a convex function $f \in \mathcal{A}$, it follows from Alexander's theorem that $zf'(z) = f(z) * (z/(1-z)^2)$ is a starlike function. In view of the identity $f(z) = f(z) * (z/(1-z))$, it is evident that the classes of convex and starlike functions can be unified by considering functions f satisfying $f * g$ is starlike for an appropriate fixed function $g \in \mathcal{A}$. Thus convolution and subordination can be used to define a more general class of analytic functions

$$\mathcal{ST}(g,h) := \left\{ f \in \mathcal{A} : \frac{z(f*g)'(z)}{(f*g)(z)} \prec h(z) \right\},$$

where g is a fixed function in $\mathcal{A}$, and h a suitably normalized analytic function with positive real part in $\mathbb{D}$. In particular, let $\mathcal{ST}(h) := \mathcal{ST}(z/(1-z), h)$ and $\mathcal{CV}(h) := \mathcal{ST}(z/(1-z)^2, h)$ are the classes introduced by Ma and Minda [5]. For $h(z) = (1 + (1-2\alpha)z)/(1-z)$, $0 \leqslant \alpha < 1$, $\mathcal{ST}(h)$ and $\mathcal{CV}(h)$ are respectively the familiar classes $\mathcal{ST}(\alpha)$ and $\mathcal{CV}(\alpha)$ of starlike functions of order α, and convex functions of order α.

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Close-to-convex functions are univalent functions; a very simple subclass of such functions are those functions f satisfying $\mathrm{Re} f'(z) > 0$. Other subclasses of close-to-convex functions include the classes of starlike functions with respect to either symmetric points, conjugate, or symmetric conjugate points. A function $f \in \mathcal{A}$ is starlike with respect to symmetric points, conjugate, or symmetric conjugate points in $\mathbb{D}$ if it satisfies respectively the conditions

$$\mathrm{Re}\left(\frac{zf'(z)}{f(z)-f(-z)}\right) > 0, \quad \mathrm{Re}\left(\frac{zf'(z)}{f(z)+\overline{f(\bar{z})}}\right) > 0 \quad \text{and} \quad \mathrm{Re}\left(\frac{zf'(z)}{f(z)-\overline{f(-\bar{z})}}\right) > 0.$$

The class of starlike functions with respect to symmetric points as well as with respect to n-ply symmetric points were introduced by Sakaguchi [10], while El-Ashwah and Thomas [4] investigated the classes of starlike functions with respect to conjugate points and symmetric conjugate points. By using subordination, Ravichandran [8] unified the classes of starlike, convex and close-to-convex functions with respect to n-ply symmetric points, conjugate points and symmetric conjugate points, and obtained several convolution properties. These works were recently extended for multivalent functions by Ali et al. [2].

Though the convolution of two univalent (or starlike) functions need not be univalent, it is well-known [9] that the classes of starlike, convex and close-to-convex functions are closed under convolution with convex functions. By using the convex hull method [9] and the method of differential subordination [7], Shanmugam [11] introduced and investigated convolution properties of various subclasses of analytic functions, whereas Ali et al. [1] and Supramaniam et al. [12] investigated these properties for subclasses of multivalent starlike and convex functions. Similar problems were also investigated for meromorphic functions in [3,6,13]. Motivated by the works in [2,3,6,8,11], in this paper, certain subclasses of meromorphic p-valent functions in the punctured unit disk $\mathbb{D}^* := \{z \in \mathbb{C} : 0 < |z| < 1\}$ defined by means of convolution with a given fixed meromorphic p-valent function is introduced, and their closure properties under convolution are investigated.

2. Meromorphic multivalent functions with respect to n-ply points

Let $\mathcal{M}_p$ denotes the class of all meromorphic p-valent functions of the form

$$f(z) = \frac{1}{z^p} + \sum_{n=1}^{\infty} a_n z^{n-p} \quad (p \geqslant 1), \tag{2.1}$$

that are analytic in the punctured open unit disk $\mathbb{D}^*$. Analogous to classes of starlike and convex analytic functions, classes of meromorphic p-valent starlike and convex functions, and other related subclasses of meromorphic p-valent functions, are expressed in the form

$$\mathcal{MST}_p(g,h) := \left\{ f \in \mathcal{M}_p : -\frac{1}{p}\frac{z(f*g)'(z)}{(f*g)(z)} \prec h(z) \right\},$$

where g is a fixed function in $\mathcal{M}_p$, and h a suitably normalized analytic function with positive real part. For instance, the class of meromorphic p-valent starlike functions of order α, $0 \leqslant \alpha < 1$, defined by

$$\mathcal{MST}_p(\alpha) := \left\{ f \in \mathcal{M}_p : -\mathrm{Re}\frac{1}{p}\frac{zf'(z)}{f(z)} > \alpha \right\},$$

is a particular case of $\mathcal{MST}_p(g,h)$ with $g(z) = 1/(z^p(1-z))$ and $h(z) = (1 + (1-2\alpha)z)/(1-z)$.

In this section, four classes $\mathcal{MST}_p^n(g,h)$, $\mathcal{MCV}_p^n(g,h)$, $\mathcal{MCC}_p^n(g,h)$ and $\mathcal{MQC}_p^n(g,h)$ of meromorphic p-valent functions with respect to n-ply points are introduced and the convolution properties of these new subclasses are investigated. These new subclasses extend the classical classes of meromorphic multivalent starlike, convex, close-to-convex and quasi-convex functions, respectively.

In the sequel, let the function $g \in \mathcal{M}_p$ be fixed, and h be a convex univalent function with positive real part satisfying $h(0) = 1$. Let $n \geqslant 1$ be any integer, $\epsilon^n = 1$ and $\epsilon \neq 1$. For $f \in \mathcal{M}_p$ of the form (2.1), let the function f_n be defined by

$$f_n(z) := \frac{1}{n}\sum_{k=0}^{n-1} \epsilon^{n+pk} f(\epsilon^k z) = z^{-p} + a_{n-p}z^{n-p} + a_{2n-p}z^{2n-p} + \cdots.$$

Definition 2.1. The class $\mathcal{MST}_p^n(h)$ consists of functions $f \in \mathcal{M}_p$ satisfying $f_n(z) \neq 0$ in $\mathbb{D}^*$ and the subordination

$$-\frac{1}{p}\frac{zf'(z)}{f_n(z)} \prec h(z).$$

Similarly, the class $\mathcal{MCV}_p^n(h)$ consists of functions $f \in \mathcal{M}_p$ satisfying $f_n'(z) \neq 0$ in $\mathbb{D}^*$ and the subordination

$$-\frac{1}{p}\frac{(zf'(z))'}{f_n'(z)} \prec h(z).$$

The class $\mathcal{MCC}_p^n(h)$ consists of functions $f \in \mathcal{M}_p$ satisfying the subordination

$$-\frac{1}{p}\frac{zf'(z)}{\phi_n(z)} \prec h(z),$$

for some $\phi \in \mathcal{MST}_p^n(h)$ with $\phi_n(z) \neq 0$ in $\mathbb{D}^*$. The class $\mathcal{MQC}_p^n(h)$ consists of functions $f \in \mathcal{M}_p$ satisfying the subordination

$$-\frac{1}{p}\frac{(zf'(z))'}{\varphi_n'(z)} \prec h(z),$$

for some $\varphi \in \mathcal{MCV}_p^n$ with $\varphi_n'(z) \neq 0$ in $\mathbb{D}^*$. The general classes $\mathcal{MST}_p^n(g,h)$, $\mathcal{MCV}_p^n(g,h)$, $\mathcal{MCC}_p^n(g,h)$ and $\mathcal{MQC}_p^n(g,h)$ consist of functions $f \in \mathcal{M}_p$ for which $f * g$, respectively belongs to $\mathcal{MST}_p^n(h)$, $\mathcal{MCV}_p^n(h)$, $\mathcal{MCC}_p^n(h)$ and $\mathcal{MQC}_p^n(h)$.

If $g(z) := 1/z^p(1-z)$, then the class $\mathcal{MST}_p^n(g,h)$ coincides with $\mathcal{MST}_p^n(h)$, and the class $\mathcal{MCC}_p^n(g,h)$ coincides with $\mathcal{MCC}_p^n(h)$. If $p=1$ and $n=1$, then the classes $\mathcal{MST}_p^n(g,h)$, $\mathcal{MCV}_p^n(g,h)$, $\mathcal{MCC}_p^n(g,h)$ and $\mathcal{MQC}_p^n(g,h)$ reduced, respectively to $\mathcal{MST}(g,h)$, $\mathcal{MCV}(g,h)$, $\mathcal{MCC}(g,h)$ and $\mathcal{MQC}(g,h)$ introduced and investigated in [6]. The notation $\mathcal{MST}_p(h)$ will be used for the class $\mathcal{MST}_p^1(h)$.

For $\alpha < 1$, the class $\mathcal{R}_\alpha$ of prestarlike functions of order α is defined by

$$\mathcal{R}_\alpha := \left\{ f \in \mathcal{A} : f * \frac{z}{(1-z)^{2-2\alpha}} \in \mathcal{ST}(\alpha) \right\},$$

while $\mathcal{R}_1$ consists of $f \in \mathcal{A}$ satisfying $\operatorname{Re} f(z)/z > 1/2$. The well-known result that the classes of starlike functions of order α and convex functions of order α are closed under convolution with prestarlike functions of order α is a consequence of the following:

Theorem 2.1 [9, Theorem 2.4]. *Let $\alpha \leqslant 1$, $\phi \in \mathcal{R}_\alpha$ and $f \in \mathcal{ST}(\alpha)$. Then*

$$\frac{\phi * (Hf)}{\phi * f}(\mathbb{D}) \subset \overline{co}(H(\mathbb{D})),$$

for any analytic function $H \in \mathcal{H}(\mathbb{D})$, where $\overline{co}(H(\mathbb{D}))$ denote the closed convex hull of $H(\mathbb{D})$.

By making use of Theorem 2.1, we prove the following:

Theorem 2.2. *Let h be a convex univalent function satisfying*

$$Reh(z) < 1 + \frac{1-\alpha}{p} \quad (0 \leqslant \alpha < 1),$$

and $\phi \in \mathcal{M}_p$ with $z^{p+1}\phi \in \mathcal{R}_\alpha$.

1. *If $f \in \mathcal{MST}_p^n(g,h)$, then $\phi * f \in \mathcal{MST}_p^n(g,h)$.*
2. *If $f \in \mathcal{MCV}_p^n(g,h)$, then $\phi * f \in \mathcal{MCV}_p^n(g,h)$.*
3. *If $f \in \mathcal{MCC}_p^n(g,h)$ with respect to a function $\varphi \in \mathcal{MST}_p^n(g,h)$, then $\phi * f \in \mathcal{MCC}_p^n(g,h)$ with respect to $\phi * \varphi \in \mathcal{MST}_p^n(g,h)$.*
4. *If $f \in \mathcal{MQC}_p^n(g,h)$ with respect to a function $\varphi \in \mathcal{MCV}_p^n(g,h)$, then $\phi * f \in \mathcal{MQC}_p^n(g,h)$ with respect to $\phi * \varphi \in \mathcal{MCV}_p^n(g,h)$.*

Proof. (1) We first show that if $f \in \mathcal{MST}_p^n(h)$, then $\phi * f \in \mathcal{MST}_p^n(h)$. Let $f \in \mathcal{MST}_p^n(h)$, and define the functions H and ψ by

$$H(z) := -\frac{zf'(z)}{pf_n(z)} \quad \text{and} \quad \psi(z) := z^{p+1}f_n(z).$$

Thus for any fixed $z \in \mathbb{D}$,

$$-\frac{zf'(z)}{pf_n(z)} \in h(\mathbb{D}).$$

Replacing z by $\epsilon^k z$ and using

$$f_n(\epsilon^k z) = \epsilon^{-pk} f_n(z),$$

it follows that

$$-\frac{\epsilon^{k(1+p)} zf'(\epsilon^k z)}{pf_n(z)} \in h(\mathbb{D}).$$

Since $h(\mathbb{D})$ is a convex domain, this yields

$$-\frac{1}{n}\sum_{k=0}^{n-1}\frac{\epsilon^{k(1+p)}zf'(\epsilon^k z)}{pf_n(z)} \in h(\mathbb{D}),$$

and since

$$f_n'(z) := \frac{1}{n}\sum_{k=0}^{n-1}\varepsilon^{k(1+p)}f'(\varepsilon^k z),$$

it follows that

$$-\frac{zf'_n(z)}{pf_n(z)} \in h(\mathbb{D}), \quad \text{or} \quad -\frac{zf'_n(z)}{pf_n(z)} \prec h(z).$$

Hence $f_n \in \mathcal{MST}_p(h)$. Now $\mathrm{Re}\,h(z) < 1 + \frac{1-\alpha}{p}$ yields

$$\mathrm{Re}\frac{z\psi'(z)}{\psi(z)} = \mathrm{Re}\frac{zf'_n}{f_n} + p + 1 > \alpha. \tag{2.2}$$

Inequality (2.2) shows that the function ψ belongs to $\mathcal{ST}(\alpha)$. A computation shows that

$$-\frac{z(\phi*f)'(z)}{p(\phi*f)_n(z)} = \frac{(\phi*(-p^{-1}zf'))(z)}{(\phi*f_n)(z)} = \frac{(\phi*(Hf_n))(z)}{(\phi*f_n)(z)} = \frac{(z^{p+1}\phi(z))*(H(z)\psi(z))}{(z^{p+1}\phi(z))*(\psi(z))}.$$

Since $z^{p+1}\phi \in \mathcal{R}_\alpha$ and $\psi \in \mathcal{ST}(\alpha)$, Theorem 2.1 yields

$$\frac{(z^{p+1}\phi(z))*(H(z)\psi(z))}{(z^{p+1}\phi(z))*(\psi(z))} \in \overline{co}(H(\mathbb{D})).$$

The subordination $H \prec h$ implies

$$-\frac{z(\phi*f)'(z)}{p(\phi*f)_n(z)} \prec h(z).$$

Thus $\phi*f \in \mathcal{MST}_p^n(h)$. The general case follows from the fact that

$$f \in \mathcal{MST}_p^n(g,h) \iff f*g \in \mathcal{MST}_p^n(h).$$

If $f \in \mathcal{MST}_p^n(g,h)$, then $f*g \in \mathcal{MST}_p^n(h)$, and therefore $\phi*f*g \in \mathcal{MST}_p^n(h)$, or equivalently $\phi*f \in \mathcal{MST}_p^n(g,h)$.

(2) The identity

$$-\frac{(z(g*f)'(z))'}{p(g*f)'_n(z)} = -\frac{z(g*-p^{-1}zf')'(z)}{p(g*-p^{-1}zf')_n(z)}$$

shows that $f \in \mathcal{MCV}_p^n(g,h)$ if and only if $-\frac{zf'}{p} \in \mathcal{MST}_p^n(g,h)$, and by the result of part (1), it is clear that $\phi*\left(-\frac{zf'}{p}\right) = -\frac{z}{p}(\phi*f)'(z) \in \mathcal{MST}_p^n(g,h)$. Hence $\phi*f \in \mathcal{MCV}_p^n(g,h)$.

The proofs of the remaining parts run along similar lines, and are therefore omitted. □

Remark 2.1.

1. The conclusion of Theorem 2.2 can be written in the following equivalent forms:

$$\mathcal{MST}_p^n(g,h) \subset \mathcal{MST}_p^n(\phi*g,h), \quad \mathcal{MCV}_p^n(g,h) \subset \mathcal{MCV}_p^n(\phi*g,h),$$
$$\mathcal{MCC}_p^n(g,h) \subset \mathcal{MCC}_p^n(\phi*g,h), \quad \mathcal{MQC}_p^n(g,h) \subset \mathcal{MQC}_p^n(\phi*g,h).$$

2. When $n = 1$ and $p = 1$, various known results are easily obtained as special cases of Theorem 2.2. For instance, the result [6, Theorem 3.3] is easily deduced from Theorem 2.2 (1), while [6, Theorem 3.6] follows from Theorem 2.2 (2). If $g(z) = 1/[z(1-z)]$, then the result [6, Corollary 3.5] follows from Theorem 2.2 (1). Similarly, the result [6, Theorem 3.7] follows from Theorem 2.2 (3), while [6, Corollary 3.12] is a special case of Theorem 2.2 (4).

3. Meromorphic multivalent functions with respect to *n*-ply symmetric, conjugate and symmetric conjugate points

In this section, it is assumed that p is an odd number. As before, it is assumed that the function $g \in \mathcal{M}_p$ is a fixed function and the function h is convex univalent with positive real part satisfying $h(0) = 1$. The classes $\mathcal{MSTS}_p^n(h)$, $\mathcal{MSTC}_p^n(h)$, $\mathcal{MSTSC}_p^n(h)$ of meromorphic p-valent starlike functions with respect to n-ply symmetric, n-ply conjugate and n-ply symmetric conjugate points are defined by the subordination

$$-\frac{1}{p}\frac{zf'(z)}{F_n(z)} \prec h(z),$$

where F is given, respectively by

$$F(z) = \frac{f(z) - f(-z)}{2}, \quad F(z) = \frac{f(z) + \overline{f(\bar{z})}}{2}, \quad \text{or} \quad F(z) = \frac{f(z) - \overline{f(-\bar{z})}}{2}.$$

The corresponding convex classes $\mathcal{MCVS}_p^n(h)$, $\mathcal{MCVC}_p^n(h)$, and $\mathcal{MCVSC}_p^n(h)$ are defined by

$$-\frac{1}{p}\frac{(zf'(z))'}{F_n'(z)} \prec h(z),$$

with the corresponding F given above. For a given g, a function f belongs to the classes $\mathcal{MSTS}_p^n(g,h)$, $\mathcal{MSTC}_p^n(g,h)$, or $\mathcal{MSTSC}_p^n(g,h)$ if and only if $f * g$ belongs to the corresponding class $\mathcal{MSTS}_p^n(h)$, $\mathcal{MSTC}_p^n(h)$, or $\mathcal{MSTSC}_p^n(h)$. The classes $\mathcal{MCVS}_p^n(g,h)$, $\mathcal{MCVC}_p^n(g,h)$, and $\mathcal{MCVSC}_p^n(g,h)$ are defined similarly.

Theorem 3.1. *Let h and ϕ satisfy the conditions of Theorem 2.2.*

1. *If f is in $\mathcal{MSTS}_p^n(g,h)$ (or in $\mathcal{MCVS}_p^n(g,h)$), then $\phi * f$ is, respectively in $\mathcal{MSTS}_p^n(g,h)$ (or in $\mathcal{MCVS}_p^n(g,h)$).*
2. *Let ϕ has real coefficients. If f belongs to any one of the classes $\mathcal{MSTC}_p^n(g,h)$, $\mathcal{MCVC}_p^n(g,h)$, $\mathcal{MSTSC}_p^n(g,h)$, or $\mathcal{MCVSC}_p^n(g,h)$, then $\phi * f$ belongs to the same class.*

Proof. We only show that if f is in $\mathcal{MSTS}_p^n(h)$, then so is $\phi * f$. The proof of the other claims is similar, and therefore omitted. Define the functions H and Ψ by

$$H(z) := -\frac{zf'(z)}{pF_n(z)} \quad \text{and} \quad \Psi(z) := z^{p+1}F_n(z).$$

Thus for any fixed $z \in \mathbb{D}$,

$$-\frac{zf'(z)}{pF_n(z)} \in h(\mathbb{D}), \tag{3.1}$$

where $F(z) = \frac{f(z)-f(-z)}{2}$. Replacing z by $-z$ in 3.1 and taking the convex combinations, it follows that

$$-\frac{zF'(z)}{pF_n(z)} \prec h(\mathbb{D}).$$

This shows that the function $F \in \mathcal{MST}_p^n(h)$, and the proof of Theorem 2.2 now shows that $F_n \in \mathcal{MST}_p(h)$. Since h is a convex function with $\text{Re}\,h(z) < 1 + \frac{1-\alpha}{p}$, it follows that

$$\text{Re}\frac{z\Psi'(z)}{\Psi(z)} = \text{Re}\frac{zF_n'(z)}{F_n(z)} + p + 1 > \alpha,$$

and hence $z^{p+1}F_n \in \mathcal{ST}(\alpha)$. Since $z^{p+1}\phi \in \mathcal{R}_\alpha$ and $\psi \in \mathcal{ST}(\alpha)$, Theorem 2.1 yields

$$\frac{(\phi * HF_n)(z)}{(\phi * F_n)(z)} = \frac{(z^{p+1}\phi(z)) * (H(z)\psi(z))}{(z^{p+1}\phi(z)) * (\psi(z))} \in \overline{co}(H(\mathbb{D})),$$

and because $H(z) \prec h(z)$, it follows that

$$-\frac{2}{p}\frac{z(\phi * f)'(z)}{(\phi * f)_n(z) - (\phi * f)_n(-z)} = \frac{(\phi * HF_n)(z)}{(\phi * F_n)(z)} \prec h(z).$$

Hence $\phi * f \in \mathcal{MSTS}_p^n(h)$. □

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